Modelling of the temperature change in vertical ground heat exchangers with single U-tube installation

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Abstract: - One of the major problems of ensuring optimal working of ground source heat pump systems is a heat transfer around vertical ground heat exchanger. The working of vertical U-tube can be understood as a heat exchanger between the ground and the heat carrying medium. In our case this heat carrying medium is fluid, which transfers the extracted heat from the ground to the heat pump. In winter we extract heat from the ground and in summer we transfer heat to the ground. In our paper we propose a simple calculation model to calculate the temperature change in vertical ground heat exchangers with single U-tube installation.

Key-words: - heat pump, U-tube, heat transfer, heat flow, thermal resistance, system theory.

1 Introduction

In the Carpathian basin, but mainly on the territory of Hungary the crust of the earth is thinner than the average; therefore its geothermal features are very good. Under the ground surface in the earth core levels from the decomposition of radioactive isotopes heat is produced. Its flow directed towards the surface is geothermal energy. The global average of the geothermal gradient is 33 m/°C, while in Hungary it is only 18-22 m/°C. The average value of the heat flow from the inner core of the ground is 80-100 mW/m² according to the heat flow map of Hungary, which is almost the double of the average value measured on the mainland [1].

The primary side heat sources of heat pumps operating with water-water sources are the following: underground waters and heat of the earth (geothermal energy). The geothermal energy is extracted from the ground by ground heat exchangers with U-tubes. The installation of U-tubes can be vertical and horizontal. In our paper we deal with heat extraction of vertically installed single U-tubes. For the vertical U-tubes boreholes are made with a diameter of 75 – 150 mm, with a depth range 40 – 200 m. In these boreholes single and double U-tubes are installed. (Fig. 1) After the installation the ground heat exchanger is filled with bentonite grout. This grout ensures better heat transfer and it blocks the underground waters [2].

2 Review of heat transfer modelling in the case of U-tubes

In a descending and ascending branch of the U-tubes, the fluid gets warm and forwards the heat to the heat pump through a heat exchanger. The modelling of this heat transfer...
transfer is a complex problem. The process of heat transfer is affected by many variables, such as ground temperature, ground humidity, the structure of the ground and the thermal features, furthermore the location of underwater. There are many authors, who deal with these problems, such as Zeng [2], Kalman [4], Kavanaugh [5], Yavusturk and Splitter [6]. During the modelling the heat transfer can be regarded as a steady or unsteady state. Theoretically steady state never occurs during the heat extraction process. Several months after steady operation, the heat transfer process is steady with good approximation. Among others, Zeng [2] describes short term unsteady processes.

If we take the processes of heat transfer and the working of U-tubes as steady, then for the description of heat transfer between the U-tube and the ground we can use the following very simple formula,

\[ T_f(t) - T_g(t) = q_b(t)R_b, \]  

where \( R_b \) is overall thermal resistance, which includes the resistance of heat transfer in the ground and grout furthermore the resistance of the heat transfer between U-tube and the fluid [3].

The process of warming of fluid can be described with the following formula

\[ \frac{q(t)H}{m \cdot c_p} = T_f(t) - T_i(t), \]  

where \( H \) is the borehole depth, \( q(t) \) the heat flux, \( m \) the mass flow rate, \( c_p \) the specific heat capacity of water, \( T_f(t) \) the fluid temperature, \( T_i(t) \) the initial temperature of the fluid and \( T_w(t) \) the temperature of the fluid in the function of depth (H).

The main problem in the modelling is determining \( R_b \) the overall heat transfer thermal resistance.

3 Simple calculating method for the heat transfer in single U-tubes

Optimizing the heat pump systems is performed by system theory models [7]. The operation of geothermal heat pump systems is affected by ground temperature and heat transfer processes in the ground, because the ground temperature determines the maximum extractable heat capacity. It basically determines the coefficient of performance (COP). Therefore we lay a big emphasis on modelling this process, i.e. on obtaining exact numerical values of the temperature change in the ascending branch of the U-tube. By knowing the rate of this warming, we can make an exact calculation for the borehole depth in function of required capacity of the unit.

In our paper we use a simple calculating model to determine the temperature change and the extractable maximum heat capacity. In our calculations we use steady and unsteady models.

By setting up our model, we use the following hypotheses:

1. In 10 m depth, the ground temperature is not affected by the outdoor temperature changes, so the season changes are not influencing parameters. In 10 m depth the ground temperature is 10 °C.
2. The ground temperature change is linear; in 100 m depth we assume 16 °C.

3.1 Bases of the calculation model

The temperature change of the fluid is described by the following differential equations:

For the descending branch of the U-tube

\[ \frac{dP}{dH} = s \pm \frac{T_1 - T_2(H)}{R_1} \pm q', \]

for the ascending branch of the U-tube

\[ \frac{dP}{dH} = s \pm \frac{T_1 - T_2(H)}{R_2} \pm q', \]

where \( H \) is the borehole depth, \( T_1 \) and \( T_2 \) describes the temperature of the fluid in the function of depth (H). \( q' \) in equations (3), (4) shows the mutual influence of the U-tube (Fig. 2). The mutual influence can be calculated by the following equation [8]:

\[ q' = \frac{2\pi}{\lambda \cdot (T_1 - T_2)} \cdot \frac{2D \cdot d}{\cosh^2 \left[ \frac{4D^2 - D^2 - d^2}{2D \cdot d} \right]}, \]

Fig. 2: Mutual influence of U-tube in endless space

In equation (5) \( T_1 \) and \( T_2 \) describe the fluid temperature in each part of the U-tube, \( D \) and \( d \) represent the diameter of the U-tube (in our case \( D=d \)), \( l \) the distance between the parts of the U-tube and \( \lambda \) represents the heat conductivity of the grout. We study the descending and ascending temperature change in a separate coordinate system (Fig. 3).

The previously shown equations add up to a system of linked differential equations. The linked differential equations contain two unknown functions \( T_1(H) \) and \( T_2(H) \). These equations are solved by applying the
method of serial approach as follows. In the 0th approach we neglect the mutual interaction of the branches of the U-tube and we solve the equations (3) and (4) separately. The solutions are as follows:

\[ T_1(H) = s \cdot R_1 - E \cdot m \cdot c_v \cdot R_1 + F + E \cdot H + e^{\frac{H}{R_1 \cdot m \cdot c_v}} \cdot C \]  

(6)

\[ T_2(H) = s \cdot R_2 - E \cdot m \cdot c_v \cdot R_2 + F_1 + E_1 \cdot H + e^{\frac{H}{R_2 \cdot m \cdot c_v}} \cdot C_1 \]  

(7)

These two solutions are shown in coordinate systems (Fig. 3).

In the following phase we correct the obtained functions for \( T_1(H) \) and \( T_2(H) \) so that we take into account the interactions of the U-tube parts according to the (5) equation. In the equation we substitute the functions \( T_1(H) \) and \( T_2(H) \) with the obtained results in the 0th approach and we solve again the equations (3) and (4).

We proceed numerically, by \( \Delta H \) steps from 10 m to 100 m and vice versa from 100 m to 10 m. We continue this method and the function correction recursively.

In the previously shown calculation method the appropriate solution calculated for values \( R_1 \) and \( R_2 \) is problematic. In the following chapter we give an exact method to obtain solution for these thermal resistances.

4 Determining \( R_1 \) and \( R_2 \) thermal resistances considering the unsteady operation of the U-tubes

Since the U-tubes extract heat from the ground while working, the temperature of the ground around the U-tube declines simultaneously and the quantity of extractable heat gradually declines, too. This phenomenon can be modelled with the method shown by Carslaw-Jaeger [9].

According to the outer radius of the U-tube the heat flux in the function of time is:

\[ \dot{q} = -\lambda_{ground} \left( \frac{\partial T}{\partial r} \right)_{r_0} = \frac{4\pi \lambda_{ground}}{r_0 \pi} \int_0^\infty e^{-m^2 r^2} \frac{du}{u J_0^2(t_0 u) + Y_0^2(t_0 u)} \]  

(8)

Integral (8) for lower values of the Fourier number approximately is:

\[ \dot{q} = \frac{\lambda_{ground} T_0}{r_0} \left( (\pi \cdot Fo)^{\frac{1}{2}} + \frac{1}{2} - \frac{1}{4} \left( \frac{Fo}{\pi} \right)^{\frac{1}{2}} + \frac{1}{8} Fo \right) ... \]  

(9)

for larger values of Fo numbers is:

\[ \dot{q} = \frac{2Fo \lambda_{ground}}{r_0} \left( \frac{1}{\ln(4Fo) - 2\gamma} - \frac{\gamma}{[\ln(4Fo) - 2\gamma]^2} \right) ... \]  

(10)

\((\gamma = 0.57 \text{, Euler number})\)

As \( T_0 \) is a beyond temperature (the difference between the temperatures of the fluid and the distant ground) the unsteady heat transfer thermal resistance can be defined by the following:

\[ R_b = \frac{T_0}{\dot{q}} = \frac{r_0}{2 \cdot \lambda_{ground} \left( \frac{1}{\ln(4Fo) - 2\gamma} - \frac{\gamma}{[\ln(4Fo) - 2\gamma]^2} \right) ...} \]  

(11)

It is demonstrable that for the larger values of Fo the value of \( R_b \) changes very slowly, with a good approximation it can be considered as constant in a fixed period of time.

With the above stated equations we can calculate the value of the thermal resistance between the ground and all of the U-tube in different depths and the amount of the heat flux arriving to the walls of the U-tube in the function of time. It is demonstrable that the process of ground temperature decreasing is very slow. After 1 year of operation the heat transfer can be defined as a steady state. The change of the Fo number in the function of time is shown in Table 1.

<table>
<thead>
<tr>
<th>( \tau ) [s]</th>
<th>10 s</th>
<th>1 hour</th>
<th>1 day</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Fo )</td>
<td>0.019618899</td>
<td>7.062803667</td>
<td>169.50729</td>
</tr>
</tbody>
</table>

Table 1: Fo number change in the function of time
Table 2 shows the values of unsteady thermal resistance $R_b$, which are calculated by equation (11) and with the average value of heat conduction $\lambda_{\text{ground}} = 2.09 \text{ W/mK}$.

Table 2: Unsteady thermal resistance $R_b$ change in the function of time

<table>
<thead>
<tr>
<th></th>
<th>10 s</th>
<th>1 hour</th>
<th>1 day</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_b$ [mK/W]</td>
<td>0.011</td>
<td>0.012</td>
<td>0.023</td>
</tr>
<tr>
<td>$\tau$ [s]: 1 month</td>
<td>0.036</td>
<td>0.058</td>
<td>0.067</td>
</tr>
</tbody>
</table>

The overall unsteady thermal resistance can be obtained if to the results shown in Table 2 are added to the thermal resistance of the plastic U-tube pipe, which value is 0.085 mK/W. The values of the thermal resistances between the fluid and the U-tube pipe are neglected. The values of the overall unsteady thermal resistances are shown in Table 3.

Table 3: Overall thermal resistance $R_r$ change in the function of time

<table>
<thead>
<tr>
<th></th>
<th>10 s</th>
<th>1 hour</th>
<th>1 day</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_r$ [mK/W]</td>
<td>0.096</td>
<td>0.097</td>
<td>0.108</td>
</tr>
<tr>
<td>$\tau$ [s]: 1 month</td>
<td>0.121</td>
<td>0.143</td>
<td>0.152</td>
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</table>

5 Calculated results for an operating single U-tube

Hereby I propose a computation sample. Its basic data are as follows:

- The outer diameter of U-tube pipes is 32 mm;
- The absolute roughness of inner walls of U-tube is 0.00015 m;
- The outer diameter of boreholes is 140 mm;
- After placing the U-tube in the borehole, the inner space is filled by bentonite to stop the porosity;
- The fluid flow in the U-tube is turbulent;
- The distance between the descending and ascending branches of the U-tube is 3.3 cm.

In the examples (3), (4) and (5) we calculated the outgoing temperature change from the U-tube and the extracted heat from the ground with the help of equations and following the method of serial approach for the periods $\tau = 1$ day, 1 year and 10 year. Values of overall thermal resistances $R_1$ and $R_2$ are taken from Table 3. The following tables (Table 4 – 12) show iteration and improvement of the results step by step from the 0th approach to the 2nd approach. The iteration is finished at the 2nd approach. Calculated values in the tables (Table 4 – 12) show fluid warming, from 10 m to 100 m in the descending part, and from 100 m to 10 m, in the ascending part of the U-tube. $Q$ [kW] represents the extractable heat from the U-tube under the given conditions.

Table 4: 0th approach, $\tau = 1$ day

<table>
<thead>
<tr>
<th>m [kg/s]</th>
<th>H [m]</th>
<th>0.95</th>
<th>0.53</th>
<th>0.32</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
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</tr>
<tr>
<td>50</td>
<td>3.79</td>
<td>4.36</td>
<td>5.14</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>6.34</td>
<td>7.71</td>
<td>8.93</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>5.34</td>
<td>6.29</td>
<td>7.66</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4.78</td>
<td>5.49</td>
<td>6.13</td>
<td></td>
</tr>
<tr>
<td>Q [kW]</td>
<td>9.71</td>
<td>11.87</td>
<td>9.58</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: 1st approach, $\tau = 1$ day

<table>
<thead>
<tr>
<th>m [kg/s]</th>
<th>H [m]</th>
<th>0.95</th>
<th>0.53</th>
<th>0.32</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>4.36</td>
<td>5.34</td>
<td>6.89</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>5.34</td>
<td>7.03</td>
<td>8.90</td>
<td></td>
</tr>
<tr>
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<td>5.76</td>
<td>7.71</td>
<td>8.93</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5.43</td>
<td>5.81</td>
<td>5.66</td>
<td></td>
</tr>
<tr>
<td>Q [kW]</td>
<td>9.71</td>
<td>11.87</td>
<td>9.58</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: 2nd approach, $\tau = 1$ day

<table>
<thead>
<tr>
<th>m [kg/s]</th>
<th>H [m]</th>
<th>0.95</th>
<th>0.53</th>
<th>0.32</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
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<td>3.0</td>
<td>3.0</td>
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</tr>
<tr>
<td>50</td>
<td>4.31</td>
<td>5.34</td>
<td>6.89</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>5.49</td>
<td>7.45</td>
<td>9.69</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>6.01</td>
<td>7.73</td>
<td>9.49</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5.50</td>
<td>5.91</td>
<td>5.73</td>
<td></td>
</tr>
<tr>
<td>Q [kW]</td>
<td>9.98</td>
<td>6.47</td>
<td>3.66</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: 0th approach, $\tau = 1$ year

<table>
<thead>
<tr>
<th>m [kg/s]</th>
<th>H [m]</th>
<th>0.95</th>
<th>0.53</th>
<th>0.32</th>
</tr>
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<tbody>
<tr>
<td>10</td>
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<tr>
<td>50</td>
<td>3.6</td>
<td>4.05</td>
<td>4.67</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>4.52</td>
<td>5.58</td>
<td>6.95</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>5.35</td>
<td>6.85</td>
<td>8.64</td>
<td></td>
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<td>10</td>
<td>5.75</td>
<td>7.39</td>
<td>9.14</td>
<td></td>
</tr>
<tr>
<td>Q [kW]</td>
<td>10.98</td>
<td>9.75</td>
<td>8.25</td>
<td></td>
</tr>
</tbody>
</table>
6 Conclusions

The results shown in the tables (Table 4 – 12) are presented in Fig. 4 – 7. From the calculated results the following conclusion can be made. The out-going temperature of the fluid $T_2$ (H = 10 m) at every period of time in the function of mass flow has a maximum, which can be found in the interval 0.4 – 0.5 kg/s. However, the extractable heat does not have a maximum.

In the case of each mass flow value, the warming of the temperature stops at around 50 m depth in the ascending branch of the U-tube, after which the temperature of the fluid is decreasing while moving toward the surface. From the calculation we can see that with the increase in mass flow the quantity of extractable heat is increasing as well. We suggest using thermal resistance calculation with equation in practice (11) following by Carslaw-jaeger’s [9] model. Using equation (11) is theoretically proven. The accuracy of calculation is however affected by how precise information we have of the heat conductivity of the ground in the surroundings of the U-tube. Our results presented hereby correspond by size with the results calculated by GLD 3.0 [10] software $R_1 = 0.124$ mK/W and with the results calculated by researchers Zeng, Diao and Fang [2].
Fig 6: Change of the extractable heat in the function of time at different mass flow

Fig 7: Change of the out-going temperature in the function of time at different mass flow

Symbols:
\( T_1 \) – Descending fluid’s temperature;
\( T_2 \) – Ascending fluid’s temperature;
\( T_0 \) – Beyond temperature;
\( m \) – Mass flow;
\( H \) – Depth;
\( c_v \) – Specific heat,
\( A \) – Surface;
\( \lambda \) – Coefficient of Heat Conductivity;
\( s \) – Friction’s heat capacity;
\( q \) – Heat flow;
\( Q \) – Heat Capacity;
\( \vartheta \) – Heat flux;
\( \text{Fo} \) – Fourier number;
\( \tau \) – Time;
\( r \) – Radius;
\( D, d \) – diameter of the U-tube;
\( R_1 \) – Descending pipe’s thermal resistance;
\( R_2 \) – Ascending pipe’s thermal resistance;
\( R_c \) – Overall unsteady Thermal Resistance;
\( \gamma \) – Euler’s number;
\( E, F, E_1, F_1 \) – Integral constants;

References: