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Modeling Volatility of the KLCI Daily Returns

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Abstract- Volatility is a central concept in financial engineering. It may be simply defined as the standard deviation of return values. A frequent modeling assumption is that volatility is constant. Unfortunately in many financial time series volatility appears to be anything but constant. This paper reports the results of an effort in modeling stock market volatility as a Generalized Autoregressive Conditional Heteroscedastic (GARCH) process.

Keywords- GARCH, Stationarity, Heteroscedasticity, Volatility Clustering, Simulation.

I. Introduction

VOLATILITY may be simply defined as the standard deviation of return values. A frequent modeling assumption is that volatility is constant. Unfortunately in many financial time series volatility appears to be anything but constant. Figure 2 is a plot of 3,916 daily returns of the Kuala Lumpur Composite Index (KLCI). Immediately evident are the different regions where the daily returns (and therefore local volatility) are more and less extreme. This existence of the so-called volatility clustering has suggested the need for alternative ways to define volatility or to make volatility assumptions within a model. Prominent among these alternatives is GARCH.

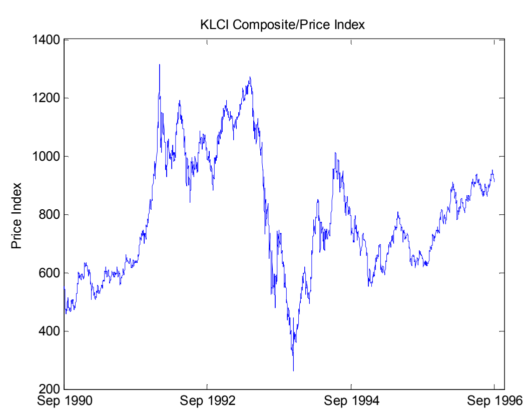

Fig. 1 Plot of KLCI Daily Index

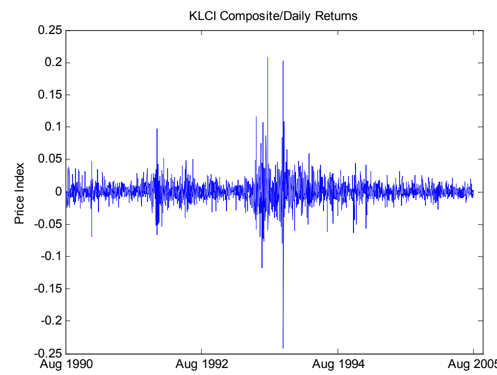

Fig. 2 Plot of KLCI Daily Returns

A GARCH(p, q) model begins with a stationary time series y_t with an assumed form $y_t = c + \varepsilon_t$. In GARCH, the random variables ε_t are allowed to have a dependency structure: the conditional distribution ε_t of given p previous values is Gaussian with mean zero and time indexed, and potentially non-constant variances σ_t^2, i.e., $\varepsilon_t | \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \approx N(0, \sigma_t^2)$. The GARCH(p, q) model for σ_t^2 then has the assumed form $\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2$. The essence of GARCH modeling is to estimate all the model parameters; the time series constant c, the volatility constant a_0, the a_i's, and the b_i's. Estimation of these $p + q + 2$ parameters is a nonlinear process and is followed by a statistical evaluation of their significance. For a typical financial time series, the choice of p and q is not intuitive. Experimentation may be made with different values of p and q. However, a starting point of p = q =1 is often itself an excellent model.

II. Modeling KLCI Daily Returns As A GARCH(1, 1) Process: Pre-Estimation Analyses

The sample consists of 3,916 daily observations of the KLCI for August 31, 1990 through August 31, 2005. Its plot in Figure 1 exhibits non-stationarity, thus giving the need to convert it to a daily returns series. The returns series is thus $y_t = \ln\left(\frac{p_t}{p_{t-1}}\right)$, where p_t is the KLCI index at time t. Its plot in Figure 2 exhibits stationarity. The ACF and PACF plots of are shown in Figure 3 and Figure 4, respectively. Figure 2 shows a pattern of volatility clustering in the return series with high amplitude oscillations at several points. The highest jump in amplitude is between August 1990 and August 1994. This clustering yields evidence of the feasibility of modeling the conditional variance as a GARCH process.. Hence, the Ljung-Box and Engle's tests were performed to see whether the volatility clustering is in fact due to the presence of heteroscedasticity in the variance of the series, i.e., to check for the presence of the GARCH effect.

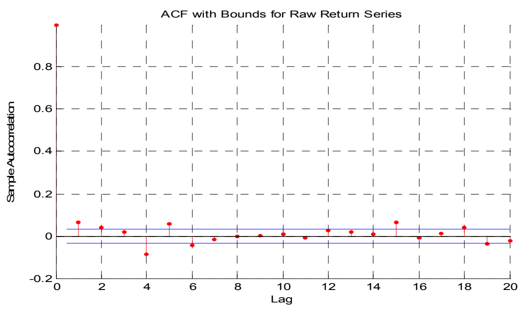

Fig. 3 ACF of Return Series

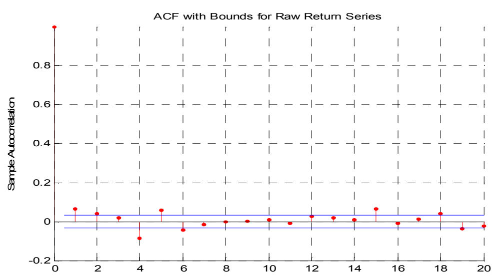

Fig. 4 PACF of Return Series

The correlation in the return series is checked by examining the sample autocorrelation function (ACF) and partial-autocorrelation (PACF) function under the assumption that no autocorrelation ahead of lag zero. Figures 3 and 4 present quite similar results. The autocorrelations are significantly different from zero at lag 1, 5 and 15, while the partial autocorrelations are significant at lags 4, 6 and 19. Thus the ACF and PACF indicate the presence of autocorrelation characteristics of the return series.

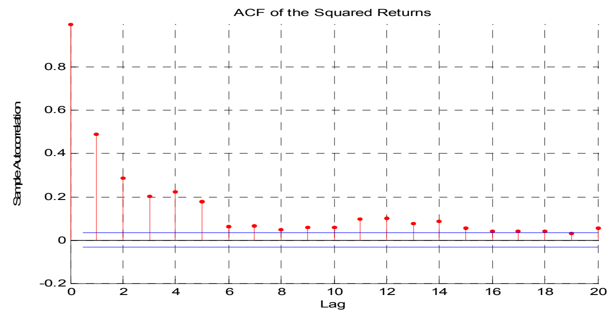

Fig. 5 ACF of Squared Return Series

Significant correlation in the return series implies the existence of correlation in the variance process, and hence a justification for a GARCH model. Figure 5 shows correlation in the variance process. The ACF seems to die out more sluggish starting from lag 2. This result reveals the presence of a non-stationary variance process in the return series.

A. Ljung-Box-Pierce Q-Test (LBQ)

Under the null hypothesis of no serial correlation, the Q-test statistic is asymptotically Chi-Squared (Box, Jenkins and Reinsel, 1994). The result of the LBQ test is summarized in Table 1. The test is applied for 10, 15, and 20 lags of the ACF at the 0.05 level of significance. The results indicate the presence of serial correlation in the series.

Table 1: Ljung-Box-Pierce Q-Test on Return Series

Lags	Test Statistic	p-value
10	76.0	0.0000
15	98.5	0.0000
20	113.5	0.0000

Results of the application of the LBQ test on the squared returns (Table 2) for 10, 15, and 20 lags of the ACF at the 0.05 level of significance indicates the presence of serial correlation.

Table 2: Ljung-Box-Pierce Q-Test on Squared Return Series

Lags	Test Statistic	p-value
10	1797.7	0.0000
15	1935.8	0.0000
20	1969.3	0.0000

B. Engle's ARCH Test

Under the null hypothesis that a series is a random sequence of Gaussian disturbances, Engle (1982) showed that the test statistic is asymptotically Chi-Squared. The test was applied for 10, 15, and 20 lags at the 0.05 level of significance. The result shows significant evidence in support of the presence of the GARCH effects.

Table 3 Engle's ARCH test

Lags	Test Statistic	p-value
10	1034.0	0.0000
15	1053.8	0.0000
20	1062.9	0.0000

III. Model Estimation

The results in Table 3 has indicated the presence of the GARCH effect in the returns series, hence the modeling of the series as a GARCH process is warranted. To estimate the model parameters the Maximum Likelihood Estimation (MLE) was used. . In this case, the baseline GARCH (p,q) regression model is defined as:

$$y_t = c + \varepsilon_t \quad t = 1,\ldots..T \quad \text{such that}$$

$$\varepsilon_t | \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \approx N\left(0, \sigma_t^2\right), \quad (1)$$

$$\text{and} \quad \sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2 \quad (2)$$

The equations in (1) and (2) is based on the following constraints:

$$\sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_i \sigma_{j-i}^2 < 1,\ a_0 \geq 0,\ a_i \geq 0,\ b_j \geq 0$$

Table 4 exhibits the results of fitting a GARCH(1,1) to the returns series.

Table 4 The Estimated GARCH(1,1) Model Parameters

Coefficient	Value	Standard Error	t-Statistic
c	0.00045	0.00015	2.9775
a_0	1.7943e-006	1.6653e-007	10.7742
a_1	0.89762	0.00382	234.9927
b_1	0.09567	0.00491	19.5039

Hence, a GARCH(1,1) process representing the returns KLCI series can simply be expressed as:

$$y_t = 0.00045 + \varepsilon_t;$$

$$\sigma_t^2 = 0.0000018 + 0.89762 \varepsilon_{t-1}^2 + 0.09567 \sigma_{t-1}^2 \quad (3)$$

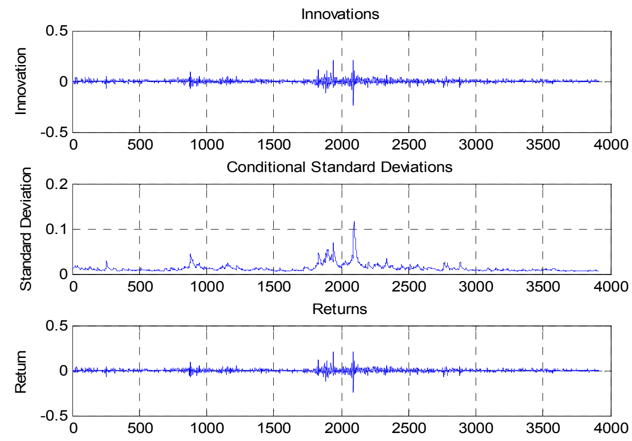

Fig. 6 Plot of Innovations, Conditional Standard Deviations and Returns of KLCI

IV. Comparing Residuals (Innovations), Conditional Standard Deviations And Returns

The residuals derived from the fitted model, the conditional standard deviations and the observed returns were plotted. In Figure 6 the innovations series (top plot) and the returns series (bottom plot) exhibit volatility clustering particularly between t = 1750 and t = 2250. Further, the sum for the estimated parameters $a_1 + b_1$ = 0.89762 + 0.0957 is 0.9933, which is quite close to the non-stationary boundary given by the constraints of equation 2. However, a plot of the standardized residuals (residuals divided by their conditional standard deviation) in Figure 7 indicates that standardized residuals is more stable and the series shows little clustering as compared to the plot of innovations in Figure 6. Further, the ACF plot of the squared standardized innovations shows that they are not significantly different from zero at all lags except for lag 1. This indicates no correlation in the squared standardized innovations as compared to the plot in Figure 3.

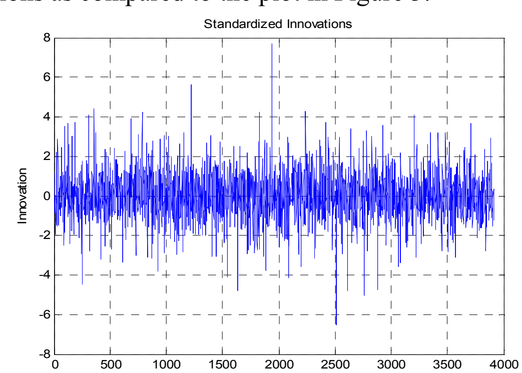

Fig. 7 Plot of Standardized Residuals

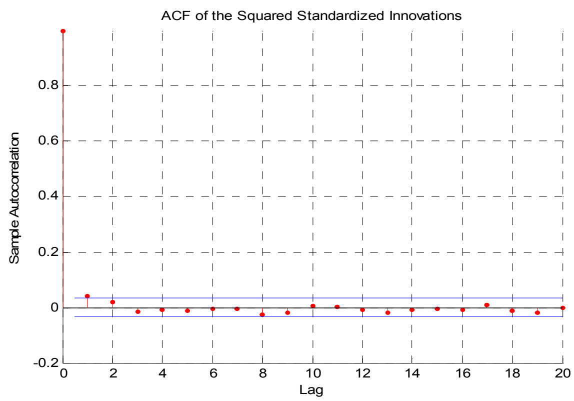

Fig. 8 Plot of the ACF of the Squared Standardized Residuals

In order to quantitatively check whether GARCH effects are present in the residuals, the LBQ and Engle's ARCH tests were carried out in the series of standardized innovations. The results of both tests are given in the Tables 4 and 5. Both tests were applied for 10, 15, and 20 lags at the 0.05 level of significance. Tables 4 and Table 5 gives evidence of no serial and no GARCH effects in the residuals, respectively.

Table 4: LBQ test on the the standardized innovations

Lags	Test Statistic	p-value
10	18.3070	0.1554
15	24.9958	0.3353
20	31.4104	0.4775

Table 5: Engle's ARCH test on the standardized innovations

Lags	Test Statistic	p-value
10	18.3070	0.1828
15	24.9958	0.3789
20	31.4104	0.5331

V. Simulation Of KLCI Returns Series.

Figure 9 is a plot of the respective values obtained from the simulation of a single realization (path) for return series based upon the estimated model in Equation (3), its innovations and conditional standard deviations. A total of 1,000 observations were simulated by assuming that there are 250 trading days per year.

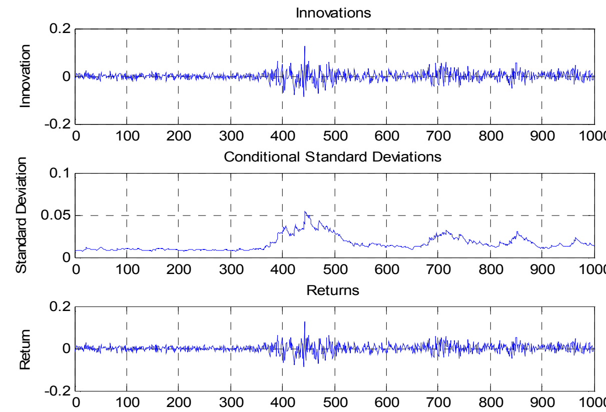

Fig. 9 Plots of residuals (innovations), conditional standard deviation and simulated returns

VI. Comparison Of Forecasts With Simulation Returns

The estimated model in Equation (3) was used to compute forecasts for the return series for 30 days into the future. Figure 10 compares the results from forecasted standard deviations of future residuals and its counterpart derived from the simulation. The result shows that both forecast and simulated standard deviations of the residuals exhibits similar results. However, results of forecasted conditional mean of the return series compared with the simulated results in Figure in 11 shows large differences.

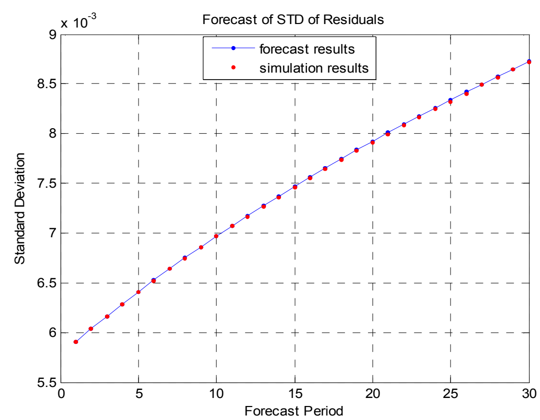

Fig. 10 Forecast and simulation of the conditional standard deviations of residuals (innovations)

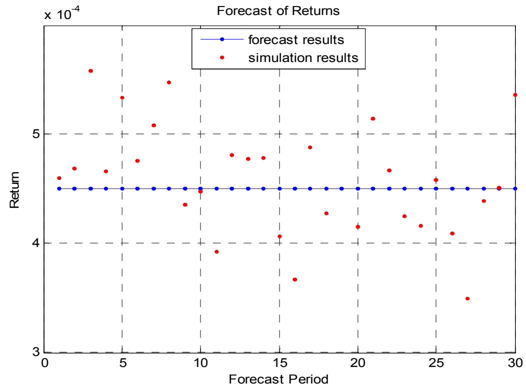

Fig. 11 Forecast and simulation of the conditional mean of the KLCI return series

VII. Fitting A Model To A Simulated Return Series

A GARCH(1,1) specification for the variance model was used with the constant term c given the value of 0 and the other parameters specified as in Table 6.

Table 6 The Estimated GARCH(1,1) Model Parameters

Parameter	**Value**
c	0.000
a_0	0.005
a_1	0.300
b_1	0.100

The model is then fitted into a simulated return series, using 2000 simulated values of the innovations, ε_t, conditional variance, σ_t^2 and returns y_t as a GARCH(1,1) process. The parameters of the simulated return y_t series were then estimated and then compared to those of the earlier estimates. Table 7 exhibits the coefficients obtained. They are found to be quite close to the set of coefficients in Table 6.

Table 7: The Estimated GARCH(1,1) Model

Coefficient	**Value**	**Standard Error**	**t-Statistic**
c	9.0012e-005	0.0020	0.0446
a_0	0.0050	0.0018	2.8393
a_1	0.29872	0.2239	1.3341
b_1	0.08548	0.0275	3.1131

VIII. CONCLUSION

This paper reports an effort in modeling stock market volatility as a simple GARCH process using a sample of 3,916 daily observations of the KLCI. Initial test results on the returns series indicated a need for modeling them as a GARCH process. Tests on the residuals indicate fair performance of the estimated model, which is fairly supported by the simulation results.

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