

# Optimal Bidding Curves for an Energy Service Provider in Iran Electricity Market

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*Abstract--* During the last few years in the competitive energy market, participants have used stochastic programming based asset allocation models in their processes. In the demand side of electricity market, the customers' behavior will be stochastic, so the actual consumption load is random. Therefore we can suppose that all decisions about load price and volume hold a certain amount of uncertainty. In this paper we present an approach for solving a stochastic linear programming model and then drawing bidding curves for an energy service provider. This approach is independent to large amount of historical information about the most relevant market variables and variables related to bidding or behavior of the other market participants, where an optimization process finds the optimal bid curve for the given prices and a single hour, by using the output of the scenario generation algorithm and the GAMS software. The model is illustrated using a case study with data from Iran electricity market and the results show that the proposed algorithm is efficient and satisfactory bidding curves are obtained.

*Index Terms--* Demand side bidding, scenario tree generation, stochastic optimization

## 1. INTRODUCTION

The electric power industry in I.R.of Iran has undergone revolutionary changes in recent years. The essence of these changes is the deregulation of the industry and introduction of competition both in generation and distribution. Instead of centralized resource allocation and operation, electric power is now sold by generation suppliers and purchased by energy service providers to meet the forecasted needs of their customers, all or partially through competition.

The Iran electricity market has 40 distribution companies to serve a population of about 70 millions. Distribution companies provide energy services to their customers as a large retailer. They submit price insensitive bids to the day-ahead market. This is due to the fact that consumers are not exposed to short-term price fluctuations, with monthly (or more often) metering being the norm. The bids are more or less close to expected demand, which is estimated by

statistical techniques. They get their margins from buying wholesale and selling to end users. Many challenging issues arise under the new competitive market environment. Among them, bidding in different energy and service markets and optimization of bidding strategies have become important daily tasks for power service providers.

Due to the fact that demand side markets are generally less mature and usually implemented much later than the supply market. In Iran, purchase allocation and bidding approaches for energy service providers<sup>1</sup> are much less studied and there is a little historical information or data. In this manner, in other countries with the restructuring of the electricity market, researchers have studied many different approaches for topics such as, bidding history, bidding strategies, optimal bidding strategy or development of last techniques, but the main

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<sup>1</sup> Energy Service Provider (ESP) such as distribution company or retailer

drawback of these studies is the energy service providers' viewpoint.

Though, constructing bidding curves for a retailer in the Norwegian Electricity Market has been studied by Fleten and Pettersen [1], with the greatest nicety. They propose a stochastic linear programming model for constructing piecewise-linear bidding curves to be submitted to Nord Pool. Stochastic programs need to be solved with discrete distributions and they are created by scenario generation. The simplest idea for generating scenario is referred in [2], this idea is to use past data that is in comparable circumstances and assign those equal probabilities. This can be done by just using the raw data or through procedures such as vector autoregressive modeling or bootstrapping which samples from the past data, in contrast to previous studies, our approach uses expert's forecasts and viewpoint. So when there is no reliable data, one can use her or his market view (examples include the 1981 Markowitz and Perold, and the 1988 Shapiro papers) or governmental regulations. Abaffy et al (2000) and Dupařova et al (2000) survey scenario estimation and aggregation methods that represent a larger number of scenarios by a smaller number [2].

This research is motivated by the needs of an energy service provider in the Iran electricity market, such as Distributions Company or retailer. But we believe that the bidding model also can be applied by power marketers elsewhere, given a pool accepting piecewise linear bids and where there is a short-term balancing market whose prices affect the retailer costs whenever he experiences an imbalance between the load planned day-ahead and the realized load.

In this approach we consider the case of price-sensitive end users and ESPs usually submit price-insensitive bids to the spot market. This is due to the fact that consumers are not exposed to short-term price fluctuations, with monthly (or less often) metering being the norm. The bids are more or less close to expected demand, which is estimated by statistical techniques.

The paper is organized as follows. Section II explains the market rules that are relevant for demand-side bidding in the Iranian daily market. Section III shows the overall optimization procedure to construct the optimal bidding curve for electric daily markets. Section IV shows a realistic case study based on the Iranian daily market. And Section V presents several conclusions derived from this work and future research.

## 2. MARKET RULES

There are two important markets for physical exchange of electricity in Iran: the day-ahead market and the regulating market organized by the independent system operator. In the day-ahead market, producers and energy service providers or retailers submit price-quantity bids for buying and selling electricity every day before noon. Market clearing prices are determined through auction trade for each delivery hour. The trading horizon is 12- 36 hours ahead – the next day 24 hour period. The System price and the area prices are calculated after all participants bids have been received before gate closure at 12:00.

The regulating market is used by the system operator to ensure real-time balance between supply and demand. Only producers with an ability to ramp up or down significantly on 15-min notice are allowed to participate. Whenever there is a load greater than was committed in the day-ahead market, there is a need for up regulation and vice versa. The independent system operator has collected bids for such up and down regulations for each participant and chooses to use the cheapest feasible source for such ramping. In the other hand, the system operator wants the day-ahead market to reflect the physical conditions. In short, they do not like demand-side speculation in the regulating market. Therefore, they are inclined to take measures toward ESPs that are suspected of bidding too high or too low volumes in the day-ahead market on purpose. So the specification of the volume deviation penalty function is necessarily ad hoc. If the realized load is larger or lower than one standard deviation from its expected value, market operator starts penalizing. Later in the full model, this risk factor will be considered.

## 3. METHODOLOGY

In our method an optimization process finds the optimal bid points for the hourly forecasted demand and several scenarios of market clearing price. It has a modular structure to allow for maximum flexibility of the model. There are three main modules, as depicted in Fig. 1.

- Optimum decision model and constraints;
- Model of scenario generator;
- Stochastic programming.

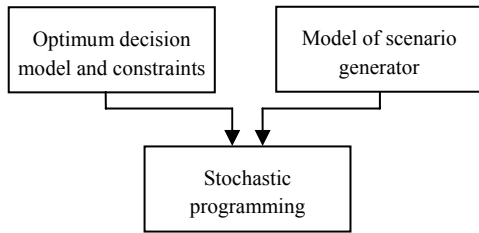


Fig. 1: Overall optimization process algorithm description.

### 3.1 Optimum decision model and constraints

The optimum decision model together with the constraints constitutes the core of the problem that has to be solved and varies with respect to the specific characteristics of each individual application.

The objective function in the implemented model is the ESP's Max profit. For, the profit is bounded, the loss is minimized. Therefore the main part of the objective function contains the probable losses, and the market rules construct the objective functions constraints.

We assume that the bidding actions of this retailer do not influence prices in the electricity markets, i.e., the retailer are a price taker operating in a competitive market.

We define  $\beta$  and  $\pi$  to be the regulating market price and the day-ahead ("spot") price, respectively. Based on the brief explanation of the regulating market design given in the introduction, we see that during up regulation  $\beta - \pi > 0$ , during down regulation  $\beta - \pi < 0$ , and when there is no regulation,  $\beta - \pi = 0$ . Then the retailer's cost of purchasing electricity may be written as

$$C = \pi y + (\xi - y)\beta = \xi\beta - y\delta \tag{1}$$

where  $\xi$  is the load,  $y$  is the volume knocked down in the day-ahead market and  $\delta = \beta - \pi$  is the difference between the regulating price and the spot price. Also observe that the day-ahead volume  $y$  is the only variable that is controlled by the retailer and the parameters  $\xi$ ,  $\beta$ , and  $\pi$  are exogenous.

The first term  $\xi\beta$  does not include any decision variables. Hence, the ESP or retailer should seek to  $\min E\{-y\delta\}$  which is equivalent to minimizing the expected is regulating market loss  $E\{(\xi - y)\delta\}$ . This relation denotes the money lost by purchasing the

excess demand in the regulating market instead of in the spot market. Note that regulating market loss may well be negative, implying that the retailer may make extra profits in the regulating market.

Since the expected cost turns out to be negative in the case study, we have chosen to turn this around, so that the model's objective is to maximize expected regulating market profit or  $\max E\{y\delta\}$ . This expression is the first and main part of the objective function in the optimization model.

The second part of objective function is an index called volume deviation risk. Since all derivatives are quoted with respect to the day-ahead price, the system operator wants the day-ahead market to reflect the physical conditions. In short, they do not like demand-side speculation in the regulating market. Therefore, they are inclined to take measures toward retailers that are suspected of bidding too high or too low volumes in the day-ahead market on purpose.

The most important items in objective function and constrains are risk, load and bid points, that we consider in the model.

**Risk:** Modeling of risk is dependent on the views of the decision maker. Decision makers perceive risk as the potential for downside losses. A way of accommodating this in a model is to have target levels for financial performance at different stages. The extent to which these targets are not met is called target shortfall [3], one would progressively penalize target shortfalls in the objective, e.g. in the form of a piecewise linear cost function as shown in Fig.2. Let we define variables  $\omega_1^+, \omega_2^+, \dots, \omega_m^+$  and  $\omega_1^-, \omega_2^-, \dots, \omega_m^-$ . When the ESP is up regulated,  $\sum_{m \in M} \omega_m^+ = \xi - y$  is positive, and when he is down regulated,  $\sum_{m \in M} \omega_m^- = y - \xi$  is positive. Now, we let and denote the marginal cost of piece on the volume deviation risk function for positive and negative deviations, respectively.

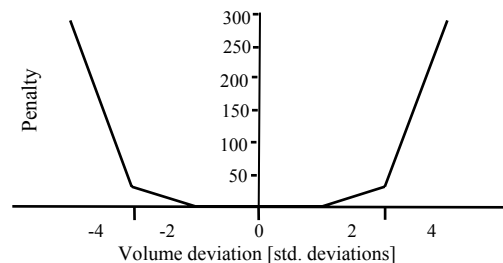


Fig. 2: Volume deviation penalty function

Then, the volume deviations more the higher we get, are penalized in the objective function by adding the term

$$-V \sum_{m \in M} (T_m^+ \omega_m^+ + T_m^- \omega_m^-) \quad (2)$$

where  $V$  quantifies the ESP's aversion to the volume deviation risk, relative to other objective terms.

**Load:** In this study, the customers' load  $\xi$  is considered price flexible. We assume that for some price  $\pi_0$ , the customers have an expected load  $E[\xi] = \xi_0$ . Then the expected load at price  $\pi$  is derived by (3) where  $\eta$  is the price elasticity.

$$E[\xi] = \xi_0 \left( \frac{\pi}{\pi_0} \right)^\eta \quad (3)$$

It means that, when the price of the load falls, the quantity consumers demand typically raises. Price elasticity is the ratio of the relative change in quantity demanded to the relative change in price and for most goods this ratio is negative. Fig.3. shows some price-load curves with varying price elasticity of demand.

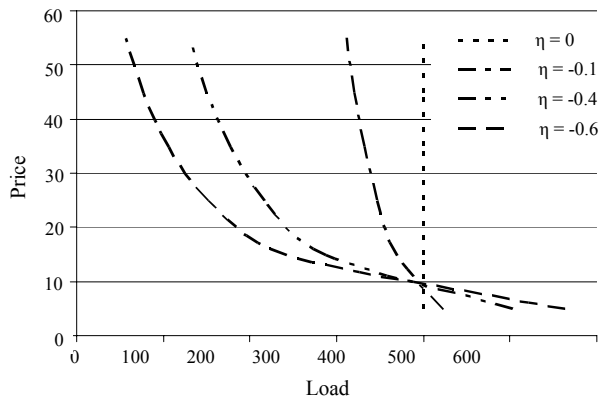


Fig.3: Price-Load curves with varying price elasticity of demand

**Bid points:** To obtain a piecewise-linear strictly decreasing curve for the day-ahead auction, which is consistent with the bidding rules on the market, the ESP submits  $n$  price-volume pairs  $(P_0, x_0), (P_1, x_1), \dots, (P_n, x_n)$ , where  $P_0 \leq P_1 \leq \dots \leq P_n$  and  $x_0 \geq x_1 \geq \dots \geq x_n$  to the pool. A linear interpolation between the pairs  $(P_i, x_i)$  and  $(P_{i+1}, x_{i+1}), i = 1, \dots, n-1$  gives the resulting  $n-1$  line segments that decide at what price  $\pi$  and volume  $y$  the retailer is dispatched.

Fig. 4 shows a bidding curve with five line segments. The ESP submits the points  $(P_i, x_i), i = 0, \dots, 5$  to the pool, and the bidding curve emerges from a linear interpolation between those points.

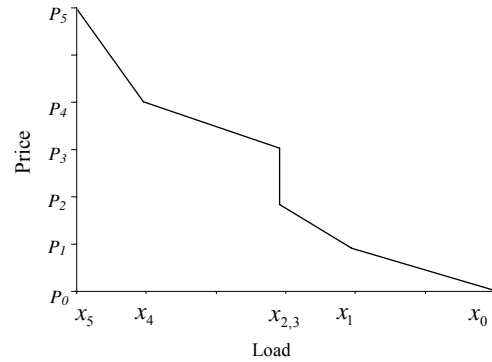


Fig. 4: An example of a bidding curve with five line segments

**Objective function:** Now using the maximum expected regulating market profit and the volume deviation approach to risk, the objective function looks as follows.

$$\max \left\{ y\delta - V \sum_{m \in M} (T_m^+ \omega_m^+ + T_m^- \omega_m^-) \right\} \quad (4)$$

Subject to

$$y = \left( 1 - \frac{\pi}{P_{i+1} - P_i} + \frac{P_i}{P_{i+1} - P_i} \right) x_i + \left( \frac{\pi}{P_{i+1} - P_i} - \frac{P_i}{P_{i+1} - P_i} \right) x_{i+1} \quad (5)$$

The problem in the objective function is to select the volumes  $x_i$  corresponding to  $P_i$ , we consider the constraint by (5) to make the relation between  $P_i$  and  $x_i$  linear in  $x_i [1]$ .

### 3.2 Model of scenario generator

The second element is the model of randomness. The correct description of the future is the key to modeling and decision-making under uncertainty. Scenario analysis is an effective tool to model the dynamics of the uncertainty.

Once the distribution is established, different scenarios that follow the underlying probability distribution are used to represent the uncertainty. Thus, by executing a scenario generator procedure, the random parameters of the decision model can be instantiated [4]. We generated scenarios for this

model by using the method described in [5]. Short summary of this process is stated below.

The algorithm produces one period scenario trees with specified correlations matrix and the first four moments (mean, variance, skewness and kurtosis) of the marginal distributions. It achieves this by using two transformations, one correcting the moments and the other correcting the correlations. Since each transformation distorts the results of the other one, they are repeated iteratively, alternating between the two. The algorithm stops when the error of both the moments and correlations is below a given threshold, or when a maximal number of iterations is reached.

Correlations are corrected using a variant of the standard method for generating correlated normal variates, based on Cholesky decomposition of the correlation matrix. Moments of the marginal distributions are corrected one at a time, using a cubic transformation

$$y = a + bx + cx^2 + dx^3 \tag{6}$$

Finding the parameters  $a, b, c, d$  is the most challenging part of the algorithm, as it requires solving a system of four implicit nonlinear equations in four unknowns. In this approach, one needs the historical data to determine the target moments. Fig.5 shows the moments matching cycle.

In [1], the authors have generated scenarios for spot price  $\pi$ , for load prediction error  $\varepsilon$ , and for the difference between the regulating market price and the spot price  $\delta$ . For  $\pi$  and  $\delta$ , they have used hourly spot prices and regulating prices for the Trondheim region in the period 10 March 1997–16 December 2003 and removed the prices in weekends and holidays. Ultimately, they have generated scenarios by using more than 30 000 entries for  $\pi$ ,  $\delta$  and  $\varepsilon$  and the method described in [5]. They have calculated the first four moments and correlation by using historical data.

The main idea of our approach is the assumption that, the ESP hasn't a large amount of information about the market variables and prices. While this assumption satisfies in Iran electricity market. Therefore, we let the ESP as a decision maker, specify his market expectations by any statistical properties that are considered relevant for the problem to be solved, and construct the scenario tree so that these statistical properties are preserved.

Expressing market expectations can be done in many ways. We have chosen to let the ESP or

decision maker supply percentiles for the marginal cumulative probability distribution functions for all uncertain variables such as  $\pi$  and  $\delta$ , see Fig. 6. An approximating cumulative distribution function is fitted to these percentiles. The properties that are listed in Table 1 are calculated from the function that is fitted to the percentiles.

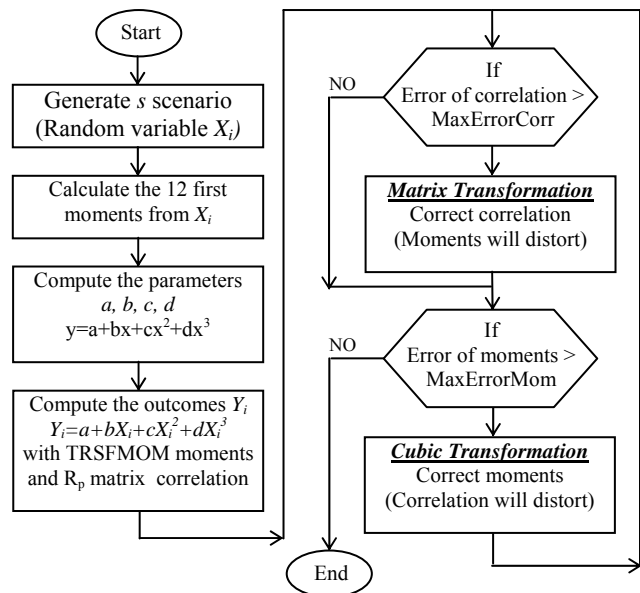


Fig. 5: Flowchart of moments matching cycle.

Table 1 Percentiles of the Marginal Cumulative Distributions

	0%	5%	25%	50%	75%	95%	100%
Market price (Rls.)	150	250	360	390	430	580	720

The approximating cumulative distribution functions are found using a NAG (Numerical Algorithms Group) C Library routine for interpolating data. This method does not guarantee that the second derivative changes sign only once, in the case of Fig. 6 causing a somewhat peculiar form near the top of the distribution. However, the resulting function is monotonic, so we are guaranteed that the curve will have the properties of a cumulative distribution function, and that the user specified percentiles are fit exactly (including the 0% and 100% points).

Probability distribution functions (pdf) are fitted to the percentiles, and then from the pdf, we can calculate all marginal moments.[6]

### 3.3 Stochastic programming

As defined by the stochastic programming community - COSP at [7] - Stochastic programming is a framework for modeling optimization problems that involves uncertainty. Stochastic programs need to be solved with discrete distributions. Usually, we are faced with either continuous distributions or data. Hence, we need to pass from the continuous distributions or the data to a discrete distribution suitable for calculations.

More formally, stochastic programming is a branch of operations research that tries to suggest an approach to deal with uncertainty. Instead of suggesting an objective function such as  $f(x)$  (in linear programming  $xc$ ) in which the decision variable  $x$  is considered to have only one realization as part of the objective function, the stochastic programming approach defines a stochastic variable  $\xi \in \Omega$  and a new objective function  $f(x, \xi)$ . Therefore, the new objective function value is dependent on a different realization of  $\xi$  and therefore includes the effect of a stochastic process when evaluating the decision at the variable  $x$ .

The purpose of a scenario generator is to discretize the distribution capture of all the various possible values of  $\xi$  and introduce uncertainty into the model. The output of the scenario generation is then used numerous times as the input for the optimization model. Fig.7 indicates the general workflow of this process.

As indicated, we have generated scenarios to solve a stochastic programming. Therefore, we may change relations (3), (4) and (5) by including the subscripts  $s$  to point out that  $y$  and  $\pi$  are scenario dependent.

$$\xi_s = \xi_0 \left( \frac{\pi_s}{\pi_0} \right)^\eta + \varepsilon_s \quad (7)$$

$$\max \sum_{s \in S} \rho_s \left\{ y_s \delta_s - V \sum_{m \in M} (T_m^+ \omega_{ms}^+ + T_m^- \omega_{ms}^-) \right\} \quad (8)$$

subject to

$$y_s = \left( 1 - \frac{\pi_s}{P_{i(s)+1} - P_{i(s)}} + \frac{P_{i(s)}}{P_{i(s)+1} - P_{i(s)}} \right) x_{i(s)} \quad (9)$$

$$+ \left( \frac{\pi_s}{P_{i(s)+1} - P_{i(s)}} - \frac{P_{i(s)}}{P_{i(s)+1} - P_{i(s)}} \right) x_{i(s)+1}$$

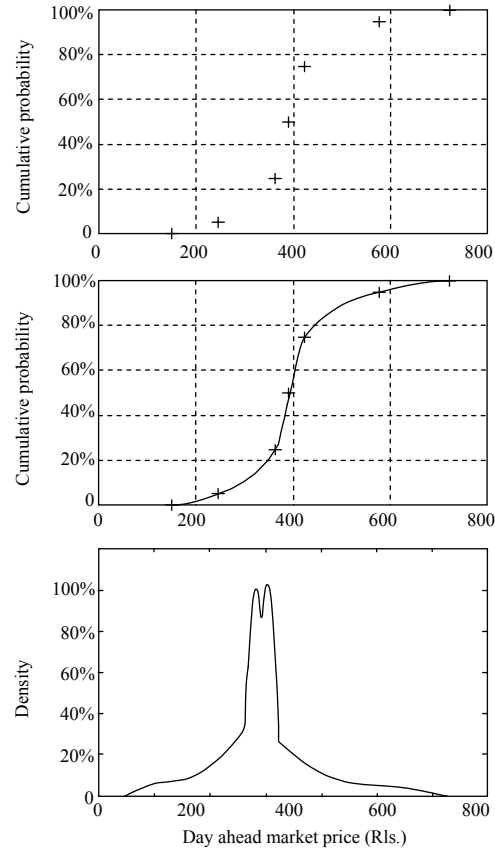


Fig.6: (i) The ESP percentiles for day ahead market price. (ii) A cumulative distribution function is fitted to the percentiles. (iii) The probability distribution function for day ahead market price.

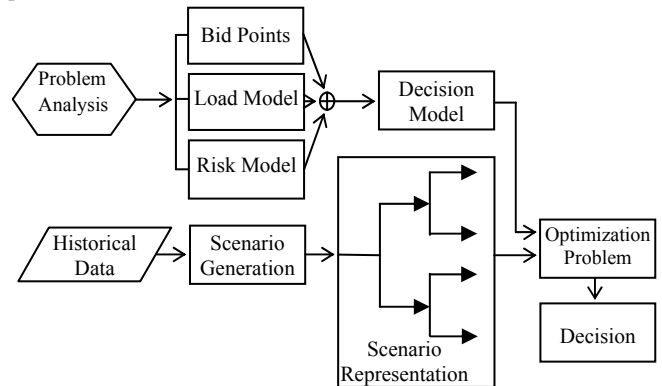


Fig. 7: Problem solving steps

## 4. CASE STUDY

The case study and numerical simulation of construct bidding curves is performed based on the actual data of the Iran power market. Since this market no longer exists, the historical data aren't still enough for analysis. Therefore we do as follow:

Step 1: Expressing our market expectations for  $\pi$  and  $\delta$ .

We have supplied our percentiles of spot price and the difference between the regulating market price and the spot price for the TSW region<sup>1</sup>, based on our experiences and prospect for the future. Then we have fitted an approximated cumulative distribution to the percentiles by using a NAG C library routine and derive the marginal distributions, as shown in table 2 and Fig. 8.

Table 2 Percentiles of the Marginal Cumulative Distributions of  $\pi$  and  $\delta$

	0%	5%	25%	50%	75%	95%	100%
$\pi$	450	480	520	580	650	740	800
$\delta$	-300	-250	0	85	150	250	350

Step 2: Determining the target moments and correlation

For load prediction error, we have used measured and estimated load for the TSW region for every hour during the period 20 March–21 September 2007. We have subtracted estimated load from measured load and obtained 4440 entries for  $\varepsilon$ . [8]

Given the fitted cumulative distribution and historical data, we calculate the first four moments into account. Table 3 shows statistical properties of random variables  $\pi$ ,  $\delta$  and  $\varepsilon$ .

Thus, we have specified the correlation between all stochastic variables. The correlation between  $\pi$  and  $\delta$  was estimated to be -0.45. Apart from that, the correlations were all rather low. We have generated scenarios with varying correlation and compared results. Fig 9 shows sets of scenario for varying correlations. Increase in correlation caused growth of dispersion of scenarios.

Table3 Statistical Properties Derived from the Marginal Distributions in Figures 8

	Expected value (%)	Standard deviation (%)	Skewness	Kurtosis
$\pi$	591.7	82.4	4.9	2.39
$\delta$	76.1	133.8	-0.75	2.93
$\varepsilon$	8.09	15.7	-0.74	2.97

<sup>1</sup> Tehran South West Power Distribution (TSWPD), in charge of electric power distribution has started its activities since 1997 having a commission to provide energy and desirable services. TSWPD undertakes a considerable responsibility to supply electricity to 708834 customers in an area of 855 km<sup>2</sup> situated in south western part of the capital city of Tehran.

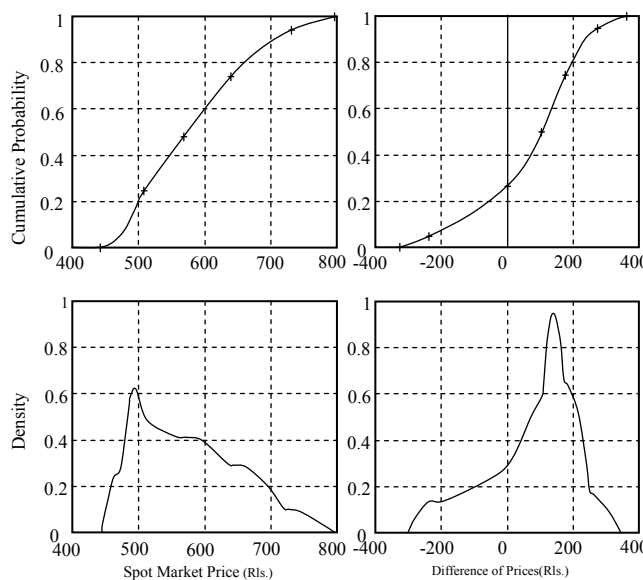


Fig 8: The fitted cumulative distribution functions and the derived density functions for the spot price and the difference between the regulating market price and the spot price.

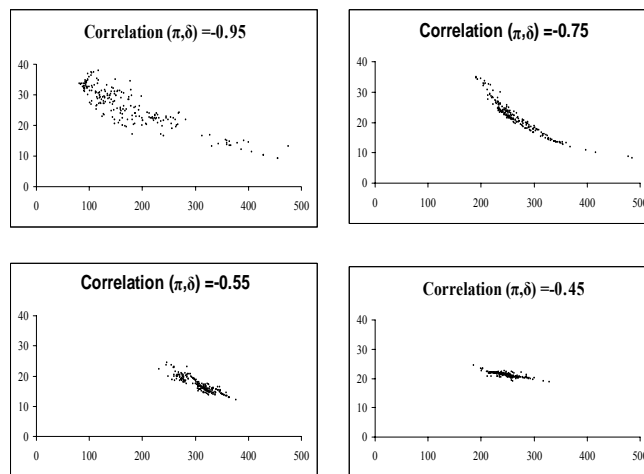


Fig 9: Generating scenario for varying correlations.

### Step 3: Generating scenarios

Based on the descriptive statistics presented in Table 3, we generated 1296 scenarios by using the method described in §III B.

### Step 4: Specifying the volume deviation penalty function

The volume deviation risk has a major impact on ESP bidding decisions; therefore, the specification of the volume deviation penalty function in the objective function is necessary. We have chosen to work with the piecewise linear penalty function, as described in §III A.

Step 5: Modeling and solving

We modeled and solved the linear program using GAMS software. A typical problem was solved in about 12 seconds on a 1-GHz Pentium III PC with 524 MB RAM. An example of an optimal bidding curve is shown in Fig. 10. This curve has 7 line pieces. Also shown is the expected load curve and the load-price scenarios.

By using a price elasticity of demand of  $\eta = -0.6$ , Fig. 11 shows the effect of varying the number of fixed price points. If the number of price points is decreased, the result is a cruder bidding curve. Notice also that the bidding curves are cruder at prices that are less likely, e.g., above 900 Rls./MWh.

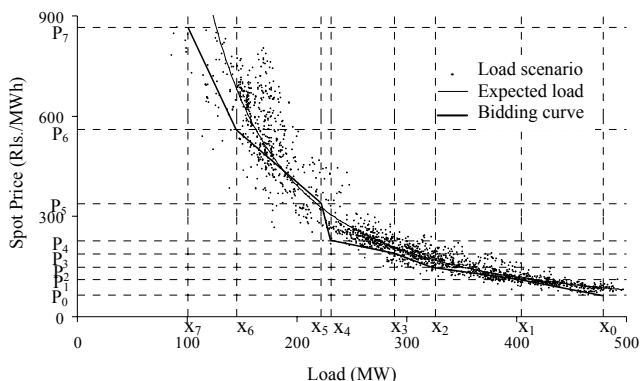


Fig. 10: An 7 segments bidding curve, expected load curve and load scenario

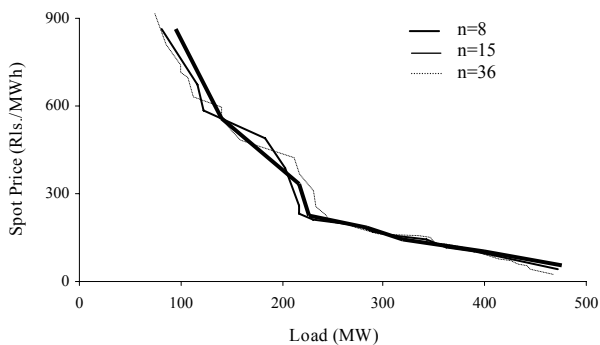


Fig.11: Bidding curves with a varying number of fixed price points.

When we do not know what the price elasticity of demand will be in a future we have to estimate this for the given hour for which a bidding curve is to be constructed. Thus, we can construct bidding curves with a range of elasticities.

5. CONCLUSION

In this study, a method of constructing piecewise linear bidding curve is designed for each hour of the day. We propose a stochastic linear programming

model and build a bidding curve based on several scenarios of MCP. We then considered the ESP's or retailer's aversion to the volume deviation risk to offer for each hour according to their risk attitude. The important suppositions of this work are the rules of day ahead market, end users with price sensitive demand, price taking retailer and making use of two way communication technology.

The mention for other factors causing retailer object function is left for future work. For example, encouragement consumers to participate in electricity trading by demand side bidding rules can make a supplier alternative for retailer. This could enable electricity prices to be reduced in the short term.

ACKNOWLEDGMENT

This work was supported by TSWPD. We highly appreciate the technical advice received from their staff. The views and opinions of the authors do not necessarily state or reflect those of TSWPD or any other utility or agency.

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