Analysis of inter-area mode oscillations using a PSS for Variable Speed Wind Power Converter and Modal decomposition.

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Abstract: This paper shows a simple approach to damping inter-area oscillations using a PPS for a variable-speed wind power converter. The proposed control is evaluated performing several simulations on modified version of 10-generator, 39-bus New England power system. The simulation represent a three-phase short-circuit, and the damping control system is applied with different gains. The selection of the variable-speed wind power converter’s gain is carried out through modal decomposition and the effectiveness of the proposed approach is demonstrated for a fault at bus 4.

Key-Words: Linearization, critical mode, eigenvalues, Wind Power Converter, damping, inter-area oscillations, participation factor, modal decomposition, amplitude ratio.

1 Introduction
Large scale wind power integration has a significant impact on the operation of power systems, including on level of oscillations damping [1]. It has been demonstrated that wind generation has some impacts on the damping levels of a power system, and the nature of the impacts depend on factors like wind generator conversion technology such as variable-speed or fixed-speed wind power converters, and also network configuration [2]. Power system stabilizer (PSS) is still one of the most cost effective solutions capable of providing supplementary damping. However, with high penetration of the system in wind, these devices may no longer be able to provide the necessary additional damping.

PSS for Variable-speed wind power converter can be used to improve the overall damping. It is the objective of this research to analyze the contribution of variable-speed turbine to the problem of the stability and to take advantage of the control strategies that employ the flexibility provided by these machines to increase the damping of oscillations. The study was conducted using modal analysis, participation factor and modal decomposition using PSS/E software package.

2 Test System
The 10-generator, 39-bus New England power system shown in figure 1 has been modelled; this system includes 10 generators and 3 Wind Parks, and comprises 107 states variables. Models were implemented in the PSS/E software package.

Figure 1. 10-generator, 39-bus New England Power System

The NE 10-generator, 39-bus system has some adjustment in loading and generation in order to create a critical case. Nine generators are represented by the two-axis model and one generator (generator number 10) is modelled by classical model. The generators are round rotor with d and q axis transient and subtransient effects represented. The exciters are IEEE type 1.

For transient stability simulations, the real and reactive power portions of the load are typically modelled as constant current and constant admittance. The real power portions of the loads are modelled as 60% constant admittance and 40% constant current; the reactive power portions of the loads are modelled as 50% constant admittance and 50% constant current.
2.1 Control design of the PSS for variable-speed wind power converter (WPSS).

The modelling of the variable-speed wind turbine is depicted in figure 2. The wind turbine model includes a block which implements an actuator disk model. Its input variables are the wind speed, the mechanical rotor speed and the pitch angle, and its output variable is the mechanical power. Besides, there are a number of additional blocks such as: A pitch angle controller, a rotor speed controller, wind speed model, and PSS for variable-speed wind power converter [4].

The proposed control is based on the ability of variable speed wind turbines to perform an active control power which is decoupled from reactive power control and from rotor mechanical speed. During normal operation, when frequency deviation is null, the control signal \( P_{\text{pssw}} \) will be zero. Reactive power reference \( Q_{\text{ref}} \) is supposed to be zero, this is, wind is turbine operating at unit power factor. Although different control strategies may be used here, there is no reason to suppose that they would have major effects on the results of the study. It should also be noted that this control technique uses only local variables, so that it does not involve any telecommunication issue.

3 Problem Formulation

3.1 State Space Model

To model the behavior of dynamic systems [7], quite often a set of \( n \) first order nonlinear ordinary differential equations are used. This set commonly has the form:

\[
\dot{x}_i = f_i(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \quad i = 1, 2, \ldots, n \tag{3}
\]

Where \( n \) is the order of the system and \( r \) is the number of inputs. If the derivatives of the state variables are not explicit functions of the time, equation (3) may then be reduced to:

\[
\dot{x} = f(x, u) \tag{4}
\]

Where \( n \) is the order of the system, \( r \) is the number of inputs and \( x, u \) and \( f \) denote column vectors of the form:

\[
\begin{align*}
  x &= \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \\
  u &= \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \\
  f &= \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}
\end{align*}
\]

The state vector \( x \) contains the state variables of the power system; the vector \( u \) contains the system inputs and \( \dot{x} \) encompasses the derivatives of the state variables with respect to time. The equation relating the outputs to the inputs and the state variable can be written as:

\[
y = g(x, u) \tag{6}
\]

The state concept may be illustrated by expressing the swing equation of a generator in per-unit torque as follows:

\[
2Hd^2\delta \frac{d^2}{dt^2} = T_m - T_e - K_D\Delta\omega_r 
\]

Where \( H \) is the inertia constant at the synchronous speed \( \omega_s \), \( t \) is time, \( \delta \) is the rotor angle, \( T_m \) and \( T_e \) are the per-unit mechanical and electrical torque, respectively, \( K_D \) is the damping coefficient on the rotor and \( \Delta\omega_r \) is the per-unit speed deviation. Now, expressing (7) as two-first-order differential equations yields:

\[
\begin{align*}
  \frac{d\Delta\omega_r}{dt} &= \frac{1}{2H} (T_m - T_e - K_D\Delta\omega_r) \tag{8} \\
  \frac{d\delta}{dt} &= \omega_s\Delta\omega_r \tag{9}
\end{align*}
\]
3.2 Linearization
Small signal stability is the ability of the power system to maintain synchronism when subjected to small disturbances. In this context, a disturbance is considered to be small if the equations that describe the resulting response of the system may be linearized [8]. For the general state space system, the linearization of (4) and (6) about operating point \( x_o \) and \( u_o \) yields the linearized state space system given by:

\[
\begin{align*}
\Delta x &= A\Delta x + B\Delta u \\
\Delta y &= C\Delta x + D\Delta u
\end{align*}
\]  

(10)

Here, \( \Delta x \) is the \( n \) state vector increment, \( \Delta y \) is the \( m \) output vector increment, \( \Delta u \) is the \( r \) input vector increment, \( A \) is the \( nxn \) state matrix, \( B \) is the \( nxr \) input matrix, \( C \) is the \( mxn \) output matrix and \( D \) is the \( mxr \) feed-forward matrix. Specifically, \( \Delta x = x - x_o \), \( \Delta y = y - y_o \) and \( \Delta u = u - u_o \). As an example, (8) and (9) are linearized about the operating point \((\delta_o, \omega_o)\), yielding:

\[
\begin{align*}
\frac{d}{dt}\Delta \omega_i &= \frac{1}{2H}(\Delta T_m - K_S\Delta \delta - K_D\Delta \omega_i) \\
\frac{d}{dt}\Delta \delta &= \omega_o \Delta \omega_i
\end{align*}
\]  

(12)

(13)

Where \( K_S \) is the synchronizing torque coefficient.

3.3 Eigenvalues and Stability Analysis
Once the state space system for the power system is written in the general form given by (10) and (11), the small-signal stability of the system can be calculated and analyzed [9]. The analysis performed follows traditional root-locus (or root-loci) methods using PSS/E software package. First the eigenvalues \( \lambda_i \) are calculated for the \( A \)-matrix, which are the non-trivial solutions of the equation

\[
A\Phi = \lambda\Phi
\]

(14)

Where \( \Phi \) is an \( nxn \) vector. Rearranging (14) to solve for \( \lambda \) yields:

\[
\text{Det}(A-\lambda I) = 0
\]

(15)

The \( n \) solutions of (15) are the eigenvalues \((\lambda_1, \lambda_2, \ldots, \lambda_n)\) of the \( nxn \) matrices \( A \). These eigenvalues may be real or complex and are of the form \( \sigma \pm j\omega \). If \( A \) is real, the complex eigenvalues always occur in conjugate pairs.

The stability of the operating point \((\delta_o, \omega_o)\), may be analyzed by studying the eigenvalues. The operating point is stable if all the eigenvalues are on the left-hand side of the imaginary axis of the complex plane; otherwise it is unstable, figure 3. If any of the eigenvalues appear on or to the right of this axis, the corresponding modes are said to be unstable, as is the system. This stability is confirmed by looking at the time dependent characteristic of the oscillatory modes corresponding to each eigenvalues \( \lambda_i \), given by \( e^{\lambda_i t} \). The latter shows that a real eigenvalue corresponds to a non-oscillatory mode. If the real eigenvalues is negative, the mode decays over time. The magnitude is related to the time of decay: the larger magnitude, the quicker the decay. If the real eigenvalue is positive, the mode is said to have aperiodic instability [10].

Figure 3. Eigenvalues and associated response

For \( \omega = 0, \sigma < 0 \) damped unidirectional response
For \( \omega \neq 0, \sigma < 0 \) damped oscillatory response
For \( \omega \neq 0, \sigma = 0 \) oscillation response of constant width
For \( \omega = 0, \sigma > 0 \) oscillatory response with the oscillation grows without limit
For \( \omega = 0, \sigma > 0 \) unidirectional response from growing monotonous.

On the other hand, the conjugate-pair complex eigenvalues \((\sigma \pm j\omega)\) each correspond to an oscillatory mode. A pair with a positive \( \sigma \) represents an unstable oscillatory mode since these eigenvalues yield an unstable time response of the system. In contrast, a pair with a negative \( \sigma \) represents a desired stable oscillatory mode. Eigenvalues associated with an unstable or poorly damped oscillatory mode are also called dominant modes since their contribution dominates the time response of the system. It is quite obvious that he desired state of the system is for all of the eigenvalues to be in the left-hand side of the complex plane.

Other information that can be determined from the eigenvalues is the oscillatory frequency and the damping factor. The damped frequency of the oscillation in Hertz is given by:

\[
f = \frac{\omega}{2\pi}
\]  

(16)

And the damping factor (or damping ratio) is given by:

\[
\xi = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}
\]  

(17)

3.4 Eigenvectors and Modal Matrices
Given any eigenvalue \( \lambda_i \), the \( n \)-column vector \( \Phi_i \) which satisfies

\[
A\Phi_i = \lambda_i \Phi_i
\]

(18)
Is called the right eigenvector of \( A \) associated with the eigenvalue \( \lambda_i \). Quite similarly, the n-row vector 
\[
\Psi_i A = \lambda_i \Psi_i
\]
(19)

Is called the left eigenvector associated with the eigenvalue \( \lambda_i \). For convenience, it is assumed here that the eigenvectors are normalized so that:
\[
\Psi_i^T \Phi_1 = 1
\]
(20)

To continue the eigenanalysis of the matrix \( A \), the following modal matrices are introduced:
\[
\Phi = [\Phi_1 \Phi_2 \cdots \Phi_n]
\]
(21)

\[
\Psi = [\Psi_1^T \Psi_2^T \cdots \Psi_n^T]^T
\]
(22)

\( \Lambda = \text{Diagonal matrix with eigenvalues as diagonal elements} \)
(23)

The relationships (18) and (20) can be written in a compact form as:
\[
A \Phi = \Phi \Lambda
\]
(24)

\[
\Psi \Phi = 1, \text{ yielding } \Psi = \Phi^{-1}
\]
(25)

Once the oscillatory modes have been identified and the modal matrices constructed, an analysis is performed to find the specific rotor-angle modes. These modes provide the largest contribution to the low frequency oscillations, the rotor-angle modes can be identified by analyzing the right and left eigenvectors in conjunction with the participation factors.

### 3.5 Participation Factor

Originally proposed in [8], a matrix called the participation matrix, denoted by \( P \), provides a measure of association between the states variables and the oscillatory modes. It is defined as:
\[
P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}
\]
(26)

With
\[
P_i = \begin{bmatrix} p_{i1} \\ p_{i2} \\ \vdots \\ p_{in} \end{bmatrix} = \begin{bmatrix} \Phi_{i1} \Psi_{i1} \\ \Phi_{i2} \Psi_{i2} \\ \vdots \\ \Phi_{in} \Psi_{in} \end{bmatrix}
\]
(27)

The element \( p_{i1} = \Phi_{i1} \Psi_{i1} \) is called the participation factor, and gives a measure of the participation of the \( i \)th state variable in the \( i \)th mode, and vice versa. The use of the participation factor will be presented in the analysis of the “oscillation profile” of the Power System.

### 4 Modal decomposition

The main idea behind modal decomposition is that any linear system response can be decomposed into a summation of terms like the following:
\[
\text{Output} = \sum (A_i e^{\sigma_i t}) + \sum (B_j e^{\sigma_j t} \cos(w_j t + \phi_j))
\]
(28)

That is, a summation of exponential and damped (or undamped) sinusoidal terms. Each exponential component is associated with one real eigenvalue \( \sigma \). Similarly, each sinusoidal term is associated with two complex conjugate eigenvalues, of real and imaginary \( \sigma_i \) and \( \pm w_j \) parts, respectively.

Participation and phase of each term is dictated by its respective \( A_i \), or \( B_j \) and \( \phi_j \) coefficient. The larger a particular \( A_i \) or \( B_j \) coefficient is relative to those of other modes, the more dominant mode is the particular mode. The less negative (or more positive) the associated \( \sigma_i \) or \( \sigma \), exponent is, the longer the mode will linger relative to other better damped modes. The order of the system is defined as the sum of the number of eigenvalues, real and complex, which describe its response.

Linear systems theory indicates that there are many roots or eigenvalues as there are states in the dynamic model. For large dynamic models, this would result in tens of thousand s of terms in Equation (28). However, the theory also says that due to pole-zero cancellations, only a few of those roots are excited by a particular disturbance, and even fewer are observable on any particular output signal. In other words, for the majority of modes, \( A_i \) or \( B_j \) are either zero or negligible. Further, of the remaining modes, most are well damped and tie out quickly.

As a consequence, it is common to observe that well into a simulation, only a few modes subsist, showing either a single lightly damped oscillation (two complex conjugate modes), or , in some instances, two or more oscillation modes (normally indicated by the presence of beating).

It is those few remaining modes that these analysis techniques are intended to capture. It is their associated \( A_i \) and \( \sigma_i \), or \( B_j \cdot \sigma_i \cdot w_j \) and \( \phi_j \) factors that the techniques will produce. Two alternative algorithms are offered in the program; one employing Prony analysis techniques; the other utilizing least-square approximations.

Prony techniques have been described in a number of technical publications [1]. They are characterized by providing an exact model (barring numerical imprecisions) of the simulation results under consideration. This precision however can prove to be a hindrance when analyzing nonlinear cases. Consider, for example, a case with a single lightly damped oscillation mode whose frequency changes slightly within the time window of interest. Because it is designed to exactly fit the simulation results with a linear model, Prony techniques will go to great lengths to derive modes of similar frequencies, some of which dominating the initial part of the time window, while others dominating the later parts of the window. The result in such cases can be quite confusing and may require significant judgment in their interpretation.
For case where Prony analysis fails to render the expected results, least square techniques are an attractive alternative. Under the least square mode of operation, the analyst would typically select a relatively low model order n to describe the system. The algorithm would then “initialize” by exactly fitting the first 2n points in the selected time window. Following initialization, it analyzes the remaining data by recursively adjusting the initial model so as to render an n\textsuperscript{th} order least-square approximation of the time window of interest. Execution time can be significantly larger than for the Prony techniques, particularly when high-order models are required and/or when a large number of time steps are selected. Two options are given under least square analyses. In one option, only eigenvalues are determined with least squares, while eigenvectors are calculated by Prony analysis to fit the initial 2n points. This option is the faster of the two and will render more exact eigenvector calculations at initial time. Under the second least-square option, both eigenvalues and eigenvectors are calculated with least squares. This option is more time consuming but will render a better fit throughout the time window of interest. Whatever technique is employed, two indices and graphical techniques are provided to assess the adequacy of the model. The two indices are:

**Percent error:** Calculated as the ratio (multiplied by 100), throughout the time window, of the summation of absolute values of differences between actual data and the first and the first actual point. In other words, it is the absolute value of error, in percentage of the average deviation relative to the first data point.

**Signal/Noise Ratio:** The Square of differences between actual data and first data point (signal) and between model and actual data (noise) are summated throughout the time window. The square root of the ratio is calculated and expressed in decibels (20 times the decimal logarithm).

### 5.1 Effect of the gain in the eigenvalues

Starting from the case with an operating point sufficiently small signal stable, the effect of increasing the wind power converter's gain has been analyzed. It is observed that the gain has small effect in a few eigenvalues of the system, but the operating point is still sufficiently small-signal stable, and then the eigenvalues of the New England power System at each gain are computed, figures 4 and 5.

![Figure 4 Operating Point sufficiently small-signal stable](image1)

![Figure 5 Effect of the gain in the eigenvalues](image2)

### 5.2 Participation factor calculations

Figure 6 shows the main eigenvalues of the simulations, when varying the gain, these eigenvalues have oscillation frequency among 0.1 and 1 Hz, they are known as inter-area oscillations and figures 7 to 12 show the participation factors.

![Figure 6 Interarea mode oscillations](image3)
6 Simulations in PSS/E

6.1 Simulations results with no damping control
A three-phase short-circuit at bus 4 has been simulated, with a duration of 150 ms. The topology of the grid before and after the fault are the same. Figure 16 shows the active power flow between buses 39-9, 27-26, 17-16, 16-19, 4-14, and 14-13. It can be seen three inter-area mode oscillations, poorly damped oscillations.

Figure 17 shows the active power production of the wind park. This production remains basically constant, because the active power reference at the wind park is independent from the grid conditions. Only during, and immediately after the fault, active power output decreases as a result of voltage decay, because of current limitation in the electronic converters.

6.2 Simulations results with damping control
Figures 18, 19, 20 and 21 show inter-area power flow, the control loop gain, which is 35, 60, 120 and 240 respectively. It can be seen how inter-area power oscillations are damped by the proposed control.
Figures 20, 21, 22, 23, 24 and 25 show the active power production of the wind park under the same circumstances. It can be seen at figure 28 which corresponds to a gain \( K = 60 \), how wind power production is modified after the fault in order to damp power system oscillations. The variation in the power production is not very large compared to the total production, and could be performed by modern wind energy converters.

Figure 29 which corresponds to a gain \( K = 120 \), shows larger oscillations in power production, which represent an additional effort to the wind power converter control. However, it can be seen that, immediately after the fault, the wind power production always decrease, which will result in a slight increase in rotor speed and, consequently, in the kinetic energy stored in the rotor and the blades. Thus, the windmill will be able to use this kinetic energy to perform the required active power control.

Figures 26, 27, 28 and 29 show the modal components of line 39-9 power flow, only inter-area mode oscillations are shown.
7 Conclusion
It has been shown how wind power converter can damp oscillations by means of a simple control loop. It should be noted that this control technique uses only local variables, so that it does involve any telecommunication issue.

References:


