Stability of Some Driving Systems Analyzed by Means of Roots Geometric Locus

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Abstract: - This paper analyzes the stability of the driving systems fitted out with asynchronous motors. In order to do this the used equations systems (written in the two axes theory) and the principle of the study method are presented. A Matlab program has been conceived with their help; this program has allowed to obtain some significant results and conclusions regarding the influences of the system parameters on stability.

Key-Words: - Stability, geometric locus of roots, driving system, static converter, asynchronous motor, parameters.

1 Introduction

Nowadays, the utilization on an ever larger scale of the asynchronous motor as an execution element in driving systems, has imposed an ever ampler approach, in speciality papers, of problems concerning the dynamic regime of it and, implicitly, of stability problems. In order to carry out an adequate study in this field, it is imposed to use an adequate mathematical model. The model is optimum from the drive point of view when it generates an as simple as possible structure of the control system. In this situation, because of the great number of equations and parameters occurring in the case of the induction motors speed adjustment by modifying the supply frequency, some simplifications have been necessary. But this fact leads to a model which does not reflect correctly the dynamic behaviour of the system. So, it is imposed to use an as complete as possible model, materialized in a great number of differential equations. For writing this mathematical model, which is to facilitate the study of the influence of the filtration circuit parameters of a voltage and frequency static converter on the stability of the VFSC-induction motor assembly, the structure of such a system will be taken into account; this structure is depicted in figure 1.

The notations have the following meaning:

- R - rectifier;
- F - filtration circuit;
- I - inverter;
- IR - industrial robot.

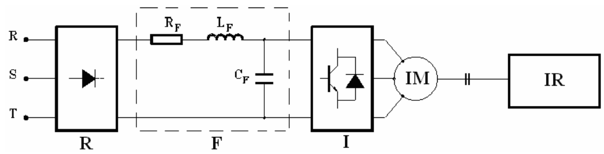

Fig. 1. Structure of driving system.

2 Mathematical Model

The mathematical model of the converter-asynchronous motor assembly will be established further on. Thus, in accordance with references, the two axes voltage equations of a squirrel cage induction machine, written in operational, have the following matrix form:

$$\begin{vmatrix} U_{ds}(s) \\ U_{qs}(s) \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} R_s + sL_s & -L_s\omega_s & sL_{sh} & -L_{sh}\omega_s \\ L_s\omega_s & R_s + sL_s & L_{sh}\omega_s & sL_{sh} \\ sL_{sh} & -L_{sh}\omega_r & R_r^{/} + sL_r^{/} & -L_r^{/}\omega_r \\ L_{sh}\omega_r & sL_{sh} & L_r^{/}\omega_r & R_r^{/} + sL_r^{/} \end{vmatrix} \cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{dr}^{/}(s) \\ I_{qr}^{/}(s) \end{vmatrix} \quad (1)$$

where s is the Laplace operator.

The equation (1) gets the following form, by processing it adequately and by considering in addition that the stator voltage phasor is oriented after the real axis ($u_{qs} = 0$):

$$\begin{vmatrix} sI_{ds}(s) \\ sI_{qs}(s) \\ s\left[I_{ds}(s) + \frac{L_{sh}}{L_s} I_{dr}^{\prime} \right] \\ s\left[I_{qs}(s) + \frac{L_{sh}}{L_s} I_{qr}^{\prime} \right] \end{vmatrix} =$$

$$= \begin{bmatrix} -\frac{\frac{R_s}{L_s}+\frac{R_r^{\prime}}{L_r^{\prime}}}{1-\frac{L_{sh}^2}{L_s L_r^{\prime}}} & \omega_r & \frac{\frac{R_r^{\prime}}{L_r^{\prime}}}{1-\frac{L_{sh}^2}{L_s L_r^{\prime}}} & \frac{\omega}{1-\frac{L_{sh}^2}{L_s L_r^{\prime}}} \\ -\omega_r & -\frac{\frac{R_s}{L_s}+\frac{R_r^{\prime}}{L_r^{\prime}}}{1-\frac{L_{sh}^2}{L_s L_r^{\prime}}} & -\frac{\omega}{1-\frac{L_{sh}^2}{L_s L_r^{\prime}}} & \frac{\frac{R_r^{\prime}}{L_r^{\prime}}}{1-\frac{L_{sh}^2}{L_s L_r^{\prime}}} \\ -\frac{R_s}{L_s} & 0 & 0 & \omega_s \\ 0 & -\frac{R_s}{L_s} & -\omega_s & 0 \end{bmatrix} \cdot$$

$$\cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{ds}(s) + \frac{L_{sh}}{L_s} I_{dr}^{\prime} \\ I_{qs}(s) + \frac{L_{sh}}{L_s} I_{qr}^{\prime} \end{vmatrix} + \begin{vmatrix} \frac{1}{L_s\left(1-\frac{L_{sh}^2}{L_s L_r^{\prime}}\right)} \\ 0 \\ \frac{1}{L_s} \\ 0 \end{vmatrix} U_{ds}(s) \qquad (2)$$

The following notations will be used further on:

- time constant of the stator winding;
- time constant of the rotor winding;
- leakage coefficient of the machine windings;

$$I_{df}(s) = I_{ds}(s) + \frac{L_{sh}}{L_s} I_{dr}^{\prime}(s)$$

$$I_{qf}(s) = I_{qs}(s) + \frac{L_{sh}}{L_s} I_{qr}^{\prime}(s) \qquad (3)$$

The relation (2) becomes with these notations:

$$s \cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} =$$

$$= \begin{vmatrix} -\left(\frac{1}{T_s}+\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & \omega_r & \frac{1}{T_r\sigma} & \frac{\omega}{\sigma} \\ -\omega_r & -\left(\frac{1}{T_s}+\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & \frac{-\omega}{\sigma} & \frac{1}{T_r\sigma} \\ -\frac{1}{T_s} & 0 & 0 & \omega_s \\ 0 & -\frac{1}{T_s} & -\omega_s & 0 \end{vmatrix} \cdot$$

$$\cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} + \begin{vmatrix} \frac{1}{L_s\sigma} \\ 0 \\ \frac{1}{L_s} \\ 0 \end{vmatrix} \cdot U_{ds}(s) \qquad (4)$$

The motion equation is added to this system:

$$\frac{J}{p}\cdot\frac{d\omega}{dt} = M + M_m \qquad (5)$$

where

$$M = \frac{3}{2} p L_{sh} (i_{qs} i_{dr}^{\prime} - i_{ds} i_{qr}^{\prime}) \qquad (6)$$

There result from (3):

$$I_{dr}^{\prime}(s) = \frac{L_s}{L_{sh}}\left[I_{df}(s) - I_{ds}(s)\right]$$

$$I_{qr}^{\prime}(s) = \frac{L_s}{L_{sh}}\left[I_{qf}(s) - I_{qs}(s)\right] \qquad (7)$$

The following relation results by applying Laplace transformation to the equation (5) and by replacing then and given by (7):

$$\frac{J}{p} s\omega - M_m = \frac{3}{2} p L_s \left[I_{qs}(s) I_{df}(s) - I_{ds}(s) I_{qf}(s)\right] \qquad (8)$$

The filtration circuit will be simulated further on.

This one is described by the following equations (in operational):

$$U_R(s) = (R_F + sL_F)\cdot I_R(s) + U_F(s)$$

$$I_R(s) = I_F(s) + C_F \cdot sU_F(s) \qquad (9)$$

The previous system can be also written in the form:

$$s\begin{vmatrix} U_F(s) \\ I_R(s) \end{vmatrix} = \begin{vmatrix} 0 & \dfrac{1}{C_F} \\ -\dfrac{1}{L_F} & -\dfrac{R_F}{L_F} \end{vmatrix} \cdot \begin{vmatrix} U_F(s) \\ I_R(s) \end{vmatrix} + $$

$$+\begin{vmatrix} -\dfrac{1}{C_F} \\ 0 \end{vmatrix} \cdot I_F(s) + \begin{vmatrix} 0 \\ \dfrac{1}{L_F} \end{vmatrix} \cdot U_R(s) \qquad (10)$$

It will be also considered that the inverter behaves like a common amplifier without delay time, characterized by the modulation factor k_m :

$$k_m = \frac{U_{ds}(s)}{U_F(s)} \qquad (11)$$

The relation (12) can be written by considering that is always equal to zero and by replacing adequately in accordance with the relation (11):

$$I_R(s) \cong I_F(s) = k_m I_{ds}(s) \qquad (12)$$

In conclusion, the mathematical model of the converter-induction machine assembly has the following form, by taking into account the relations (4), (8) and (10):

$$s\begin{vmatrix} \Delta I_{ds}(s) \\ \Delta I_{qs}(s) \\ \Delta I_{df}(s) \\ \Delta I_{qf}(s) \\ \Delta U_F(s) \\ \Delta I_R(s) \\ \Delta\omega(s) \end{vmatrix} =$$

$$= \begin{vmatrix} -(\frac{1}{T_s}+\frac{1}{T_r})\cdot\frac{1}{\sigma} & \omega_r & \frac{1}{T_r\sigma} & \frac{\omega}{\sigma} & \frac{k_m}{L_{s\sigma}} & 0 & \frac{I_{qf0}}{\sigma} \\ -\omega_r & -\left(\frac{1}{T_s}+\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & -\frac{\omega}{\sigma} & \frac{1}{T_r\sigma} & 0 & 0 & -\frac{I_{df0}}{\sigma} \\ -\frac{1}{T_s} & 0 & 0 & \omega_s & \frac{k_m}{L_s} & 0 & 0 \\ 0 & -\frac{1}{T_s} & -\omega_s & 0 & 0 & 0 & 0 \\ -\frac{k_m}{C_F} & 0 & 0 & 0 & 0 & \frac{1}{C_F} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{L_F} & -\frac{R_F}{L_F} & 0 \\ k_c I_{qf0} & -k_c I_{df0} & -k_c I_{qs0} & k_c I_{ds0} & 0 & 0 & 0 \end{vmatrix} \cdot$$

$$\cdot\begin{vmatrix} \Delta I_{ds}(s) \\ \Delta I_{qs}(s) \\ \Delta I_{df}(s) \\ \Delta I_{qf}(s) \\ \Delta U_F(s) \\ \Delta I_R(s) \\ \Delta\omega(s) \end{vmatrix} + \begin{vmatrix} 0 & I_{qsD} & \frac{U_{FD}}{L_{s\sigma}} \\ 0 & -I_{dsD} & 0 \\ 0 & I_{qfD} & \frac{U_{FD}}{L_s} \\ 0 & -I_{dfD} & 0 \\ 0 & 0 & -\frac{I_{dsD}}{C_F} \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} \Delta U_R(s) \\ \Delta\omega_r(s) \\ \Delta k_m(s) \end{vmatrix} \qquad (13)$$

where $k_c = -p^2 \cdot \dfrac{L_s}{J}$.

The system is therefore described by seven equations which can be written again in the known compact form:

$$s|Y(s)| = |A|\cdot|Y(s)| + |B|\cdot|U(s)|. \qquad (14)$$

where the input quantities are $\Delta U_R(s)$, $\Delta\omega(s)$ and $\Delta k_m(s)$.

3 Results and Conclusions

A Matlab program has been conceived with the help of the mathematical model detailed before; its results are presented further on.

In order to emphasize the filtration capacitor influence on stability, a numerical analysis of the problem has been performed for several values of the capacitor .

Figure 2 has been obtained by running the computation program.

A few values of the seven roots corresponding to four values of the filtration capacitor are presented further on in order to emphasize the emerging conclusions.

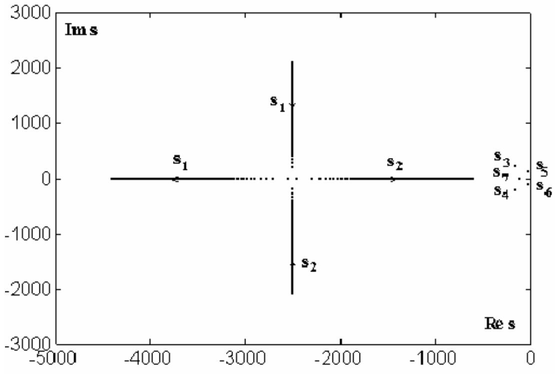

Fig. 2. Geometric locus of roots for C_F= 470 μF - 1,88 mF (step 5 μF).

Table 1. Roots corresponding to the filtration capacitor.

	C_F			
	470 μF	940μF	1,41 μF	1,88 mF
s_1	-2499,2 +2094,7i	-3465,2	-4144,4	-4394,9
s_2	-2499,2 -2094,7i	-1532,9	-853,6	-603,2
s_3	-155,5 +218,4i	-155,5 + 218,5i	-155,6 +218,6i	-155,5 +218,8i
s_4	-155,5 -218,4i	-155,5 -218,5i	-155,6 -218,6i	-155,5 -218,8i
s_5	-18,4 +119,2i	-18,4 +119,2i	-18,4 +119,2i	-18,5 +119,2i
s_6	-18,4 -119,2i	-18,4 -119,2i	-18,4 -119,2i	-18,5 -119,2i
s_7	-103,2	-103,2	-103,2	-103,2

Thus, while the roots s_3, s_4, s_5, s_6 and s_7 are almost "immobile", the roots s1 and s_2 displace (at the same time with the increase of the capacitor value) in accordance with the arrows. If its value does not exceed 850 μF the roots s1 and s_2 have constant real part, the system stability remaining practically unchanged. Over this value, the stability begin to decrease because of the fact that s_2 comes nearer the imaginary axis.

An increased value is imposed for C_F for having an as good as possible filtration of the rectified voltage. However, in the case of the studied converter, although there are reserved on the base board four seats for fixing the filtration capacitors, still only two capacitors of 470 μF have been connected in parallel (C_F=940 μF). One of the reasons for preferring this combination is (in accordance with the ones mentioned before) even the increased stability of the system for that case.

For studying the filtration inductivity influence the following results have been obtained (fig. 3 and table 2).

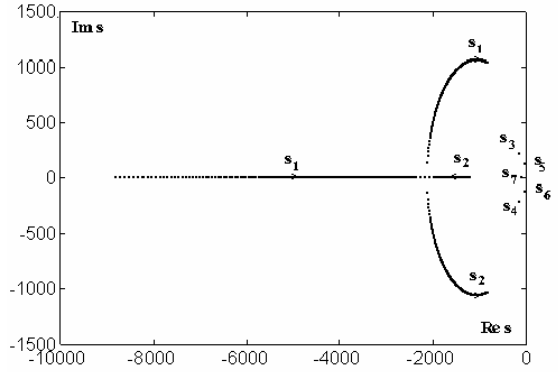

Fig. 3. Geometric locus of roots for L_F=0,1-0,6 mH (step 1 mH).

The studied system is always stable because all roots have negative real part.

In the previous representation the arrows show the sense for the filtration inductivity increase. In order to justify their senses further on we present the roots values for four particular values of the inductivity L_F.

Table 2. Roots corresponding to the inductivity L_F.

	L_F			
	0,1 mH	0,2 mH	0,3 mH	0,4 mH
s_1	-8789,7	-3465,2	-1665,8 + 876,0i	-1249,1 +1047,3i
s_2	-1208,4	-1532,9	-1665,8 - 876,0i	1249,1 -1047,3i
s_3	-155,5 + 218,5i	-155,5 + 218,5i	-155,5 + 218,5i	-155,5 + 218,5i
s_4	-155,5 - 218,5i	-155,5 - 218,5i	-155,5 - 218,5i	-155,5 - 218,5i
s_5	-18,4 + 119,2i	-18,4 + 119,2i	-18,4 + 119,2i	-18,4 + 119,2i
s_6	-18,4 - 119,2i	-18,4 - 119,2i	-18,4 - 119,2i	-18,4 - 119,2i
s_7	-103,2	-103,2	-103,2	-103,2

As one can notice from the previous table, the characteristic equation have five immobile roots in the complex plan (s_3, s_4, s_5, s_6 and s_7). The other two, depending on the filtration inductivity value, displace in accordance with the arrows indication (they come nearer the imaginary axis at the same time with its increase).

The emerging conclusion is the same as that one deduced in the case C_F=0, therefore when the filter inductivity value increases the system stability decreases.

The modulation factor influence is analyzed in the same way.

The following results have been obtained by doing as in the previous cases (fig. 4 and table 3).

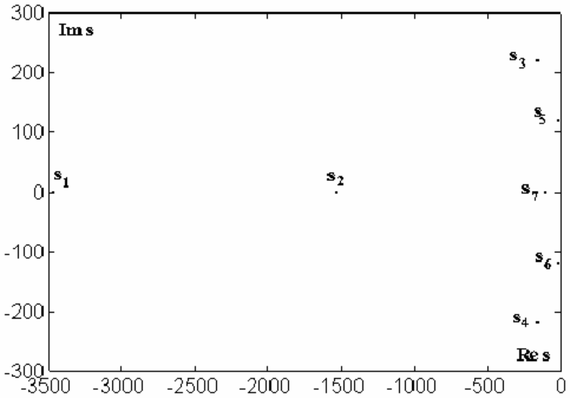

Fig. 4. Geometric locus of roots for k_m= 0-0,5 (step 0,01).

Table 3. Roots corresponding to k_m.

	k_m			
	0	0,2	0,4	0,5
s_1	-3464,8	-3465,0	-3465,5	-3465,9
s_2	-1535,2	-1534,3	-1531,4	-1529,3
s_3	-154,6 +218,4i	-155,0 +218,4i	-156,1 +218,5i	-157,0 +218,6i
s_4	-154,6 -218,4i	-155,0 -218,4i	-156,1 - 218,5i	-157,0 - 218,6i
s_5	-18,5 +119,5i	-18,5 +119,4i	-18,4 + 119,1i	-18,4 +118,9i
s_6	-18,5 -119,5i	-18,5 -119,4i	-18,4 -119,1i	-18,4 -118,9i
s_7	-103,1	-103,1	-103,2	-103,3

As it can be noticed, the modulation factor influence on stability is very small.

However, owing to the fact that at the same time with its increase the root s_2 comes nearer the imaginary axis (while the other roots are almost "fixed"), we can say that the studied system stability, in the mentioned situation, tends to decrease.

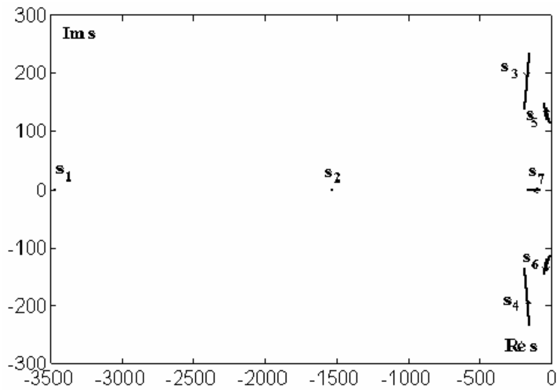

Fig. 5. Geometric locus of roots for $R_r^{/}$=4-10 Ω.

The arrows show the sense of the roots displacement in the case of the rotor winding increase, fact confirmed by data from table 4, too.

Table 4. Roots corresponding to $R_r^{/}$.

	$R_r^{/}$			
	3,5 Ω	5,5 Ω	7,5 Ω	8,5 Ω
s_1	-3464,8	-3464,8	-3464,8	-3464,8
s_2	-1535,2	-1535,2	-1535,2	-1535,2
s_3	-145,9 + 238,2i	-154,6 + 218,4i	-164,2 + 191,2i	-179,9 + 151,2i
s_4	-145,9 -238,2i	-154,6 -218,4i	-164,2 -191,2i	-179,9 -151,2i
s_5	-9,3 +114,9i	-18,5 +119,5i	-29,0 +127,5i	-38,7 +141,4i
s_6	-9,3 -114,9i	-18,5 -119,5i	-29,0 -127,5i	-38,7 -141,4i
s_7	-66,5	-103,1	-135,1	-156,4

Observation

The induction machine parameters influences on stability when considering the converter parameters too, can also be studied with the help of the relations (13). The conclusions are identical with the ones obtained by using the method shown in references.

The roots geometric locus when modifying the rotor winding resistance is depicted further on for exemplification.

The conclusion emerging from the analysis of these results is identical with the one from references, therefore at the same time with the increase of the rotor winding resistance the studied system stability also increases.

Observation

The characteristics depicted in the previous figures have been obtained for the case when: C_F=940 μF, L_F=0,3mH, f=50 Hz, U_{F0}=522 V, I_{ds0}=1,84 A, I_{qs0}=0 A (values measured on the converter CEGELEC VNTV 4004).

It has also been considered that the machine operates without load. This means that it can be considered that (by neglecting the rotor current components over the stator current components) I_{df0}=1,84 A, I_{qf0}=0 A.

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