VLSI: An Investigation into the Effect of Non-Uniform Distribution of Current on Crosstalk Noise

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Abstract - On-chip interconnect has become a critical dominant factor affecting the performance and reliability of high performance integrated circuits (IC) such as system-on-a-chip (SoC). As the chip dimensions increase with chip complexity, interconnects tend to get longer. The longer on-chip interconnects coupled with a decrease in wire width and wire separation, inductive effects, and more specifically, mutual inductance coupling between neighbouring wires become non-negligible. An analytical model to examine the effect of non-uniform distribution of current on the mutual inductance and capacitance between on-chip interconnects, and the subsequent effect on the noise peak voltage of the victim line, is presented.

Key-Words: - Interconnect, crosstalk, noise, distributed RLC

1 Introduction
Cross-coupling between on-chip interconnect lines may yield crosstalk noise, which can be defined as the noise voltage on signal lines caused by a change of state in neighbouring lines. The line affecting its neighbour by its switching state is often referred to as the aggressor line, while the affected one as the victim line. Crosstalk noise can lead to performance degradation and functional failure depending on the state of the conduction wire and its adjacent neighbours and of equal importance, i.e. depending on the width, peak amplitude and frequency of the generated parasitic noise [1]. Furthermore, the impact of parasitic coupling on interconnect crosstalk is not only data-dependent but is also dependent on the specific combination of each individual wire and their adjacent neighbours [2]. Therefore, the worst-case on-chip performance degradation often occurs when crosstalk between interconnects due to capacitive and inductive parasitic coupling is considered [3][4][5]. Development toward DSM technology requires that such consideration has to be taken into account to accurately and efficiently estimate interconnect crosstalk-induced noise.

Moving toward DSM integration, coupled with the use of high clock frequencies, has resulted in the average length of an interconnect line being often resistive compared to its driver resistance; therefore requiring that the inductive and distributed nature of on-chip interconnect has to be properly modelled. In order to account for the signal-distorting effects the use of high operation frequency, present interconnects require to be modelled as distributed RLC lines. A four-poles- based model for distributed RLC lines was presented in [6]. Although the accuracy has been improved in comparison with that of the two poles expression [7], no closed form solution was developed. In reference [4], an RLC model based on Fourier analysis for delay and crosstalk prediction was proposed, with higher accuracy is obtained by including more harmonics in the model. Several works have modelled capacitive crosstalk using distributed networks [8][9]. However, all above works assume uniform distribution of current across the interconnect line. Advances in DSM technology have resulted in that longer interconnect wires of small cross section are packed closer together. This may give rise to the possibility that different parts of an interconnect line − i.e. the victim line − may experience a varying degree of coupling effect from other interconnects − i.e. aggressor lines − with the consequent variation in the induced-current across the victim line. The work presented in this paper is based on the model developed in [10] to examine...
the effect of non-uniform distribution of current on the estimation of crosstalk noise.

This paper is organised as follows: Section 2 presents the mathematical modelling of a distributed RLC, including time-domain expressions of the line voltage response under finite and infinite ramp inputs. Section 3 introduces the distributed-coupled RLC transmission line model for the crosstalk noise estimation. Section 4 presents the simulation results. Conclusions are given in Section 5.

2 Mathematical Modelling

A distributed RLC interconnect of length \( d \), with driver resistance and load capacitance is depicted in Fig.1. The voltage and current expressions for the interconnect of Fig.1 are given in a matrix form as follows:

\[
\begin{bmatrix}
V_x \\
I_x
\end{bmatrix} = 
\begin{bmatrix}
\cosh(\text{dd}) & Z_c \sinh(\text{dd}) \\
\frac{1}{Z_c} \sinh(\text{dd}) & \cosh(\text{dd})
\end{bmatrix}
\begin{bmatrix}
V_o \\
I_o
\end{bmatrix}
\]

(1)

with the characteristic impedance \( Z_C \) of the distributed line is given by:

\[
Z_C(s) = \frac{l}{c} \sqrt{\frac{1}{s l + r}}
\]

(3)

where \( r, l \) and \( c \) are respectively the per unit length resistance, inductance and capacitance of the line.

Taking into account the boundary conditions at the end of the line, terminated with load capacitance \( C_L \), the transfer function of the RLC line \( H(d,s) \) is given by:

\[
H(d,s) = \frac{2}{[bcRL + Z_C L + \frac{R}{C}] s^2 + [bcRL + Z_C L + \frac{R}{C}] A^2 d} \]

(4)

Applying Maclaurin series around \( s=0 \), a fourth order approximation of \( H(d,s) \) is given as follows:

\[
H_p(s) = \frac{1}{1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4}
\]

(5)

where the \( b_i \)'s coefficients are as in [10].

With the infinite ramp-response \( v_{\text{inf}} \):

\[
v_{\text{inf}}(t) = \frac{V_{DD}}{T_R} \left[-h_1 t + \sum_{i=1}^{4} d_i e^{s_i t}\right] u(t)
\]

(6)

where \( T_R \) is the finite ramp rise time.

The time-domain response of a finite ramp \( v_{\text{fin}}(t) \) is

\[
v_{\text{fin}}(t) = v_{\text{inf}}(t) - v_{\text{inf}}(t-T_R)
\]

\[
= \frac{V_{DD}}{T_R} \left[T_R + \sum_{i=1}^{4} d_i \left(e^{s_i t} - e^{s_i(t-T_R)}\right)\right] u(t)
\]

(7)

where \( d_i \) and poles \( s_i \) represent the residues and poles of the function, respectively.

3 Worst-Case Crosstalk Noise

Depending on the layout of a highly complex integrated circuit, different sections of an interconnect line may experience different cross-coupling effects. Consequently, this may yield variation in, for instance, the amplitude of the current across the victim line. This may give rise to different coupling inductive and capacitive effects on different parts (sections) of an interconnect line, with the subsequent variation in the self and mutual inductance and capacitance of the line. The work in this paper assumes that the victim line is split into \( n \) sections \((0 < n)\), with only a subset of \( n \) having experiencing variations in the current.

The relationship between the coupled lines is given by [2]:

\[
\frac{\partial^2 v^+(x,t)}{\partial x^2} = \lambda^2 v^+(x,s)
\]

(8)
\[
\frac{\partial^2 V}{\partial x^2}(x,t) = \lambda^2 V(x,s) \tag{9}
\]

where \( V^+ \) and \( V^- \) represent the voltage at the end of the line in common and differential mode, respectively.

\[
\lambda_1 = s \sqrt{c(l + l_m^-)} \left( 1 + \frac{r}{s(l + l_m^-)} \right) \tag{10}
\]

\[
\lambda_2 = s \sqrt{(c + 2c_m^-)(l - l_m^-)} \left( 1 + \frac{r}{s(l - l_m^-)} \right) \tag{11}
\]

where \( l_m^\sim = Kl_m\lvert_{K < 2} \) and \( c_m^\sim = Qc_m\lvert_{Q < 2} \).

\( K \) and \( Q \) are multiplicative constants representing a small fractional change in the mutual inductance and capacitance.

Non-uniform distribution current may introduce a fractional increase or decrease in the nominal value - i.e. when the current is evenly distributed - of the mutual inductance and capacitance as well as the self-inductance and self-capacitance of the interconnect line. In this paper, the fractional change in current is considered to be step-wise increasing from the nominal value. For an interconnect line represented by \( n \) sections, where the length of each section is defined as the unit length:

\[
l_m^- = l_m^{-1} + l_m^{-2} + \ldots + l_m^{-n-1} + l_m^{-n} \tag{12}
\]

and

\[
c_m^- = c_m^{-1} + c_m^{-2} + \ldots + c_m^{-n-1} + c_m^{-n} \tag{13}
\]

with

\[
l_m^{-j} = K^i l_m\lvert_{K < cK}, \quad c_m^{-j} = q^i l_m\lvert_{Q < cQ} \tag{14}
\]

\( I \): integer \((i > 0)\).

To account for the non-uniform distribution of current, the transfer function of the distributed RLC line in (5) can be represented as follows:

\[
H_p(s) = \frac{1}{1 + b_T^s s + b_T^s s^2 + b_T^s s^3 + b_T^s s^4} \tag{15}
\]

with \( b_T^s = b_1 + b_\sim \lvert_{i > 0} \)

where \( b_i \)s and \( b_\sim \)s are the nominal coefficients of \( H_p \), as in [10], and the fractional change in \( b_i \)s as a result of non-uniform distribution of current, respectively. In the case of the former, the overall mutual inductance \( l_m \) and mutual capacitance \( c_m \) of the line can be represented as:

\[
l_m = \sum_{i=1}^{i=n} l_m^i, \quad c_m = \sum_{i=1}^{i=n} c_m^i \tag{16}
\]

or

\[
l_m = \sum_{i=1}^{i=n} i^i l_m^i, \quad c_m = \sum_{i=1}^{i=n} i^i c_m^i \tag{17}
\]

The overall inductance and capacitance of the line can be represented as:

\[
L = \sum_{i=1}^{i=n} l^i + \sum_{i=1}^{i=n} i^i l_m^i, \quad C = \sum_{i=1}^{i=n} i^i c_m^i + \sum_{i=1}^{i=n} i^i c_m^i \tag{18}
\]

For the fractional change in the mutual inductance and mutual capacitance as a result of variation in the current across the interconnect line.

\[
l_m^\sim = \sum_{i=1}^{i=n} l_m^\sim, \quad c_m^\sim = \sum_{i=1}^{i=n} c_m^\sim \tag{19}
\]

Under worst-case noise, the use of equations (7) and (9) to determine the coupled lines voltage responses require the following adjustments

\[
C_{eff} = c + 2c_m^\sim, \quad l_{eff} = l - l_m^\sim \tag{20}
\]

where \( C_{eff} \) and \( l_{eff} \) represent the effective capacitance and effective inductance, respectively. The respective infinite and finite time-domain responses of the aggressor line under ramp input could be determined based on the expression of the single line as follows:

\[
v_{Ainf}(d, t) = v_{inf}(d(l_{eff}, C_{eff}), t) \tag{21}
\]

\[
v_{Afin}(d, t) = v_{fin}(d(l_{eff}, C_{eff}), t) \tag{22}
\]

Equally, under this worst-case scenario, the time-domain response of the noise peak voltage at end of the victim line is given by:

\[
v_v(d, t) = \frac{1}{2}(v_{fin}^+(d, t) - v_{fin}^-(d, t)) \tag{23}
\]

### 4 Simulation Results

To assess the effect of non-uniform distribution of current on the crosstalk noise induced by the aggressor line on the victim line, a typical interconnect of length 2000 is used. The per length mutual inductance, resistance and capacitance are 0.246pf, 0.0015Ω and 0.000176pf respectively. The source resistance and load capacitance are 100 Ω and 0.01 pf, respectively. For comparison purposes, we used the results from obtained from Spice to examine the extent to which variation in the current across an interconnect line influence the crosstalk noise peak-voltage at the end of the victim line. Fig.3 shows a noticeable increase in the crosstalk noise voltage when the change in he mutual
inductance – caused by variations in the current – exceeds 5% of the nominal value.

Fig. 3 (a) Aggressor line Transient response, (b) Noise peak estimation for varying $c_m$.

5 Conclusion
For DSM designs, cross-coupling effects can lead to significant crosstalk problems that may affect the signal integrity and, consequently, the functionality and reliability of highly dense integrated circuits. As on-chip interconnects are packed closer together, coupled with the increase in the operating frequency, different sections of an interconnect line may experience varying coupling effects. This may give rise to variation in the current across the interconnect line, with subsequent fluctuations in its self and mutual inductance and capacitance. Therefore, ignoring non-uniform distribution of current across an interconnect line may yield an underestimation of the noise voltage and the time delay of on-chip interconnects.

References: