Identification of Distributed Parameter Systems Based on Sensor Networks and Multivariable Estimation Techniques

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Abstract: - The domain of system identification may be developed today using the powerful tool represented by the intelligent sensor networks, placed in real distributed parameter systems. The sensor networks, as a “distributed sensor”, allow the usage of multivariable estimation techniques, in different ways: classical linear methods of modelling or methods based on artificial intelligence for complex non-linear systems. The paper presents a short survey of some results obtained in the study of using of sensor networks with multivariable estimation techniques to estimate heat transfer in space.

Key-Words: - Wireless sensor networks, system identification, distributed parameter systems, neural networks, multivariable estimation techniques, auto-regression, heat distribution.

1 Introduction
This paper presents some considerations on the advantages of using multivariable estimation techniques [1, 2, 3] in the new context of the intelligent sensor networks as a “distributed sensor” in distributed parameter systems. The concepts of sensor networks, multivariable estimation models and artificial intelligence used in system estimation appear many times in the international literature in many applications and many approaches [4, 5]. The wireless distributed autonomous intelligent sensor networks represent a powerful tool in the modern measurement techniques. Being a distributed tool, they may be a distributed measurement element, so they may be used to measure time variables in the complex distributed parameter systems. The commercial sensor networks have sensors for all kind of variables from physical distributed parameters systems as: temperature, pressure, radiation or light intensity. In this application, with a large field of interest in science and engineering, all these topics: modern sensor networks, theory of distributed parameter systems, theory of system identification, classical identification multivariable linear models, neuro-fuzzy modeling, partial differential equations, modeling of dynamic systems contribute and all are converging to the same objective – identification of distributed parameter systems [6, 7, 8]. The author has published some surveys in the field of identification of distributed parameter systems based on sensor networks and artificial intelligence [9, 10, 11, 12, 13, 14].

This paper makes an approach of system identification based on modern intelligent wireless sensor networks referring to the measurements of the dynamic characteristics of the distributed parameter process to be observed and controlled. Often it is a part of the fault detection and diagnosis problem. Using of intelligent sensor networks in distributed parameter systems eliminates the classical disadvantages of the multivariable estimation methods. The whole objective of the identification problem is to formulate a multivariable mathematical model, of these unknown complex processes, which adequately describes the distributed parameter system dynamics, in time and space. One of difficulties that appear when the multivariable system observer is face with this problem, is that identification must be achieved in the presence of system’s normal operating signals and noise disturbances which appear in many places in the same time on a large distributed space. In this case using a distributed sensor network any test measurement that have to be performed upon the system does not disturb the normal operation of the control. This possibility allows off-line and also on-line identification. Another advantage is that the time taken for system identification is relatively the shortest possible ever.
is the temperature at the time the level surface for of internal thermal conductivity of the object. The where there are no heat sources:

\[ \frac{\partial}{\partial t} \left( \frac{\partial \theta}{\partial \tau} \right) + \frac{\partial}{\partial \tau} \left( \frac{\partial \theta}{\partial \tau} \right) + \frac{\partial}{\partial \tau} \left( \frac{\partial \theta}{\partial \tau} \right) \]

The law of heat propagation through an object in the point P\(\{15, 16, 17\}\):

\[ dQ = k \left| \frac{\partial \theta(P, t)}{\partial n} \right| dt d\sigma \]  

where \(k\) is a proportionality factor, called coefficient of internal thermal conductivity of the object. The vector grad \(\theta\) has its direction along the normal at the level surface for \(\theta=ct\), in the sense of \(\theta\) rising.

The heat sources in the object have a distribution given by the function:

\[ F(t, P) = F(t, x, y, z) \]  

If the object is homogenous \(a = \sqrt{k / \gamma \rho} = ct\). and the equation (2) is written:

\[ \frac{1}{a^2} \frac{\partial^2 \theta}{\partial t^2} = \frac{\partial}{\partial x} \left( \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \theta}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial \theta}{\partial z} \right) \]  

The initial conditions or of the limit conditions have physical significance. The equation

\[ \rho c \frac{\partial \theta}{\partial t} = \text{div}(k \text{ grad } \theta) + F(t, P) \]  

does not determine completely the state of the object \(K\). We must take in considerations the initial state of the object, the temperature distribution in the object at the moment \(t=0\):

\[ \theta(x, y, z, t)_{t=0} = f(x, y, z) \]  

called initial conditions.

We consider the parabolic partial derivative equation [1, 2, 3]:

\[ \frac{\partial \theta}{\partial t} = a \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \]  

where \(\theta(x, y, t)\) is the temperature at the time moment \(t\) in the point \(P(x, y)\) of coordinates \(x, y\) in a finite space \(S(xOy)\), from a plane of coordinate system \(xOy\); \(a\) is a constant parameter, given by the relation:

\[ a = \frac{k}{\gamma \rho} \]  

where \(k\) is a proportional factor, the inner thermal conductibility coefficient or the coefficient of heat conduction of the space \(S\) in which there is taken place the heat transfer, \(\gamma\) is a proportional coefficient, the specific heat or heat capacity of space \(S\), \(\rho\) is the density of the space \(S\). This equation describes the heat transfer in a plane. With it asymmetric cases may be analyzed in a general approach. In the steady state case the equation (7) becomes:
\[
\left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) = 0
\]  

(9)

A case study with a large utility in practice is considered, in which the space of heat transfer is a square of dimensions \( l \times l \). The limit and initial conditions of the equation (1) may be:

\[
\begin{align*}
\theta(0, y, t) &= \theta_{xa}, \ y \in [0, l], t \in [0, T], \\
\theta(x, 0, t) &= \theta_{y0}, \ x \in [0, l], x \in [0, T], \\
\theta(x, y, 0) &= 0, \ x \in [0, l], y \in [0, l], \\
\theta(x, y, t) &= \theta_{ix}, \ y \in [0, l], t \in [0, T], \\
\theta(x, l, t) &= \theta_{il}, \ x \in [0, l], x \in [0, T],
\end{align*}
\]  

(10)

Boundary conditions must be imposed for the equation (7). The boundary conditions are of two more important types. When the temperature on the boundary is specified we are speaking about Dirichlet conditions and when the heat flux and the heat transfer coefficient are specified there are Neumann conditions. If in the space \( S \) heat sources appear, the heat equation (1) is:

\[
\frac{\partial \theta}{\partial t} = a \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + Q(x, y, t)
\]  

(11)

where \( Q(x, y, t) \) is a variable heat generated by a source, in space and time. With the equation (5) and the conditions (4) we may describe temperature variations in a plane on a square space with known dimensions, when the initial and limit conditions are known. At the equation (9) a last term \( k_c (\theta_{ext} - \theta) \) may be added, to describe the transversal heat transfer from the external medium, where \( \theta_{ext}(x, y, t) \) is the external temperature, variable in space and time. The heat equation may be solved by numerical computation [18]. There are many programs to solve the parabolic equation of heat transfer presented as a common partial differential equation problem, using the finite element method. The solid geometries of the distributed parameter systems may be analysed in space. The limit and initial conditions of the equation (9) may be introduced as Neumann and Dirichlet boundary conditions. For the above case study we are choosing Neumann condition on top: the heat source \( Q = 1 \) and a heat transfer coefficient \( q = 0.1 \). For the left, right and down side of the space we are choosing Dirichlet conditions: weight \( h = 1 \) and temperature \( \theta = 0 \). Triangular meshes are used in equation solving [19]. The comparison study is made analysis some different resulted meshes for different number of sensor placed in space.

### 3 Discrete Approximation

We may associate to the equation (5) a system with finite differences [1]. For this purpose the space \( S \) is divided into small pieces of dimension \( l_p \):

\[
l_p = l / n
\]  

(12)

In each small piece \( S_{pi,j}, i=1,...,n, j=1,...,n \) of the space \( S \) the temperature could be measured at each moment \( t_k \) in a characteristic point \( P_{i,j}(x_i, y_j) \), of coordinate \( x_i \) and \( y_j \). Let it be \( \theta_{i,j} \) the temperatures in the point \( P_{i,j}(x_i, y_j) \) at the moment \( t_k \). It is a general known method to approximate the derivatives of a variable with small variations. In the equation with partial derivatives there are derivatives of first order, in time, and derivatives of second order in space. So, theoretically, we may approximate the temperature derivative in time with a small variation in time, with the following relation:

\[
\frac{\partial \theta}{\partial t} \approx \frac{\theta_{i,j}^{k+1} - \theta_{i,j}^{k}}{t_{k+1} - t_k}
\]  

(13)

Also, we may approximate the summation of the temperature second derivatives in space, with small variations in space and to obtain the following relations:

\[
\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \approx \frac{\theta_{i+1,j}^{k} + \theta_{i-1,j}^{k} + \theta_{i,j+1}^{k} + \theta_{i,j-1}^{k} - 4 \theta_{i,j}^{k}}{l_p^2}
\]  

(14)

We may consider the temperature is measured as samples at equal time intervals with the value

\[
h = t_{k+1} - t_k
\]  

(15)

called sample period, in a sampling procedure, with a digital equipment. Combining the equations (12 - 15) in the equation (7) a system with differences results:

\[
\theta_{i,j}^{k+1} = \theta_{i,j}^{k} + \frac{ah}{l_p^2} (\theta_{i+1,j}^{k} + \theta_{i-1,j}^{k} + \theta_{i,j+1}^{k} + \theta_{i,j-1}^{k} - 4 \theta_{i,j}^{k})
\]  

(16)

The sampling period \( h \) may be chosen according to different theories for signal sampling. For example, we must choose it without obtaining alias errors at specific frequencies. In practice the sampling period is taken according to the capacity.
of digital equipment to manipulate data, the time constants of the system, assuring the conditions of stability, controllability and observably for the discrete time model. An interesting value for $h$ is proposed in [1], to eliminate the term $\theta_{i,j}^k$:

$$h = \frac{l^2}{4a}$$

so:

$$\theta_{i,j}^{k+1} = \frac{1}{4} \left( \theta_{i-1,j}^k + \theta_{i+1,j}^k + \theta_{i,j-1}^k + \theta_{i,j+1}^k \right)$$

The relation (18) gives a discrete approximation of temperature value $\theta_{i,j}^k$ in every characteristic points $P_{i,j}$ of the space $S$, for $i, j = 1, \ldots, n$, at the every moment $t_k, k>1$, from the temperature values $\theta_{i,j}^{k-1}, \theta_{i-1,j}^k, \theta_{i+1,j}^k, \theta_{i,j-1}^k, \theta_{i,j+1}^k$, at the antecedent moment $t_{k-1}, k>0$, of the adjacent points $P_{i-1,j}, P_{i,j-1}, P_{i+1,j}$ and $P_{i,j+1}$. Relations of the approximation error and for stability are given in literature. The approximation (12) is a linear system with differences of $n \times n$ order.

In the discrete model (12) we must take in consideration the position points $P_{i,j}$ in the discrete space of the sources $Q_{i,j}$ at each time moment. For transversal heat transfer from the surroundings the term $q_m (\theta_{\text{ext}} - \theta_{i,j}^k)$ must be added, where $q_m$ is the convective heat transfer coefficient and $\theta_{\text{ext}}$ is the external temperature. In the discrete model we must take account on each point with a contact with the exterior: $q_m (\theta_{\text{ext}} - \theta_{i,j}^k)$ at each time moment.

4 Sensor Network
4.1 Basic Structure
A sensor network is made by hundreds or thousands of ad-hoc tiny sensor nodes spread across the heat transfer space $S$. Sensor nodes collaborate among themselves, and the sensor network provides information anytime, by collecting, processing, analysing and disseminating temperature measured data. Sensor network is working as a distributed sensor. The constructive and functional representation of a sensor network is presented in Fig. 1. The sensor networks have different structures. The star networks (point-to-point), are networks in which all sensors are transmitting directly with a central data collection point. The mesh networks are networks in which sensors can communicate with each other. In mesh networks sensor nodes can relay messages from other sensor nodes, there is no need for repeaters. Software controls the flow of messages through network with self-configuration.

4.2 Practical Measurements
A Crossbow sensor network was used in practice. It has the following components: a starter kit, a MICA2 2,4 GHz wireless module, and an MTS320 sensor board. Their nodes (Fig. 2) are:

![Fig. 1 A sensor network with mobile access](image1)

New nodes automatically detected and incorporate. The number and the place point of the sensor nodes may be discussed according to the desired accuracy of temperature estimation using different identification methods. Based on the study from this paper we may see that the number is an important data in network design, but the way to place these sensors, the points of their placement according to the heat isotherms in space has the same importance, maybe more.

![Fig. 2 A sensor from the network](image2)
network monitorization and real time graphics and MoteWorks for nod programming in MesC language.

The transient temperature characteristics measured for temperature monitorization in a chamber are presented in Fig. 3.

Fig. 3 Temperature transient characteristics measured with the sensor network

The user interface allows some facilities, as: administration, searching, connections options and so on.

5 Multivariable estimation techniques

5.1 General model and estimation structure

The present paper considers that a multivariable autoregressive (AR) model can efficiently approximate the time evolution in space of the measured values provided by each and every sensor within the coverage area. An autoregressive model describes the evolution of a variable measured over the same sample period as a linear function of past evolutions. This kind of systems evolves due to its "memory", generating internal dynamics. The AR model definition is:

\[ x(t) = A_1 x(t-1) + \ldots + A_n x(t-n) + \xi(t) \]  \hspace{1cm} (19)

where \( x(t) \) is a vector of the series under investigation (in our case is the series of values measured by the sensors from the network):

\[ x = [x_1 \ x_2 \ \ldots \ x_m]^T \]  \hspace{1cm} (20)

and \( A_i \) are the matrix of auto-regression coefficients, \( n \) is the order of the auto-regression and \( \xi(t) \) is a vector containing the noise components that is almost always assumed to be a Gaussian white noise. By convention all the components \( x_1(t), \ldots, x_m(t) \) of the multivariable time series \( x(t) \) are assumed to be zero mean. If not, another term \( (A_0) \) is added in the right member of equation (1). Based on the model (1), (2) the coefficients \( A_i \) may be estimated in case that the time series \( x(t), x(t-1), \ldots, x(t-n) \) is known (recursive parameter estimation), either predict future values \( \hat{x}(t) \) in case that \( A_i \) coefficients and past values \( x(t-1), \ldots, x(t-n) \) are known (AR prediction). The method uses the time series of measured data provided by each sensor and relies on an autoregressive multivariable predictor placed in base stations (Fig. 4).

Fig. 4 Multivariable estimation structure

The principle is the following: the sensor nodes will be identified by comparing their output values \( x(t) \) with the values \( \hat{x}(t) \) predicted using past/present values provided by the same sensors. After this initialisation, at every instant time t the estimated values \( \hat{x}_A(t) \) are computed relying only on past values \( x_A(t-1), \ldots, x_A(0) \) and both parameter estimation and prediction are used as in the following steps. First the parameter matrixes \( A_i \) are estimated using a recursive parameter estimation method. There are a large number of methods for obtaining AR coefficients […]. In this case a recursive least square method may be considered an appropriate solution to solve this problem in an efficient manner since it produce the best spectral estimates. An Armax method, with zero coefficients for the inputs is used. After that, the present values \( x_A(t) \) measured by the sensor nodes may be compared with their estimated values \( \hat{x}_A(t) \) by computing the errors:

\[ e_A(t) = \| x_A(t) - \hat{x}_A(t) \| \]  \hspace{1cm} (21)

If this errors are higher than the thresholds \( e_A \) the sensors A may be considered potentially corrupted sensors. Here, based on a database containing the known models, on a knowledge-based system we may see the case as a multi-agent system which can do critics, learning and changes, taking decision based on node analysis from network topology.
There is no simple method to establish the correct model order \( n \) in case of an AR model. Two parameters can influence the decision: the type of data measured by sensors and the computing limitations. Because both of them are a priori known an off-line methodology is proposed. Realistic values are between 3 and 6.

### 5.2 Linear Model

The procedure, used to determine the linear model of the dynamical system from observed input-output \([1, 2, 3]\) is as follows. 1. The input data for estimation was a pseudo random binary signal for the powers \( P \). The output of the heat diffusion model \( x \) at these signals was computed. Means and trends of the signals were removed. The signals were filtered. 2. A set with a multivariable model with 9 state space equation for the cell and \( n=8 \) time delays for the estimate was chosen. 3. The criterion of selected the model was the residual and the fit of the estimate in the simulation. The multivariable estimation model with auto-regression has \( 9 \times 9 \times 9 = 729 \) parameters which are majority 0 and only on the diagonals of the 9 parameter matrix are nonzero values. For example, the parameters of the estimation model for \( x_A \) are given in the following equation, where \( h \) is the sampling period:

\[
\hat{x}_A(t) = 1.207x_A(t-h) - 0.302x_A(t-2h) + \\
+ 0.079x_A(t-3h) - 0.026x_A(t-4h) + \\
+ 0.019x_A(t-5h) - 0.0286x_A(t-6h) + \\
+ 0.022x_A(t-7h) - 0.0112x_A(t-8h) + \\
+ e(t)
\]  (22)

Four cases of traveling wave temperatures are taken in consideration in this case study, when four traveling temperature waves pass on the North, South, West and East sides of the cell, next to the sensor cell. For the North traveling temperature wave the signal diagram of powers is presented in Fig. 5. The response in temperature diffusion is presented in Fig. 6. An anomaly may be presumed at a sensor node \( S_A \) appearing at a time moment \( t=50 \) s. As an example the output value \( \theta_A \) has the shape from Fig. 7. On the other hand, the sensor’s estimated output value \( \hat{\theta}_A \), predicted by the above-mentioned estimator, is different from the actual value of the malicious sensor \( A \) showing that something wrong happened to sensor \( A \).

![Fig. 5 A set of power test signals](image1)

![Fig. 6 The time response of the sensors, that fit the estimate](image2)

![Fig. 7 Estimated \( \hat{\theta}_A \) and corrupted \( \theta_A \) sensor A values](image3)

The positive error \( e_A(t) \) at the sensor \( S_A \) is time variable. When it passed over the threshold \( \varepsilon_A \), imposed by the user, a decision must be taken.

### 5.3 Neural Model

The neural network used for estimation is a feedforward neural network, with continuous values [20]. It has 4 inputs, the sensor values at 4 antecedent time moments: \( x_A(t-1) \), \( x_A(t-2) \), \( x_A(t-3) \) and \( x_A(t-4) \). The structure of it is presented in Fig. 8.
According to Kolmogorov’s theorem we are using two hidden layers of neurons with biases to obtain a reduced error of approximation of the estimate. The input layer has 4 neurons, for the previous values of the measured temperatures. The output layer has one neuron for the estimated temperature. The first and the second hidden layers have a reduced number of neurons, 32 and 16 neurons, respectively. These numbers resulted after some iterative training. The activation functions of the neural network are the hyperbolic tangent function for the hidden layers and the first-order linear function for the output layer.

The procedure, used about us, to determine a model of the dynamical system from observed input-output data involves the following ingredients. The training of the neural network with a training set, which cover the entire possible scenario in the field. The input data for estimation is a time series of the temperature sensor $S_A$, as the state of the heat diffusion model. This time series is obtained using sums of the travelling temperature waves, from the North, South, West and East sides of the network, generated by the heat sources $P_i$. The temperatures are propagated through the sensor network to the sensor $S_A$. The training set was obtained using present and anterior values of the sensors, $\theta_A(t-1), \theta_A(t-2), \theta_A(t-3), \theta_A(t-4); \theta_A(t)$) taken from the transient responses of the model (11) and (12). The method chosen for training was the Levenberg-Marquardt method. The sum square error after 300 training epochs is presented in Fig. 9. At a specific time moment ($t=400$) we considered that the sensor was corrupted. Some different sets of candidate models for the model structure could be experimented.

We have chosen a 4th order estimation general model:

$$\hat{x}_A(t) = \sum_{i=1}^{4} a_i x_A(t-i) + \xi$$  \hspace{1cm} (23) $$

With weights and biases it has the expression:

$$\hat{x}_A(t) = \sum_{i=1}^{4} a_i (w, b) \cdot x_A(t-i) + \xi$$  \hspace{1cm} (24) $$

This autoregressive neural estimation is applied for sensor $S_A$ by using the time series from Fig. 10. The estimated temperature $\hat{x}_A(t) = \hat{\theta}_A(t)$ for the sensor $S_A$ is presented in Fig. 16, over the original time series $x_A(t)$.

And in Fig. 11 there is presented the error $e_A(t) = x_A(t) - \hat{x}_A(t)$. We may see that the error increases at the corruption time moment. The maximum error value is $e_{\text{Max}}(t)=e(402)=0.16$ being above the threshold value considered to be $e_\lambda=0.075$ will trigger the decision block to expel the sensor from the network.
5.4 Adaptive-Network-Based Fuzzy Inference

The neural network developed in the neuro-fuzzy modeling frame has the following general structure [21]. If the sum-prod inference is used a neural structure with product between the activation function outputs and weights and summation of the connections signals may be developed. The first layer is the input layer. The second layer represents the input membership or fuzzification layer. The neurons represent fuzzy sets used in the antecedents of fuzzy rules determine the membership degree of the input. The activation function represents the membership functions. The 3rd layer represents the fuzzy rule base layer. Each neuron corresponds to a single fuzzy rule from the rule base. The inference is in this case the sum-prod inference method, the conjunction of the rule antecedents being made with product. The weights of the 3rd and 4th layers are the normalized degree of confidence of the corresponding fuzzy rules. These weights are obtained by training in the learning process. The 4th layer represents the output membership function. The activation function is the output membership function. The 5th layer represents the defuzzification layer, with single output, and the defuzzification method is the centre of gravity.

An important attention must be given in practice to the space variation of the distributed parameter system variables, measured in the distributed parameter system space using sensor nodes. Some results of mesh generation according to the finite element method, for different numbers of sensor nodes placed in the measuring temperature space and the solid geometry of heat distribution in space are presented. Tested were done for over 15, over 75 and over 150 sensor nodes place in heat transfer space. Examples of meshes generated in the test cases, the heat propagation estimates, in the simplest case of more then 15 sensor, with temperature isotherms in 2 coordinate and the temperature estimates in isotherms in 3 coordinates are presented in Fig. 12, 13, 14.

6 Conclusion

The paper presents a short survey of some results obtained in practical application of some multivariable estimation techniques as: linear conventional AR models, neural models and adaptive network based fuzzy inference, on a general structure for identification of distributed
parameter systems, with some case studies for heat transfer in plane. Some advantages of using sensor networks were used in tests. Estimations methods were applied in the case of discovery of malicious nodes in wireless sensor networks.

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