Complex guide to human comfort, the static and dynamic heat balance the human body.

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Abstract: This paper proposes to give a brief summary of the concept, objectives and investigation methodology of comfort theory and in particular thermal comfort for building service engineers as well as the findings of international research and the goals of ongoing Hungarian research projects (mainly carried out at the Department of Building Service Engineering and Process Engineering at the Budapest University of Technology and Economics). The importance of comfort theory and its application by engineers will be highlighted. The concept of dynamic thermal sensation and the possibilities of its research will be introduced. It is only possible through the presentation of system theory, system dynamics and regulation theory.

Keywords: Thermal Comfort, Static and Dynamic PMV.

1 Introduction
The architectural and building engineering design of spaces suitable for human occupation and work aims to meet the so-called comfort requirements optimally. Human comfort requirements cover the air quality of the occupied space and its thermal attributes in addition to other ergonomic, sociological and psychological criteria. Previously, dimensioning the „human comfort” of spaces used for human occupation mainly and/or exclusively covered the specification of a pleasant air temperature and the dimensioning, installation and management of the required heating and cooling. Since then other requirements have become equally important besides air temperature: ventilation, draught criteria, wall temperatures, „cold walls”, air humidity, resultant temperature, radiation temperature, i.e. all characteristics of a microclimate and the issue of discomfort. It is obvious and has also been confirmed by research that some microclimate parameters may boost or weaken the impact of the other ones (within certain limits) or compensate for their negative or positive impact. According to the research approach used in the last two decades (comfort theory, thermal comfort – research) the analysis of the joint impact of microclimate parameters can be based on the energy balance of an active or sedentary human
who has a mechanical or thermal connection with his/her environment.

2 Classifying and measuring thermal comfort

Human response to thermal environment and the satisfaction with thermal comfort are expressed by subjective thermal sensation. Thermal comfort is linked to the heat balance of the human body: the time and the adaptation reactions required to achieve this balance and whether it is agreeable for the given person and what skin temperature and perspiration are measured.

ASHRAE expresses subjective thermal sensation using the thermal sensation scale:

+3 (hot), +2 (warm), +1 (slightly warm), 0 (neutral), -1 (slightly cool), -2 (cool), -3 (cold)

The agreeable thermal sensation falls between –1 and +1 on the scale. In an experiment conducted with several thousand live subjects the test persons had to declare (after a relaxation interval) how they felt in the examined microclimate and what value they voted for on the scale.

P. O. Fanger based the investigation of thermal sensation on writing up and calculating the energy balance of the human body and established a mathematical connection between the solutions of the energy balance and the thermal sensation scale. [1]

If the thermal impacts of the environment change, a human conducting a certain activity in a given thermal environment will try to set off the balance of heat production and heat loss by increasing or reducing heat loss in a transient process. Fanger examined only the static heat balance of the human body, after the heat equilibrium caused by the transient process set in.

3 Static thermal comfort equation

Fanger’s experiments have shown that the thermal sensation (thermal comfort) of an active person is deemed agreeable (optimal) if the following criteria are met:

- The static energy balance describing the thermal and mechanical connection between the human and his environment is zero, and the resultant of the heat produced and lost as well as the performed work is zero,
- Human skin temperature remains within a narrow range,
- Perspiration remains within a given range.

This was expressed by Fanger by the following inequalities:

\[ a < t_s < b , \quad c < E_{sw} < d . \]  \hspace{1cm} (1)

According to the experiments skin temperature in the state of agreeable thermal comfort only depends on metabolism and heat loss through perspiration is similarly linked to metabolic heat in a defined statistical connection.

Fanger found that skin temperature in the state of agreeable thermal comfort has the following regressive relationship with metabolic heat.

\[ \overline{t_s} = 35,7 - 0,032 \frac{M}{A_{dh}} \, [^\circ C] \]  \hspace{1cm} (2)

Fanger also discovered that the regressive relationship between heat loss by perspiration and metabolic heat in the state of agreeable thermal comfort is as follows:

\[ Q_{sw} = 0,42 \left( \frac{M}{A_{dh}} (1-\eta) - 58,15 \right) \, [W]. \]  \hspace{1cm} (3)

The comfort equation describing the heat balance of the human body in the state of thermal comfort is the following:

\[
\begin{align*}
&\frac{M}{A_{dh}} (1-\eta) - \\
&3.05 \cdot 10^{-4} \left[ 5733 - 6.99 \frac{M}{A_{dh}} (1-\eta) - p_s \right] - \\
&0.42 \left[ \frac{M}{A_{dh}} (1-\eta) - 58,15 \right] - \\
&1.7 \cdot 10^{-5} \frac{M}{A_{dh}} (5867 - p_s) - 0.0014 \frac{M}{A_{dh}} (34 - t_s) = \\
&3.96 \cdot 10^{-5} f_{si} \left[ (t_s + 273)^4 - (t_{new} + 273)^4 \right] + \\
&f_{di} b_t (t_s - t_a)
\end{align*}
\]  \hspace{1cm} (4)

This equation is discussed in detail by book [1] and [2] listed in the References.

4 Co-dependency of comfort parameters

Fanger processed his findings about the dependence of the solution of equation (4) and the factors influencing the thermal comfort (with using the pertaining derivatives) in diagrams [1]. Using the comfort equation derivatives describing the co-
dependency of the activity level, the clothing and the climate parameters are obtained. Derivatives express the change that is produced by the unit change on the variable in the numerator while the value of comfort equation remains zero. Two examples for the functions of derivatives:

\[
\frac{\partial L_a}{\partial m_{ct}} = 1.584 \times 10^{-7} f_{ct}(t_{mrt} + 273)^3, \quad (5)
\]

\[
\frac{\partial L_a}{\partial p_a} = -0.00305 - 0.000017 \frac{M}{A_{ho}}, \quad (6)
\]

5 PMV the complex measurement value of thermal comfort

5.1 Theoretical and empirical fundamentals of PMV

Fanger introduced the term thermal load of the human body which is calculated by differing from 0 in the comfort equation as follows [1], [2]:

\[
L = \frac{M}{A_{ho}}(1-\eta) - 3.05 \times 10^{-5} \left[5733 - 6.99 \frac{M}{A_{ho}}(1-\eta) - p_a\right] - 0.42 \frac{M}{A_{ho}}(1-\eta) - 58.15 - 0.0014 \frac{M}{A_{ho}}(34 - t_a) - 0.0036 f_{ct}(t_{mrt} + 273)^3 - f_{ct}(t_{mrt} + 273)^3 + f_{ct}(t_a - t_m)
\]

(7)

Conducting masses of experiments and using the live subjects’ votes Fanger looked for a mathematically defined function between the ASHRAE scale [1] and thermal load L on an empirical basis.

Fanger found the following regressive function between the ASHRAE scale and the L values of thermal load:

\[
\frac{\partial Y}{\partial L} = 0.303 e^{-0.036 \frac{M}{A_{ho}} + 0.028}
\]

(8)

This expression is in fact a differential equation which can be integrated to produce the dependence of the thermal comfort indicator (Y) on the thermal load of the human body (L). The integrated of the differential equation:

\[
Y = PMV = \left(0.303 e^{-0.036 \frac{M}{A_{ho}} + 0.028}\right) \left[\frac{M}{A_{ho}} \frac{(1-\eta)}{1.95 \times 3.05 \times 10^{-3} - (t_{mrt} + 273)} + f_{ct}(t_a - t_m)\right]
\]

(9)

Thermal comfort indicator Y calculated with the above equation was called PMV, the predicted mean vote by Fanger [1], [8].

The physically reasonable solutions of the thermal comfort indicator fall between -3 and +3. PMV can be used in two ways:

- PMV is calculated for different activity levels, various environmental and clothing parameters to classify our environment and thermal sensation: between -1 and +1 thermal comfort is described as slightly cool, neutral and slightly warm.

- the inverse of the above: environmental microclimate parameters are defined for the activity level and clothing linked to a specified PMV or PMV range (practically -1, +1 range) to meet the criteria of the given PMV.

The above tasks can be carried out using mathematical operations on equation (9). To perform the tasks the derivatives of the PMV function has been produced in function of the different microclimate parameters and \(I_{cl}\) characterizing the activity level and clothing. Derivatives also showed the sensitivity of PMV in function of the various factors and in case of slight changes provided us with answers about how to compensate for the impact of a given factor by modifying the value of another factor.

5.2 Sensitivity and derivatives of PMV

The (PMV) thermal sensation indicator is expressed by its derivatives. Fanger produced these derivatives numerically and introduced them with help of diagrams. The following authors presented the explicit mathematical functions of the derivatives and published them in appendix (7).
6 Dynamic heat balance of human body

In his live subject experiments Fanger placed his subjects under various activities and thermal loads where the produced and lost heat was not in balance at the initial phase of the examined interval. This was expressed by the deviation from 0 in thermal load L of the human body. Naturally if the human body was able to undergo an adaptation process the heat balance was achieved and the L thermal load of the human body turned 0. Through their votes live subjects were describing this adaptation process. Their votes obviously expressed what new core temperature was required to achieve the new L=0 heat balance and whether it was agreeable or disagreeable for them. This shows the weak point of Fanger’s theory: the regressive straight lines of skin temperatures and perspiration values considered agreeable are deemed valid thus also covering the discomfort state of heat balance.

This contradiction can be mathematically presented in the following figure:

![Figure 1](image1)

Fig. 1

To develop the so-called static PMV Fanger took his starting point as the extrapolation of the dynamic energy balance of the human body and called a hypothetical condition static PMV. In a stochastic sense there exists a dynamic PMV that classified the ongoing changes of the actual thermal load L of the human body in time and describes the changes of the human response in time as well as the response of the human body to the changes in the environment in time. This theory can grasp static conditions by recording the environmental parameters, eliminating changes in time and revealing the asymptotic conditions. To create such a simplified (and stochastically valid) theory we must start with the fact that the response of the human body to environmental changes can be described by the type of differential equation known from the regulation theory [3]:

\[ A_2v'' + A_1v' + A_0v = u \quad \text{or} \quad A_1v' + A_0v = u \]  

(10)

The variables are in theory vector variables but may be component variables as well. Variable components of \( u \) are environmental impacts affecting humans i.e. \( t_{sa}, t_{mat}, v_{air}, p_a \), while the components of \( v \) variable are the physiological responses of humans i.e. \( t_{core}, t_{skin}, t_{sw}, Q_L, Q_{SW}, Q_D, S, C, \) clothing, body mass, work intensity, other characteristics, constant parameters.

In a stochastic sense equation (10) can be written up for thermal load L in humans and for the relationships of responses given to thermal load intensity.

If the functions in time of the physiological responses given to the unit jump function of the various environmental parameters were or are available we could define the so-called temporary functions of the human body. The solution of equation (10) for a unit jump input is shown graphically on Fig. 2:

![Figure 2](image2)

Fig. 2

If there are no jump unit inputs and physiological responses given to them available in terms of environmental variable and impacts we can still define the temporary function for the physiological response of the human body in a stochastic sense, using Duhamel’s principle.

Duhamel’s principle describes the following: in the event the time function of a disturbance in the system is known then the time function for the system’s response can be expressed by the convolution integral:

\[ v(t) = \int_0^t h'(t - \tau) u(\tau) d\tau. \]  

(11)

According to equation (11) if we knew the temporary function for a human physiological response to an environmental impact affecting a person then the time function for the examined physiological response can be determined for the
time function of the examined environmental impact.

We can also do the inverse task: if we know the time function for a human physiological response to the time function of an environmental impact then using convolution integral (11) we can in theory determine the temporary function for the examined response. Our task is then to resolve an integral equation which is in fact far from being as difficult as it looks because by solving equation (10) the temporary function is

\[
h(t) = \frac{1}{A_0} \left(1 - e^{-\frac{A_1}{A_0}t}\right) \tag{12}
\]

Expression (12) is substituted into convolution integral equation (11). If value pairs \( u, v \) are available among the measurement results we can integrate expression (12) to get algebraic equations to determine \( A_0 \) and \( A_1 \) coefficients.

The above discussed examinations enable us to write up dynamic thermal sensation if thermal load \( L \) is considered the input of equation (10) and the output is \( Y \) and PMV parameters.

We would like to draw your attention to the work carried out by Muhsin Kilic and Omer Kaynakli whose publication processed the actual adaptations of the human body in time with regard to the response functions of the human body to the various runnings of the different environmental impacts in time. Some of these, from Fig. 3 to Fig. 8 are presented below [4]. The diagrams graphically show the temporary functions of heat loss, TSENS (thermal sensation index), average skin temperature, average skin wettedness under the given environmental parameters (disturbances).
We must note that unlike Fanger the above mentioned authors used the term perceived temperature recommended by ASHRAE to describe PMV:

\[ T_{SENS} = \begin{cases} 0.4685(T_b - T_{bc}) & T_b < T_{bc} \\ 4.749(T_b - T_{bc})/(T_{bc} - T_{hb}) & T_{bc} < T_b < T_{hb} \\ 4.749 + 0.685(T_b - T_{hb}) & T_{hb} < T_b \end{cases} \]

The expression defines thermal sensation on a scale of −5 to +5.

The authors developed a mathematical model to describe the dynamic heat balance of the human body, taking into account the heat storage capacity of the human body and clothing as seen on Fig. 9.

![Fig. 7](image)

Variations of forearm and foot skin temperature with time

\[ M_{st} = 80 \text{ W/m}^2, T_s = T_\text{st} = 30 \text{ C, RH = 50\%, } V = 0.1 \text{ m/s}. \]

![Fig. 8](image)

Variations of chest and pelvis skin wetness with time

\[ M_{st} = 80 \text{ W/m}^2, T_s = T_\text{st} = 30 \text{ C, RH = 50\%, } V = 0.1 \text{ m/s}. \]

Based on the diagrams presented by the authors we defined for the temporary functions the \( A_0 \) and \( A_1 \) parameters of expression \( h(t) = \frac{1}{A_0} \left( 1 - e^{-\frac{A_1}{A_0}} \right) \) belonging to the \( A_1 v' + A_0 v = u \) differential equation [Table 1 and Table 2].

![Fig. 9](image)

**Table 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( A_0 )</th>
<th>( A_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensible heat loss</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Latent heat loss</td>
<td>0.053</td>
<td>0.435</td>
</tr>
<tr>
<td>Average skin temp</td>
<td>0.053</td>
<td>0.435</td>
</tr>
<tr>
<td>Average skin wettness</td>
<td>0.053</td>
<td>0.435</td>
</tr>
<tr>
<td>M = 45% RH, 30 C</td>
<td>0.053</td>
<td>0.435</td>
</tr>
<tr>
<td>M = 50% RH, 30 C</td>
<td>0.053</td>
<td>0.435</td>
</tr>
<tr>
<td>M = 65% RH, 30 C</td>
<td>0.053</td>
<td>0.435</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( A_0 )</th>
<th>( A_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chest skin temp</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>Foot skin temp</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>TSENS</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>DISC</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>M = 45% RH, 30 C</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>M = 50% RH, 30 C</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>M = 65% RH, 30 C</td>
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<td>0.1</td>
</tr>
<tr>
<td>M = 75% RH, 30 C</td>
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<td>0.1</td>
</tr>
</tbody>
</table>
7 Conclusion

Our paper presented the fundamentals of PMV which is used to satisfy human comfort requirements optimally in spaces used for human occupation and work. We described the sensitivity of PMV, produced by deriving PMV according to the influencing parameters. Using system theory and regulation theory terms we endeavoured to develop a dynamic PMV with time.

Terms:

- $H$: internal heat production in the human body
- $Q_L$: dry respiration heat loss
- $Q_{rv}$: latent respiration heat loss
- $Q_d$: heat loss by skin diffusion
- $Q_{sw}$: heat loss by evaporation of sweat
- $Q_K$: heat transfer from the skin to the outer surface of the human clothing
- $Q_s$: heat loss by radiation
- $Q_c$: heat loss by convention
- $I_{cl}$: thermal resistance of clothing
- $L$: thermal load
- $M$: metabolic rate
- $t_b$: mean skin temperature
- $t_c$: surface temperature of clothing
- $t_a$: ambient temperature
- $t_m$: internal (core) temperature of the human body
- $t_s$: surface temperature
- $q_{conv}$: heat loss by convention
- $q_{rad}$: heat loss by radiation
- $W$: external mechanical work
- $t_l$: air temperature
- $t_{mrt}$: mean radiant temperature
- $v_{air}$: relative air velocity
- $p_a$: partial water vapour pressure
- $Q_{sw}/A_{Du}$: heat loss by skin diffusion and evaporation of sweat.

References:


