On the pre-tensioning technique of PMC-tubes for a ultralight wheel chair with applications in the medical technique

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Abstract: - The paper presents an original method to increase the loading capability of PMC (polymer matrix composite) tubes used for ultralight wheel chair for persons with locomotors disabilities. The method involves the introduction of supplementary internal stresses in thin-walls cylinders with only a few wound layers. An original device has been developed to attain this end. Various tube specimens with different disposal of reinforced material were carried out. The specimens have been heated at a proper temperature and then an elastic material was pressed at the inner of the tubes. While keeping the internal pressure, the specimens were cooled and then discharged. Then, the pre-tensioned specimens were subjected to internal pressure until weeping occurs. Using this method of pre-tension, the loading capability of PMC-tubes is increased up to 43%. A theoretical approach regarding the cross-ply and balanced angle-ply composite tubes is presented.

Key-Words: - Pre-tensioning, Internal stresses, Loading capability, PMC-tubes, Weeping pressure, Cross-ply composite, Balanced angle-ply composite.

1 Introduction

The purpose of pre-tensioning PMC-tubes for ultralight wheel chair for persons with locomotors disabilities is to introduce internal stresses in wall structure that can work against the operational stresses. These internal stresses increase the loading capability of PMC tubes and their cracking limits. To attain this aim, an original device has been designed and developed. In practice we can encounter two special cases of tubes: the cross-ply composite tube (denoted AC-tube) and the balanced angle-ply composite one (denoted \pm E-tube). The AC-tube consists from unidirectional reinforced plies with the same basic elasticity constants. The entire thicknesses t_1 (fibers on axial direction) and t_2 (fibers on circumferential direction) can be different (fig. 1). We suppose that the individual plies of cylindrical tubes are orthotropic and the wall thickness is thin. Therefore, the loadings of the tubes wall are:

$$
\sigma_C = p \cdot \frac{r}{t}, (1)
$$

\n
$$
\sigma_A = p \cdot \frac{r}{2t},
$$
\n(2)

$$
\tau_{AC} = 0,\tag{3}
$$

where A and C represent the axial respective the circumferential direction of the tube.

Fig. 1. Cross-ply composite tube (AC-tube)

For the AC-tube subjected to internal pressure, the elasticity laws for the entire wall thickness are [1, 2, 3, 4]:

$$
\begin{bmatrix} \sigma_A \\ \sigma_C \end{bmatrix} = \begin{bmatrix} t'_1 \cdot c_H + t'_2 \cdot c_\perp & c_{\perp H} \\ c_{\perp H} & t'_1 \cdot c_\perp + t'_2 \cdot c_H \end{bmatrix} \begin{bmatrix} \varepsilon_A \\ \varepsilon_C \end{bmatrix}, \quad (4)
$$

where c_{II} , c_{\perp} and $c_{\perp II}$ are the elastic constants and the relative thicknesses t'_1 and t'_2 can be expressed as following:

$$
t_1' = \frac{t_1}{t}; \quad t_2' = \frac{t_2}{t} \ . \tag{5}
$$

For the individual plies:

$$
\begin{bmatrix} \sigma_{AI} \\ \sigma_{CI} \end{bmatrix} = \begin{bmatrix} c_{II} & c_{\perp II} \\ c_{\perp II} & c_{\perp} \end{bmatrix} \begin{bmatrix} \varepsilon_{A} \\ \varepsilon_{C} \end{bmatrix},
$$
\n(6)\n
$$
\begin{bmatrix} \sigma_{A2} \\ \sigma_{C2} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ \sigma_{C1} & c_{13} \end{bmatrix} \begin{bmatrix} \varepsilon_{A} \\ \varepsilon_{C} \end{bmatrix}
$$

$$
\begin{bmatrix} \sigma_{A2} \\ \sigma_{C2} \end{bmatrix} = \begin{bmatrix} c_{\perp} & c_{\perp H} \\ c_{\perp H} & c_{H} \end{bmatrix} \begin{bmatrix} \varepsilon_{A} \\ \varepsilon_{C} \end{bmatrix}.
$$
 (7)

The tube strains are:

$$
\varepsilon_A = \frac{I - \nu_{\perp II} \cdot \nu_{\Pi \perp}}{E_{II}} \cdot \frac{\hat{\sigma}_C}{2K} \cdot \left[\left(t'_I + t'_2 \cdot \frac{E_{II}}{E_{\perp}} \right) - 2\nu_{\perp II} \right], \quad (8)
$$

$$
\varepsilon_C = \frac{I - \nu_{\perp H} \cdot \nu_{H\perp}}{E_H} \cdot \frac{\hat{\sigma}_C}{2K} \cdot \left[2 \left(t_2' + t_1' \cdot \frac{E_H}{E_\perp} \right) - \nu_{\perp H} \right], \quad (9)
$$

 $\gamma_{AC} = 0$, (10) where:

$$
K = t'_I \cdot t'_2 \left(\frac{E_{II}}{E_{\perp}} + \frac{E_{\perp}}{E_{II}} - 2 \right) + I - \nu_{\perp II} \cdot \nu_{II \perp} , \qquad (11)
$$

 E_{II} , E_{\perp} , $v_{\perp II}$ represent the basic elasticity constants and $\hat{\sigma}_C$ is the stress that acts in the circumferential direction of the composite tube. At the Poisson ratio, the first index represents the shrinkage direction and the second one is the loading direction that produces this shrinkage.

 The stresses in each ply of the composite tube are expressed as following:

$$
\sigma_{II1} = \frac{\hat{\sigma}_C}{2K} \left[t'_1 + t'_2 \frac{E_{II}}{E_{\perp}} - v_{\perp II} v_{II\perp} - 2t'_2 (v_{\perp II} - v_{II\perp}) \right], (12)
$$
\n
$$
\sigma_{II2} = \frac{\hat{\sigma}_C}{2K} \left[2(t'_2 + t'_1 \frac{E_{II}}{E_{\perp}} - v_{\perp II} v_{II\perp}) - t'_1 (v_{\perp II} - v_{II\perp}) \right], (13)
$$
\n
$$
\sigma_{\perp I} = \frac{\hat{\sigma}_C}{2K} \left[2(t'_1 + t'_2 \frac{E_{\perp}}{E_{II}} - v_{\perp II} v_{II\perp}) + t'_2 (v_{\perp II} - v_{II\perp}) \right], (14)
$$
\n
$$
\sigma_{\perp 2} = \frac{\hat{\sigma}_C}{2K} \left[t'_2 + t'_1 \frac{E_{\perp}}{E_{II}} - v_{\perp II} v_{II\perp} + 2t'_1 (v_{\perp II} - v_{II\perp}) \right]. (15)
$$

In the case of the balanced angle-ply composite tube (±E-tube), the unidirectional reinforced plies present the same mechanical properties (fig. 2). The entire fibers quantity, fibers that are disposed under the angles $\alpha = +\omega$ and $-\omega$, is half the fibers quantity disposed on axial direction. In the case of ±E-tube subjected to internal pressure, the elasticity laws for the entire wall thickness are:

$$
\begin{bmatrix} \hat{\sigma}_A \\ \hat{\sigma}_C \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_A \\ \varepsilon_C \end{bmatrix},
$$
 (16)

For the individual plies: \Box

$$
\begin{bmatrix} \sigma_{A1,2} \\ \sigma_{C1,2} \\ \tau_{ACL,2} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{22} & 0 \\ c_{131,2} & c_{231,2} & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_A \\ \varepsilon_C \\ 0 \end{bmatrix}.
$$
 (17)

Fig. 2. Balanced angle-ply composite tube (±E-tube)

From the concordance of the first two relations of the elasticity laws (16) and (17), it results:

$$
\sigma_{A1} = \sigma_{A2} = \hat{\sigma}_A; \quad \sigma_{C1} = \sigma_{C2} = \hat{\sigma}_C.
$$
 (18)
The tube strains are:

$$
\varepsilon_A = \frac{\hat{\sigma}_C}{E_H} \left(\frac{I}{2N}\right) \cdot \left[3(AF - D)\sin^2 2\omega + H - J \cdot \cos 2\omega - 2L\right], (19)
$$
\n
$$
\varepsilon_C = \frac{\hat{\sigma}_C}{E_H} \left(\frac{I}{2N}\right) \cdot \left[3(AF - D)\sin^2 2\omega + 2(H + J\cos 2\omega) - L\right], (20)
$$

$$
\gamma_{AC} = 0 \,, \tag{21}
$$

The stresses in each ply of the \pm E-tube are:

$$
\sigma_{A1,2} = \hat{\sigma}_A, \tag{22}
$$

$$
\sigma_{C1,2} = \hat{\sigma}_C, \qquad (23)
$$

$$
\tau_{ACI,2} = -\frac{1}{2N} \cdot [3AJ - (I - AB)\cos 2\omega] \sin 2\omega \cdot \hat{\sigma}_C (24)
$$

$$
\sigma_{II,2} = \frac{\hat{\sigma}_C}{2N} \{ 3[1 - (1 - 2AR)\sin^2 2\omega] - \cos 2\omega \}, (25)
$$

$$
\sigma_{II,2} = \frac{\hat{\sigma}_C}{2} \{ 3[1 - (1 - 2AP)\sin^2 2\omega] + \cos 2\omega \}.
$$
 (26)

$$
\sigma_{\perp l,2} = \frac{\sigma_C}{2N} \left\{ 3\left[I - \left(I - 2AP \right) \sin^2 2\omega \right] + \cos 2\omega \right\}, (26)
$$

$$
\tau_{\#I,2} = -\lambda + \frac{\hat{\sigma}_C}{2N} \cdot A(B + 3J\cos 2\omega)\sin 2\omega, \tag{27}
$$
\n
$$
\text{where:}
$$

$$
A = \frac{G_{\#}}{E_{II}},\tag{28}
$$

$$
B = \frac{E_{II}}{E_{\perp}} + l + 2\upsilon_{\perp II} \,, \tag{29}
$$

$$
D = \frac{1}{2} \left(\frac{E_{\text{II}}}{E_{\text{I}}} + I - 2\upsilon_{\text{I}} \right),
$$
 (30)

$$
F = 2\left(\frac{E_{II}}{E_{\perp}} - \nu_{\perp II}^2\right),\tag{31}
$$

$$
H = \frac{E_{II}}{E_{\perp}} + l(32) J = \frac{E_{II}}{E_{\perp}} - l, (33) L = 2v_{\perp II}, (34)
$$

$$
N = 2[l - (I - AB)\sin^2 2\omega], (35) P = I + v_{\perp II}, (36)
$$

$$
R = \frac{E_{\text{II}}}{E_{\perp}} + \nu_{\perp \text{II}} \,. \tag{37}
$$

2 The pre-tension method

The pre-tension method consists in the accomplishment of following successive steps. First, the PMC-tube is manufactured in the fabric-winding process. After curing, the tube is pulled-out of the mandrel. Second, the pre-tension device is positioned and fixed vertically. Third, at this stage, the tube is heated up to 10° C above the glass transition temperature T_G . In this field of temperature, the resin elasticity modules decreased quickly and the resin matrix became highly elastic.

 Fourth, the heated tube is introduced into the pretension device and then the silicone rubber is pressed at the inner of the tube. Since during the heating the matrix elasticity moduli decrease, the inner pre-tension pressure will be taken over by the fiber network. Fifth, while keeping the inner pretension pressure, the tubes cooled at the environmental temperature. Sixth, after cooling, the tube is discharged from the inner pre-tension pressure. Now, the fiber network will relax and in wall structure will remain a status of internal stresses. After these six stages, the tube is removed from the device and it is stored 24 hours in a controlled atmosphere room ($T = 20^{\circ}$ C and 50%) relatively air humidity). This is necessary to reduce the internal stresses relaxation.

3 Material and method

Six kinds of specimens have been used for the experimental tests, some of them presented in tables 1 – 5. The tubes material used in tests is a thermosetting compound based on polyester resin reinforced with EWR-300 and EWR-500 glass fabric. The tube wall structure was made in the fabric winding process. The specimens were heated at an average temperature $T = 105^{\circ}C$ in a temperature controlled oven. From every specimen type, two pieces were accomplished, one of them were subjected to pre-tension. All types of specimens were subjected to the same pre-tension pressure of 1,37 MPa given by the pre-tension device. This pressure has been kept at the inner of specimens for 900 seconds in an environmental temperature of about 2°C. After 24 hours from this operation, both the pre-tensioned and non-pretensioned tube specimens were subjected to internal pressure until weeping occurs. This weeping pressure produces irreversible damages in the tube wall structure, such as micro cracks and delamination. Finally, the weeping pressure value of the pre-tensioned specimen is compared with the

weeping pressure value of the non-pre-tensioned specimen.

Table 1. Characteristics of specimen type 1 (UP resin/Mat reinforcement)

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Matrix	UP resin
Reinforcement	E-glass fiber
Tube diameter [mm]	$80^{+0,4}$
Tube length [mm]	100
Wall thickness [mm]	$3,5-\overline{5}$
Number of plies	8
Plies thickness [mm]	$0,43 - 0,62$
Fibers volume fraction [%]	15
Pre-tension pressure [MPa]	1,37
Heating temperature $[^{\circ}C]$	105
Cooling environment	Air
Pre-tension time [s]	900
Cooling environmental	$\overline{2}$
temperature $[^{\circ}C]$	
Table 2. Characteristics of specimen type 2 (UP	
resin/EWR-300 glass fabric reinforcement)	
Matrix	UP resin
Reinforcement	E-glass fiber
Type of fabric	EWR-300
Tube diameter [mm]	$80^{+0,4}$
Tube length [mm]	100
Wall thickness [mm]	$2 - 2,6$
Number of plies	8
Plies thickness [mm]	$0,25-0,32$
Strip cutting angle [°]	0 (in length)
Fibers volume fraction [%]	35
Pre-tension pressure [MPa]	1,37
Heating temperature $[°C]$	101
Cooling environment	Air
Pre-tension time [s]	900
Cooling environmental	2
temperature [°C]	
Table 3. Characteristics of specimen type 3	
Matrix	UP resin
Reinforcement	E-glass fiber
Type of fabric	EWR-300
Tube diameter [mm]	$80^{+0,4}$
Tube length [mm]	100
Wall thickness [mm]	$2,5 - 3,6$
Number of plies	8
Plies thickness [mm]	$0,31 - 0,45$
Strip cutting angle [°]	45
Fibers volume fraction [%]	35
Pre-tension pressure [MPa]	1,37
Heating temperature $[^{\circ}C]$	100
Cooling environment	Air
Pre-tension time [s]	900
Cooling temperature $[^{\circ}C]$	1

Matrix	UP resin
Reinforcement	E-glass fiber
Type of fabric	EWR-500
Tube diameter [mm]	$80^{+0,4}$
Tube length [mm]	100
Wall thickness [mm]	$3,4-4$
Number of plies	8
Plies thickness [mm]	$0,42-0,5$
Strip cutting angle $\lceil \circ \rceil$	90 (in width)
Fibers volume fraction [%]	35
Pre-tension pressure [MPa]	1,37
Heating temperature $[°C]$	104
Cooling environment	Air
Pre-tension time [s]	900
Cooling temperature [°C]	2

Table 4. Characteristics of specimen type 4

Table 5. Characteristics of specimen type 5

The high-pressure plant presents the possibility to adjust the pre-tension pressure up to 500 MPa.

4 Results

The increase of loading capability of PMC-tubes is presented in fig. 3.

Fig.3. Increase of loading capability of PMC-tubes

These experimental results and also a finite element modeling was realized for the sustain tubs for a ultra light structure from polymeric composites.

Fig. 4 Ultralight pretension tubes used for a wheel chair structure

5 Conclusions

Using this original method of pre-tension, the increase of loading capability of PMC-tubes is situated between 20% (specimen type 1) and 43% (specimen type 3). This method emphasized a low structure endowment with internal stresses, which suppose a reduced pre-tension process due to a prudent choice of the pre-tension pressure. This pressure can cause micro cracks and other damages in the tube wall structure.

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