Static Behaviour of an Advanced Ultra-light Sandwich Composite Structure for a Wheel Chair

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Abstract: A theoretical approach of an ultra lightweight sandwich composite structure with extreme rigidity is presented. The structure features two carbon/epoxy skins reinforced with twill weave fabric, and an expanded polystyrene (EPS) core. The structure is subjected to a biaxial field of normal loads combined with a shear load. An equivalent model of this structure is presented. It has been accomplished a comparison between this structure and a similar one with glass/epoxy skins reinforced with EWR-300 fabric. Sandwich structure's strains, stresses, skins plies' strains and a comparison between the rigidities of the structure's components are presented. A theoretical approach regarding the bending of the structure is also shown.

Key-Words: sandwich structure, expanded polystyrene core, carbon/epoxy skins, twill weave fabric

1. Introduction

The structure is a sandwich with two carbon/epoxy skins reinforced with a 300 g/m^2 twill weave fabric, and an expanded polystyrene (EPS) 9 mm thick core with a density of 30 kg/m^3. The final thickness of the structure is 10.4 mm (fig. 1).

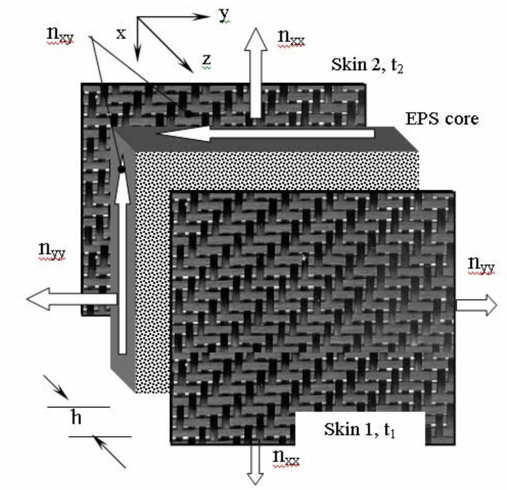

Fig. 1. The sandwich structure subjected to a biaxial field of normal loads combined with shear load

The carbon-fiber fabric used in this structure is a very high rigidity one which presents a so-called twill weave. The main feature of this weave is that the warp and the weft threads are crossed in a programmed order and frequency, to obtain a flat appearance (see fig. 2). The skins were impregnated under vacuum with epoxy resin and stacked to the core with polyurethane adhesive.

The equivalence model of the twill weave fabric skins is presented in fig. 3. According to this model, the skin with *t* thickness, reinforced with this weave can be equivalent to two *t/2* unidirectional laminas.

Data regarding the architecture of the sandwich structure:

- Structure thickness: t_s = 10,4 mm
- Skins plies number: N = 4
- Thickness of each ply: $t'_{1...4}$ = 0,35 mm
- Skins thickness: t_{skin} = 1,4 mm
- Core thickness: h = 9 mm
- Fibers disp ang of each ply: $\alpha_{1,3} = 90°$; $\alpha_{2,4} = 0°$
- Plies fibers volume fraction: $\varphi_{1...4} = 60\%$

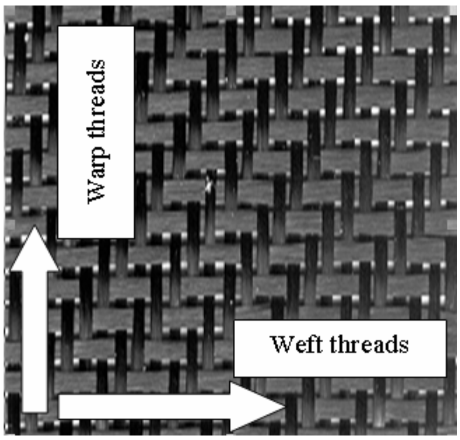

Fig. 2. The architecture of carbon/epoxy twill weave fabric skins

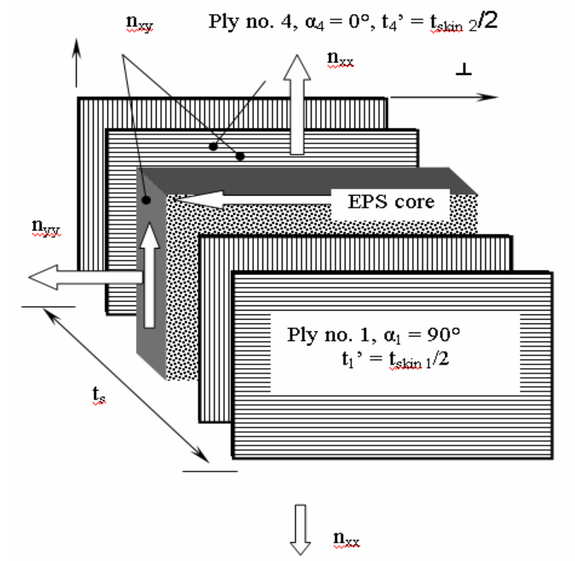

Fig. 3. The structure with an equivalence model of the twill weave fabric skins

Data regarding the features of the sandwich structure:

- Skins reinforcement: carbon fibers/glass fibers
- Fabric type: twill weave/EWR-300 glass fabric
- Specific weight of the fibers: 300 g/m^2
- Matrix type: epoxy resin
- Core type: expanded polystyrene (EPS)
- Core density $\rho_{core} = 30$ kg/m^3
- Core Young's modulus: $E_{core} = 30$ MPa
- Core Poisson's ratio: $\upsilon_{core} = 0{,}35$
- Core shear modulus: $G_{core} = 11$ MPa
- Fiber Young's modulus in longitudinal direction: $E_{F\parallel} = 540$ GPa/73 GPa for glass fibers
- Fiber Young's modulus in transverse direction: $E_{F\perp} = 27$ GPa / 73 GPa for glass fibers
- Fiber Poisson's ratio: $\upsilon_F = 0{,}3$ / 0,35 for glass fibers
- Fiber shear modulus: $G_F = 10{,}38$ GPa / 29,2 GPa for glass fibers
- Matrix Young's modulus: $E_M = 3{,}9$ GPa/3,75 GPa for glass fibers
- Matrix Poisson's ratio: $\upsilon_M = 0{,}37$ / 0,35 for glass fibers
- Matrix shear modulus: $G_M = 1{,}425$ GPa/1,39 GPa for glass fibers

Data regarding the loading of the sandwich structure:

- Normal force in x-axis direction: $n_{xx} = 100$ N/mm
- Normal force in y-axis direction: $n_{yy} = 50$ N/mm
- Shear force: $n_{xy} = 25$ N/mm

In case of a sandwich plate with dimensions: 10.4 x 2350 x 4070 mm subjected to bending with a bending moment of $m_x = 5660$ Nmm, the sandwich structure bending rigidity is computed at the theoretical approach. The structure subjected to bending is presented in fig. 4.

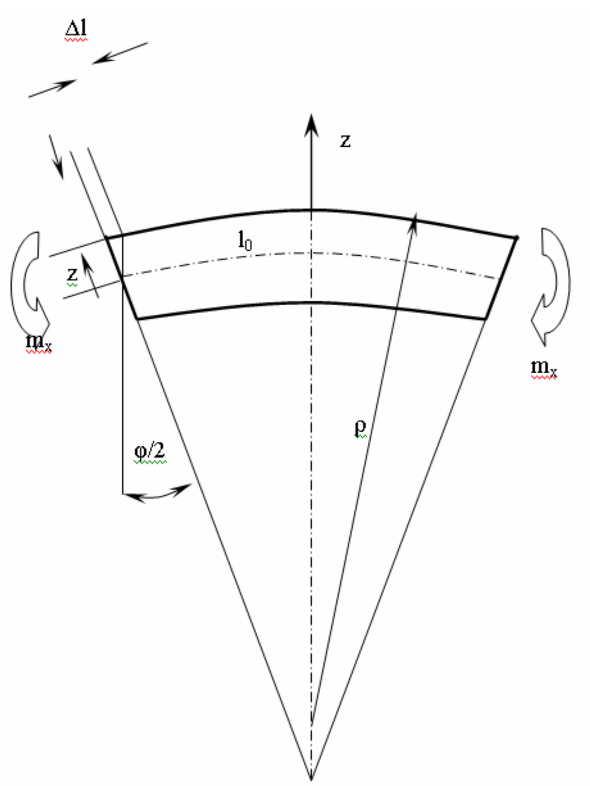

Fig. 4. The sandwich structure subjected to bending

where:
ρ – represents the curvature radius;
m_x – is the bending moment applied at the x-axis of the structure;
l_0 – the length of the neutral axis;
Δl – represents the lengthen of the structure due to the bending.

2. Theoretical approach

The core rigidities can be computed as follows [1-8, 14-17]:

$$r_{core\,11} = r_{core\,22} = \frac{E_{core}}{1 - \upsilon_{core}^2}, \quad (1)$$

$$r_{core\,12} = \frac{E_{core} \cdot \upsilon_{core}}{1 - \upsilon_{core}^2};\; r_{core\,33} = G_{core}. \quad (2)$$

The rigidities of the sandwich structure are [8]:

$$\underline{r}_{ij} = \sum_{K=1}^{N} \left(r_{ijK} \cdot \frac{t'_K}{t_{skin}} \right) + r_{core\,ij} \cdot \frac{h}{t_s}. \quad (3)$$

The sandwich structure compliances are obtained by reversing the rigidities:

$$\underline{c}_{ij} = \frac{1}{\underline{r}_{ij}}. \quad (4)$$

The longitudinal stresses in *x* respective *y* directions and the skins shear stress according to *x-y* axis are [9-13]:

$$\underline{\sigma}_{xx}=\frac{n_{xx}}{t_{skin}};\quad \underline{\sigma}_{yy}=\frac{n_{yy}}{t_{skin}};\quad \underline{\tau}_{xy}=\frac{n_{xy}}{t_{skin}}. \quad (5)$$

The stresses in skins plies are [8]:

$$\sigma_{\parallel K}=\frac{E_{\parallel K}}{1-\upsilon_{\perp\parallel K}\cdot\upsilon_{\parallel\perp K}}\cdot\varepsilon_{\parallel K}+\frac{\upsilon_{\perp\parallel K}\cdot E_{\perp K}}{1-\upsilon_{\perp\parallel K}\cdot\upsilon_{\parallel\perp K}}\cdot\varepsilon_{\perp K},$$
$$\sigma_{\perp K}=\frac{\upsilon_{\perp\parallel K}\cdot E_{\perp K}}{1-\upsilon_{\perp\parallel K}\cdot\upsilon_{\parallel\perp K}}\cdot\varepsilon_{\parallel K}+\frac{E_{\perp K}}{1-\upsilon_{\perp\parallel K}\cdot\upsilon_{\parallel\perp K}}\cdot\varepsilon_{\perp K}, \quad (6)$$
$$\tau_{\parallel\perp K}=G_{\parallel\perp}\cdot\gamma_{\parallel\perp},$$

and the core stress is:

$$\sigma_{core}=E_{core}\cdot\varepsilon_{xx}. \quad (7)$$

The relations regarding the structure's bending are:

$$\Delta l=z\cdot\frac{\varphi}{2};\quad l_0=\rho\cdot\frac{\varphi}{2};\quad \varepsilon=\frac{\Delta l}{l_0}. \quad (8)$$

The curvature of the sandwich structure can be computed as following: $\kappa=\frac{1}{\rho}$, (9)

and the structure's strain is:

$$\varepsilon=z\cdot\kappa. \quad (10)$$

The bending rigidity of the structure is [14-17]:

$$R_{bending}=\frac{m_x}{\kappa}=E_{skin}\cdot\left(\frac{t_s\cdot a^2}{2}+\frac{t_s^3}{6}\right)+E_{core}\cdot\frac{h^3}{12} \quad (11)$$

where:

E_{skin} – represents the skins Young modulus;
a – is the distance between the skins neutral axis;
h – is the core thickness;
κ – the curvature;
t_s – represents the sandwich thickness.
The results of the theoretical approach are presented in tables 1 – 10 and in fig. 5 – 8.

3. Results

Table 1. The basic elastic characteristics of the skins plies

	Carbon-fibers	Glass-fibers
Young's modulus $E_{\parallel}$ [MPa]	325560	45300
Young's modulus $E_{\perp}$ [MPa]	14100.3	15800
Poisson's ratio $\upsilon_{\perp\parallel}$[-]	0.328	0.29
Poisson's ratio $\upsilon_{\parallel\perp}$[-]	0.014	0.101
Shear modulus $G_{\parallel\perp}$ [-]	5212.4	5873

Table 2. The skins' plies transformed rigidities

	Plies 1&3 (Carbon)	Plies 2&4 (Carbon)	Plies 1&3 (Glass)	Plies 2&4 (Glass)
r_{11}[MPa]	14165.3	327061.8	16276.7	46666.8
r_{22}[MPa]	327061.8	14165.3	46666.8	16276.7
r_{33}[MPa]	5212.4	5212.4	5873	5873
r_{12}[MPa]	4646.2	4646.2	4720.2	4720.2
r_{13}[MPa]	0	0	0	0
r_{23}[MPa]	0	0	0	0

Table 3. Core rigidities

	Value
$r_{core\ 11}$ [MPa]	34.18
$r_{core\ 22}$ [MPa]	34.18
$r_{core\ 33}$ [MPa]	11
$r_{core\ 12}$ [MPa]	11.96
$r_{core\ 13}$ [MPa]	0
$r_{core\ 23}$ [MPa]	0

Table 4. Sandwich structure's rigidities

	Carbon fibers	Glass fibers
$\underline{r}_{11}$ [MPa]	170643.12	31501.3
$\underline{r}_{22}$ [MPa]	170643.12	31501.3
$\underline{r}_{33}$ [MPa]	5221.9	5882.5
$\underline{r}_{12}$ [MPa]	4656.5	4730.7
$\underline{r}_{13}$ [MPa]	0	0
$\underline{r}_{23}$ [MPa]	0	0

Table 5. Sandwich structure's compliances

	Carbon fibers	Glass fibers
$\underline{c}_{11}$ [$\cdot 10^{-6}$ MPa^{-1}]	5.86	31.7
$\underline{c}_{22}$ [$\cdot 10^{-6}$ MPa^{-1}]	5.86	31.7
$\underline{c}_{33}$ [$\cdot 10^{-6}$ MPa^{-1}]	191.5	169.9
$\underline{c}_{12}$ [$\cdot 10^{-6}$ MPa^{-1}]	214.7	211.3
$\underline{c}_{13}$ [$\cdot 10^{-6}$ MPa^{-1}]	0	0
$\underline{c}_{23}$ [$\cdot 10^{-6}$ MPa^{-1}]	0	0

Table 6. Sandwich structure's strains

	Carbon fibers	Glass fibers
Strain ε_{xx} [-]	0.00808	0.0098
Strain ε_{yy} [-]	0.01554	0.01622
Strain γ_{xy} [-]	0.00341	0.00303

Table 7. Skins plies' strains

	Plies 1&3 (Carbon)	Plies 2&4 (Carbon)	Plies 1&3 (Glass)	Plies 2&4 (Glass)
Strain $\varepsilon_{\parallel}$ [-]	0.01554	0.00808	0.01622	0.0098
Strain $\varepsilon_{\perp}$ [-]	0.00808	0.01554	0.0098	0.01622
$\gamma_{\parallel\perp}$ [-]	-0.00341	0.00341	-0.003	0.003

Table 8. Sandwich structure's stresses

[MPa]	Plies 1&3 (Carbon)	Plies 2&4 (Carbon)	Plies 1&3 (Glass)	Plies 2&4 (Glass)
$\sigma_{\parallel}$	5120	2714.8	803.2	533.8
$\sigma_{\perp}$	186.6	257.6	236	310.2
$\tau_{\parallel\perp}$	- 17.7	17.7	- 17.8	17.8
Core stress	0.24		0.29	

Table 9.Constructive features of the structure subjected to bending

	Value
ρ [mm]	11443
z [mm]	5.2
κ [10^{-6} mm^{-1}]	87.38
l_0 [mm]	4070
Δl [mm]	1.85
a [mm]	9.7
h [mm]	9

Table 10. Structure's bending properties and loading

	Value
m_x [Nmm]	5660
$R_{bending}$ [Nmm]	10741612.8

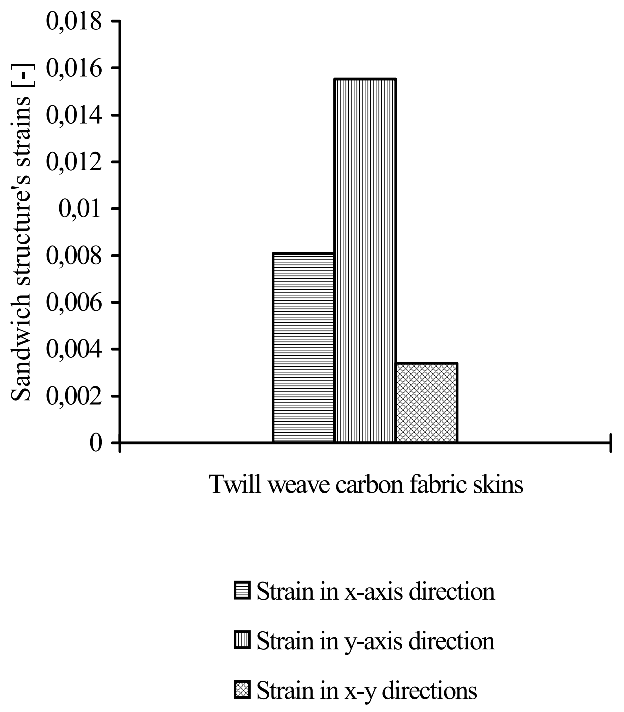

Fig. 5. Sandwich structure's strains, structure subjected to a biaxial field of normal loads combined with a shear load

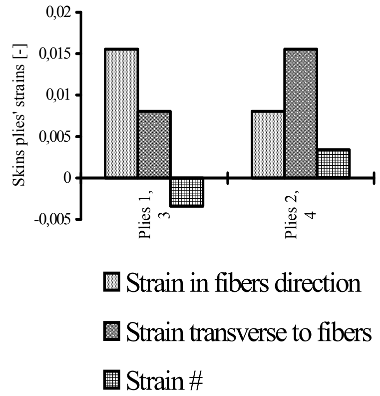

Fig. 6 Skins plies' strains, sandwich structure subjected to a biaxial field of normal loads combined with a shear load

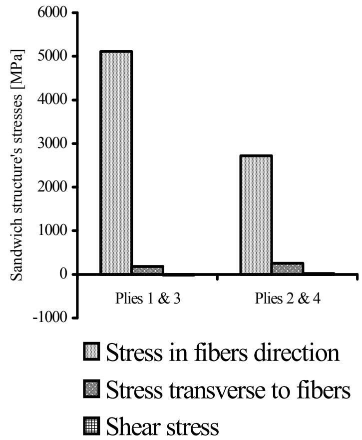

Fig. 7 Sandwich structure's stresses, structure subjected to a biaxial field of normal loads combined with a shear load

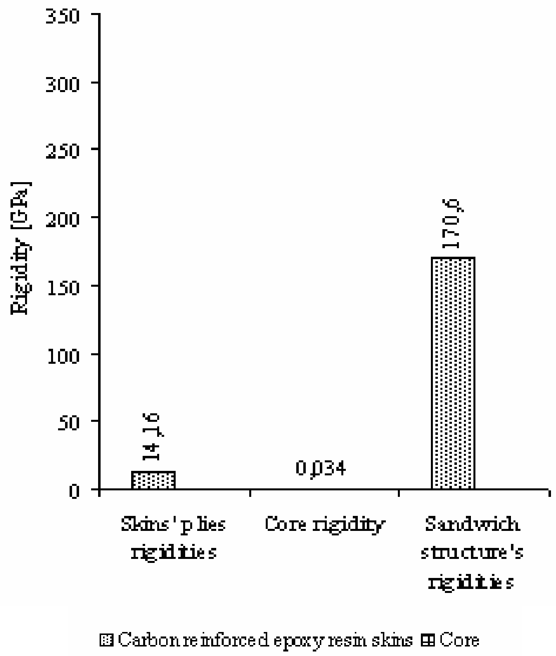

Fig. 8 A comparison between the rigidities of the structure's components

Determinations were applied during a project for the ultra-light wheel chair structure for persons with locomotors disabilities. The main part to sustain the chair was executed from sandwich composites materials. For these parts was applied the finite element analysis method.

Fig. 9 Structure for the wheel chair from polymeric composites materials

Fig. 10 Main part to sustain the chair executed from sandwich composites materials.

4. Conclusions

- The sandwich structure with two carbon/epoxy skins reinforced with twill weave fabric, and an expanded polystyrene (EPS) 9 mm thick core, fulfils following special requirements: plate dimensions: 10,4 x 2350 x 4070 mm; overall weight: maximum 10 kg.
- The sandwich structure's strains with skins based on twill weave carbon fabric reinforced epoxy resin are comparable with those of the structure with skins based on EWR-300 glass fabric/epoxy resin.
- Stresses in fibers direction in case of the sandwich structure with carbon fabric/epoxy resin reinforced skins, are up to six times higher than those existent in EWR-300 glass fabric/epoxy resin skins.
- Stresses transverse to the fibers direction in case of the sandwich structure with carbon fabric/epoxy resin reinforced skins, are 20% lower than those existent in EWR-300 glass fabric/epoxy resin skins.
- The shear stresses in carbon fabric/epoxy resin reinforced skins' plies are almost identical with those existent in EWR-300 glass fabric/epoxy resin skins' plies.
- Both sandwich structures' core stresses are almost negligible, the loading is taken-over exclusively by the two structures' skins.
- Using a 9 mm thick expanded polystyrene core (EPS), according to fig. 8, the rigidity of the sandwich structure with carbon fiber reinforced epoxy resin skins is more than ten times higher than the skins' plies rigidity.

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