Further developments on gear transmission monitoring

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Abstract – This work shows how it is possible to improve the algorithm used for the duration test. Generally, it shows some frequencies in the spectrum of signal obtained by an accelerometer. Several considerations on frequencies appears when an unexpected event occurs are presented. In the first part of the paper it is described the typical frequencies due to the vibration of the gearbox. In the second part it is illustrated the concept of harmonics, side bands and their meaning. In the third part a new index for monitoring the duration test is introduced. This index, as explained below is very important, because it gives the opportunity of checking the various aspects of the spectrum of the signal of an accelerometer. A warning starts if a damage on a gear wheel is detected.

Key-Words: - Assembly-Phase-Passage Frequency, Tooth Repeat Frequency, Harmonics, Side Bands.

1 Introduction

During some tests of duration of gear transmission performed in laboratory, we found out some fundamental characteristics on the spectrum derived from an accelerometer signal. These aspects carefully investigated led the research for the gear transmission monitoring to a further improvement. As it showed in following pages, a new index for monitoring was proposed, which can be used in parallel with the following one (see [1] for details):

\[ \text{Index} = \Delta \% (\text{Area}_{\text{cur}} / \text{Area}_{\text{ref}}) \]

It gives more strength to the monitoring because by means of its more specificity and sensitivity we can stop earlier the duration test.

2 Typical frequencies of gear transmission vibration

The vibrations of gear transmission system are characterized by certain "typical" frequencies (signatures) that can be determined before starting the monitoring. Usually, the spectrum of vibrations of a gear system is dominated by peaks belonging to typical frequencies, but in some cases, it shows other frequencies that don’t coincide with typical ones. However, this irregularity is not true; in fact these frequencies can be made in relation to the typical ones.

Below we can find the definitions of four frequencies that usually occur as dominant peaks in the spectrum of vibrations of gearbox. The formulas for the calculation of these frequencies are valid both for a cylindrical gear with straight-toothed and for those with helical teeth, and for the wheel crown gear. As we show, before using the following formulas to calculate these frequencies, it is necessary to know the number of teeth and the speed of rotation of all wheels under testing.

- **Frequency of rotation**:
  - Let’s consider a typical pair of wheels, where \( Z_1 \) is the number of teeth of drive wheel, \( n_1 \) its rotation speed (rpm), \( Z_2 \) the number of teeth of the driven wheel and \( n_2 \) its speed of rotation (rpm). We define, for both the wheels, the frequency of rotation:

  \[ f_1 = \frac{n_1}{60} \quad (1) \]
  \[ f_2 = \frac{n_2}{60} \quad (2) \]

  for the drive wheel, and

- **Geared Frequency**: it is well known that, during the rotation, teeth are subjected to collision, sliding and bending. These phenomena are repeated every time that a pair of teeth comes in contact, and therefore they appear as a vibration which fundamental frequency, characteristic of the pair of wheels, is the gear mesh frequency (GMF or \( f_{\text{mesh}} \)).

  Using the same symbolism adopted before, the gear mesh frequency is given by [2,3]

  \[ \text{GMF} = Z_1 \cdot n_1 / 60 = Z_2 \cdot n_2 / 60 \quad (3) \]

  The second equivalence concerns the fact that for a pair of gears, the gear ratio is given by
\[ \epsilon = n_1 / n_2 = Z_2 / Z_1 \quad (4) \]

and therefore,
\[ Z_1 \cdot n_1 = Z_2 \cdot n_2 \quad (5) \]

and from this formula we can infer immediately the equivalence (3). From the definition of rotational speed, it is possible to rewrite the expression of GMF
\[ GMF = Z_1 \cdot f_1 = Z_2 \cdot f_2 \quad (6) \]

Gear mesh frequency and its harmonics are the dominant components in the spectrum of vibration as well as the noise of any gear transmission system [2,4].

**Assembly-Phase-Passage frequency:** this frequency could occur in the spectrum of vibrations of a gearbox when it is disassembled (e.g., for a detection or a repair) and then reassembled. For that reason the contact of teeth is never the same.

Before showing the formula to calculate this frequency, it is necessary to introduce the concept of Phase Assembly [2,3].

Let’s take in consideration a typical pair of wheels, and let’s suppose to numerate all teeth of both wheels starting from the real point of contact, as it is showed in Fig. 1.

**Fig. 1 - Teeth numeration for a generic pair of wheels**

Fig. 2 shows the sequence of teeth involved during their contact. Each wheel has 6 teeth.

**Fig. 2 - Sequence of contact between the teeth**

During the rotation each tooth of the wheel 1 (pinion) comes in contact with a single tooth of the wheel 2 (gear). In this case, the teeth showing the same number develop during their contact, a complementary wear so that it compensate the imperfections due to manufacturing and/or assembling operation. If after a certain working period these two wheels are disassembled and then reassembled without taking the right sequence into consideration of the numbered teeth, each tooth would no longer be in contact with its "complementary" and it would bring out a changing in the vibration. We say that this pair of wheels has 6 Assembly Phases (AP).

**Fig. 3 - Sequence of contact between the teeth**

The Fig. 3 shows the sequence for two gears having respectively 6 and 7 teeth. In this case the situation is completely different. After 7 rotation, all the teeth of the pinion are in contact with all the teeth of the wheel; all teeth of both gears wear in the same way, and therefore, even if two gears have to be disassembled and then reassembled in a different way, there will be no change in vibration. This pair of gears has a single AP. Therefore, the number of AP (N_{AP}) of a pair of gearwheels can be considered as the number of
"types of wear" (wear patterns) developed by the contact of teeth of the pair of wheels. Therefore the type of wear depends on the combinations of contact between the teeth. Numerically speaking, \(N_{AP}\) is the “highest common factor” of the number of teeth for two wheels tested [2,3].

The "Assembly-frequency Phase-Passage", is given by

\[ f_{AP} = \frac{GMF}{N_{AP}} \]  

(7)

where GMF is gear mesh frequency (Hz), \(N_{AP}\) is the number of Assembly Phases for pair of wheels tested. Note that if \(N_{AP} = 1\), \(f_{AP}\) coincides with the gear mesh frequency.

Tooth Repeat frequency: this frequency (Hunting-Tooth frequency [2]) usually occurs when each of the two wheels has a defect localized on a tooth [2,3,5]. In this case, the pair of damaged teeth comes into contact one more time, with a frequency equal to [2,3]

\[ f_{TR} = \left(\frac{GMF \cdot N_{AP}}{Z_1 \cdot Z_2}\right) \]  

(8)

where:

- GMF is the gear mesh frequency (Hz)
- \(N_{AP}\) is the number of Assembly Phases
- \(Z_1\) and \(Z_2\) represent the number of teeth of the drive and driven wheel respectively.

Once more, the value of the frequency under consideration depends on \(N_{AP}\).

### 3 Harmonics and side bands

It was already observed that in the spectrum of gearbox peaks appear frequencies which are different from the typical ones. These frequencies are "unknown" and are often referable to one or more typical frequencies because:

- they are multiple of a characteristic frequency (in this case they are called typical harmonic frequency)
- or they are the sum or the difference of the typical frequencies.

The rotation frequency for each wheel usually occurs when there is a residual imbalance, which causes a vibration and therefore harmonic component around the rotation frequency [4,6]. Usually, the residual imbalance (static and/or dynamic) is always present, more or less, in all types of rotor. Static imbalance appears as a radial vibration characterized by a frequency equal to \(N/60\) (where \(N\) is the speed of rotation of the rotor expressed in revolutions per minute) and then, in the spectrum, it appears as a harmonic component at this frequency [4]. However, it is well known the non-linearity of some components (e.g., lubricant film, bearings), may change the feedback of the system to generate components at higher frequencies [6]. The amplitude of vibration due to imbalance grows with the square of the rotation speed.

Usually, in the spectrum of vibrations of a gearbox, there are also the harmonics belonging to the frequencies of rotation. In particular, a wide second harmonic of the rotation frequency is usually taken as a “signal” due to the shaft damage [2]. This points out the situations in which the shafts of a pair of gears are not perfectly parallel, or those in which the axes of the shafts of two generic machines are not perfectly coincident, in this case there are two types of problems: parallel and angular. Such situations usually are generated by errors realized during the assembling process.

The peaks due to GMF and their harmonics are usually dominant in the spectrum. In some cases, in the neighbourhood of GMF, may appear peaks due to, for example, a small eccentricity of the drive/driven wheel. The peak frequencies are:

\[ GMF \pm f_1\] or \[GMF \pm f_2.\]  

(9)

where \(f_1\) and \(f_2\) are the frequencies of drive and driven wheel respectively.

The presence of these frequencies is due to the fact that the eccentricity modulates the amplitude of the forces acting during the mesh [2,6,7]. Localized defects on the teeth, such as cracks or broken part determine the apparition of several peaks in the neighbourhood of the GMF and its harmonics, due to changing of deflection of the tooth during the mesh. This happens once for each rotation of the damaged wheel. This change could be seen as an amplitude modulation while the frequency modulation is the frequency of rotation of the damaged wheel. Therefore, the distance from the side of the peak of the GMF can identify the modulated frequency, and then the wheel generating the defect [7,8].

In [7] (Chapter 16), talking about peak side, it is stated that:

- for localized defects such as cracks or broken on teeth, the side peaks are smaller than the gear mesh frequency and they cover a very wide range of frequencies
- for distributed defects such as wear or errors in working-process, the peaks side are more extended and are more clustered around the gear mesh frequency and its harmonics.

In a gearbox, each wheel can be a source of vibration and therefore of side peaks, in order to
measure, with low error, their distance from the GMF, it is necessary that the spectra have, at least, the same resolution equal to the lower rotation frequency. However, even this kind of resolution might not be enough, to differ from neighbourhood of GMF, because of its very low value. This is a negative aspect, because it would mean a loss of information if two gears show a well localized damage [4]. In this case, if it is impossible a further improvement of the resolution, it is fundamental to carry out an analysis based on the techniques of the close-up. The frequencies $f_{AP}$ and $f_{TR}$ could appear as peaks side of the GMF, where, respectively, the pair of wheels is disassembled and reassembled in a different sequence and contemporary both the wheels are damaged.

It should be remarked that if both wheels have the number of teeth such as $N_{AP} = 1$ their $f_{AP}=GMF$, and therefore the Assembly-Phase-frequency Passage is no more distinguishable from the GMF. The Tooth-Repeat frequency, however, because of its very low value, could hardly be distinguished, especially when $N_{AP} = 1$.

4 Index of Side Bands

The idea to improve a new index for monitoring the duration test of a gearbox was born starting from the following considerations. First of all we thought to monitoring only the most important frequency (or their orders [1]). But it should be a very expensive system. A new index has been proposed to enforce the previous index in [1].

For each order the index of Side Bands is the ratio, between the "current spectrum" and the “target spectrum”. This index doesn’t overthrow the threshold level of 3. So this index, takes on account how the instantaneous spectrum is related to the target spectrum: the index should ideally be close to "1" for each order.

As it is well known in the literature, and as we reported before, the presence of side-band and the changing of the spectrum, due to typical frequencies different from the typical ones of gearbox, indicate the presence of defects. In these cases the $\text{Index\_SideBand}> 1$.

Moreover, referring to the first threshold [1]

$$\Delta \% (\text{Area}_{\text{cur}} / \text{Area}_{\text{ref}}) \quad (10)$$

meaningful changes of some orders could not stop the test (e.g., the order of bearing changes, which has no weight on the overall spectrum). In these cases, if the $\text{Index\_SideBand}> 3$ we stop the test and we also have an indication on which order is increased.

For this type of monitoring is not required the use of additional sensors and this is a great additional benefits for us. So, on the algorithm proposed in [1], we thought to modify the STEP 5 only, in which the $\text{Index\_SideBand}$ must be controlled. So the process is modified as in Fig. 4:

![Fig. 4 - How the algorithm stop the test](image)

where $\text{Index\_SB}$ stands for $\text{Index\_Side\_Band}$. Using this method the monitoring control the energy content of the whole spectrum does not overthrow the threshold. In addition an order doesn’t overthrow the other threshold.

Note that if an order growths significantly, but not the first index in [1] then the $\text{Index\_SideBand}$ should be employed in order to solve this kind of problem.

In the next paragraph it will be showed how the above index should be used for stopping earlier the test.

5 Experimental results

Taking now the same test showed in [1], we can see that the duration test is stopped to the 13% of the total test. Vice versa, by means of the index proposed in [1] the performance was less good, in fact the test-run was stopped at 16% of the total test. The diagrams reported below, obviously, show only a part of the test results. They are selected to best underline the duration test performed with this technique.

We analyze the 6th cycle (in particular, two seconds: the instants corresponding to 40s and 80s). Later it will be shown 2 seconds of the 13th cycle where the test-run was stopped.
In the Fig. 5 it is represented the target spectrum and the actual spectrum (at 40s). To check the effectiveness of the Index_SideBand, it must be done the ratio between these two spectra as it is shown in the Fig. 6.

It’s clear that the Index_SideBand doesn’t overthrow the threshold level, so the test-run can go on. In Fig. 7 it is shown the spectrum at 80s.

In order to see the Index_SideBand, in Fig. 8 it is represented the ratio between these two spectra.

It’s clear again that the test can go on.

Fig. 9 shows the 13th cycle: at the second 1 the order 1 has already overthrown the target

At this moment the order 1 of the actual spectrum overthrow the amplitude of the same order of the target spectrum. It’s quite clear looking at the Fig. 9, but it is more clear looking at the Fig. 10 in which it is reported the Index_SideBand. The threshold imposed to stop the duration test.

![Fig. 5 - Comparison at 40s](image1)

![Fig. 6 - SideBand at 40s](image2)

![Fig. 7 - Comparison at 80s](image3)

![Fig. 8 - SideBand at 80s](image4)

![Fig. 9 - Comparison at 1s](image5)
So looking at the last diagram the duration test must be stopped at the 13th cycle, with all benefits.

6 Conclusions

The new index proposed in this paper allows new performance. In fact, the algorithm is designed for detecting earlier the gearbox damage. The Index_SideBand monitoring each order gives us information about each one of these orders. So it helps to find the reasons of damage. In fact, as we showed in the above paragraphs, each event is showed like frequency. Observing the frequencies which overthrow the threshold level, we can detect the wheel to which frequency corresponds. Note that test is also stopped if one order overthrows of the threshold level. The Index_Area leads to stop the duration test only if the total contribution, given by the area of spectrum, overthrows the threshold level. The proposed algorithm, using two index, offers a better control of the test and gives more information about an unexpected event that can cause a serious damage to the gearbox.

References