# Optimal Interest Rate Derivatives Portfolio with Constrained Greeks-A stochastic Control Approach

Konstantinos Kiriakopoulos,Ph.d
Capital Markets, Director
Proton Bank
Lagoumitzi 17673, Athens
Greece
k_ kiriak@otenet.gr

George Kaimakamis
Lecturer
Hellenic Army Academy
Vari 16673, Athens
Greece
gmiamis@gmail.com

*Abstract:* This paper presents a method of managing interest rate portfolios with constrained sensitivities.The problem is formulated as a stochastic control portfolio optimization problem. The method is general enough so that it can applied equally well to trading and to risk managemnet level in a systematic way.The constraints imposed on the portfolio sensitivities (greeks) must be met at all times so that optimal positions do not contribute to unwanted risks. The method is dynamic by its nature and it can be used in a bottom up way so that additional VAR or CVAR constraints can be imposed as well.

*Key–Words:* interest rate,derivatives, portfolio sensitivities, stochastic control.

*Jel numbers:* G11,G12,G32,C61

## 1 Introduction

In this paper we present a method of managing portfolios of interest rate derivatives with specific risk-return characteristics. The derivatives can be any interest rate derivative instrument such as bonds, caps, floors, interest rate swaps, swaptions, constant maturity swaps, etc. This is accomplished through the maximization of portfolio value subject to constrained imposed on the portfolio sensitivities throughout the portfolio horizon. It is a special case of the broader case of maximization of expected utility of ternminal wealth of a portfolio of interest rate derivative instruments with constrained sensitivities[1]. Various papers try to tackle the portfolio maximization problem either under stochastic interest rate (but with no constraints imposed on portfolio value [18]) or with constraints imposed as expressed on value at risk [5],[4],[19] but with no explicit reference to the nature of the assets in the portfolio. The design of derivative portfolios with controlled risk-return characteristics has been of major importance especially in the interest rate area. We believe that the exact risk-return quantification (through constraints on the portfolio sensitivities and on the minimum portfolio appreciation rate) gives us the right means to construct portfolios with predetermined level of risk over their entire horizon and can achieve a maximum possible level of return given the risk preferences of the individual trader[2]. In this sense this method is analogous to the classical mean-variance methods applied to stocks (Markowitz, [15]). The method described is useful to risk management in order to quantify exactly the risk limits of the relevant proprietary desks, and to the proprietary desks themselves where risk taking is a normal part of the trading procedure. Also the method can be harmoniously combined with modern VAR and CVAR techniques([22],[23]) for better risk monitoring of a bank's derivative portofolio.
The problem is formulated as a portfolio maximization problem (Merton,[17]), with the additional characteristics that interest rate derivatives instead of primary securities are involved, and that constraints on the portfolio positions are time and state dependent. Employing the usual stochastic control methodology ([3]) , the utility function can be any continuous well behaved function so for practical reasons we choose the linear one (in essence we maximize the portfolio value). The algorithm proposed to solve the above problem is based directly on the discretization of the interest rate derivative securities. In practical terms this corresponds to the discretization of the interest rate, since the derivative securities and their sensitiv-

[1] A detailed treatment of the portfolio problem in the interest rate area with constraints in the sensitivities exists in [11] [12] or in risk arbitrage setting in [8]

[2] In this paper we do not distinguish the preferences of an individual trader from the preferences of the bank.

ities are calculated on each node of the interest rate discretization scheme. It has features borrowed from the stochastic control algorithms (Kushner, [13]), and from algorithms used for portfolio optimization on trees [7, 6][3].
The salient characteristic of the algorithm is that we no longer need an explicit discretization of the portfolio value. Furthermore, the algorithm averts the need to solve the problem for all possible initial portfolio values (a usual characteristic of control problems), but only for the given initial portfolio wealth. So the execution time of the algorithm is significantly reduced relative to such control algorithms. The algorithm is based on the construction of a Markov chain to approximate the portfolio process by matching the portfolio drift and variance at each node of the discretization scheme, and ultimately reduces to a single Non-Linear maximization Program (NLP).
Thus, instead of having many NLPs to solve (a feature encountered in stochastic control dynamic programming algorithms) we have one NLP but with several variables.
The paper is organised as follows: In the next section the necessary mathematical apparatus is briefly developed. In Section 3 control problem is formulated and the algorithm is developped. In section 4 a specific numerical example based on the widely used Hull and White trinomial interest rate model is presented. Conclusions follow in Section 5.

# 2 Preliminaries

We assume that the horizon of the portfolio is $T > 0$.We assume that the rate market is governed by the Ito type diffusion[4]

$$dr = \mu_r(r(t),t)dt + \sigma_r(r(t),t)dw(t) \qquad (1)$$

where $r = r(t)$ is the short rate, $\mu_r(r(t),t)$ and $\sigma_r(r(t),t)$ are the instantaneous drift and standard deviation of the rate respectively and $w(t)$ is a one-dimensional standard Brownian motion which generates the augmented right continuous filtration $\mathcal{F}_t, \quad \forall t \in [0,T]$ in the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$.
The interest rate market is assumed to be complete [10]. More precisely we assume the existence of a generic pure discount bond $B(r(t),t)$ with maturity greater than $T$, that "spans" the derivative market. We assume that we have $N$ interest rate derivative securities depending on the rate $r$ with maturities at least $T$. The vector of derivatives is denoted by

$$F(t,r(t)) = [F_1(t,r(t)), \cdots, F_N(t,r(t))]^\top.$$

where $F_i(t,r(t))$ denotes the price of the $i^{\text{th}}$ derivative at time $t$. The derivative security vector may represent any derivative depending on the rate $r$ or on the bond such as bonds, swaps, swaoptions,constant maturity swaps, caps, floors, european and/or american options depending on the rate $r$, or on the bond, etc. (In the following we may suppress the $r(t)$ and/or the $t$ from the functional expression of the derivatives according to our needs).
We denote the vector payoff rate per unit of time by $g(r(t),t)$ with the understanding that if the $i^{\text{th}}$ claim is American, then the corresponding entry in the vector will be zero after the stopping time $\tau_0^i$[5]. The terminal payoff rate is denoted by the $\mathbb{R}^N$ vector $f(r(t),t)$.
We define the **value process** of the portfolio for an initial investment $v$ to be the process given by the strong solution of the linear stochastic differential equation

$$\begin{aligned} dV^{(v,\theta)}(r(t),t) &= \theta^\top(t)dF(r(t),t) + \\ &\frac{(V^{(v,\theta)}(r(t),t) - \theta^\top(t)F(r(t),t))}{D(t)}dD(t) + \\ &\theta^\top(t)g(t)dt \end{aligned} \qquad (2)$$

The initial condition of the above S.D.E. is $V^{(v,\theta)}(r(0),0) = v$.
It is easy to see that the value process consists of positions in the securities and in the "locally riskless" bond $D(t)$ [6]. From the above S.D.E. we can deduce that the trading strategies (or equivalently the portfolio positions) are self-financing. Throughout this paper we impose the condition that $V^{(v,\theta)}(r(t),t) \geq 0 \ \forall t \in [0,T]$. This condition rules out arbitrage. The set of all strategies (or equivalently portfolio processes) such that the corresponding value process satisfies:
a. $V^{(v,\theta)}(r(0),0) = V^{(v,\theta)}(r(0),0) = v$
b. $V^{(v,\theta)}(r(t),t) \geq 0 \ \forall t \in [0,T]$
is called **admissible** for the portfolio $F$ and is symbolised by $\mathcal{A}(T,v)$. From now on the portfolio drift will be denoted by $\mu(t,V(t),\theta(t))$ and the portfolio variance by $\sigma(t,V(t),\theta(t))$. Usually the bank (and of course the individual interest rate traders) would like to construct "near" risk-free or risk-constrained portfolios. So the ideal situation would be to construct a portfolio in such a way that the risk-sensitivities of the whole portfolio would be constrained at the appropriate level over the **whole horizon** of the portfolio. In other words, the problem amounts to the choice

[3] applications of stochastic control exist in many other areas such as electrical enginnering ([24])

[4] This is assumed only for reasons of simplicity. In fact our method can be applied equally well for any type of interest rate model that is able to value interest rate derivative securities such as the widely used family of Libor Market Models [20]

[5] $\tau_0^i$ is the optimal exercise time of the $i^{\text{th}}$ derivative. If the derivative is European this is the expiration time

[6] $D(t) = \frac{1}{\beta(t)} = \exp(\int_0^t r(s)ds)$

of controls (portfolio positions) in order to have constrained portfolio delta, gamma and theta and at the same time restrict the portfolio growth from falling below a desired level. This has to be done whatever the state of the world is, or in mathematical terms "almost surely". Now we quantify the above ideas: We define the portfolio delta ($\Delta$), portfolio Gamma ($\Gamma$), portfolio theta ($\Theta$) to be the weighted sum of deltas, gammas and thetas of the individual derivatives respectively as follows[7]:

$$\begin{aligned}
\Delta(F,B,\theta)(t) &= \theta^{\top}(t)F_B(B(t)) \\
\Gamma(F,B,\theta)(t) &= \theta^{\top}(t)F_{BB}(B(t)) \\
\Theta(F,\theta)(t) &= \theta^{\top}(t)F_t(t)
\end{aligned}$$

It is a matter of preference whether we express the portfolio sensitivities with respect to the rate or the bond[8]. Short sale constraints and frictions in the market. For instance, it may be known that at a future date $t_1$, only a limited number $K_{t_1}$ of derivatives of type $i$ will be available in the market. The constraint can be described by the inequality $\theta_i(t_1) \leq K_{t_1}$, almost surely. We use two functions to encapsulate all the different kinds of anomalies or constraints imposed by the bank itself: all constraints in the form of an equality can be described by the $\mathbb{R}^{N1}$-valued function $G(t,F(t),Z(t),\theta(t)) = 0$ and all constraints that can be written in the form of an inequality can be described by the $\mathbb{R}^{N2}$-valued function $L(t,F(t),Z(t),\theta(t)) \leq 0$. So we can define the risk sensitivity set $U(F,t)$

$$U(F,t) = \left\{ \theta(t) \in \mathbb{R}^N, \begin{array}{c} |\Delta(F,B,\theta)(t)| \leq f_\delta(t) \\ |\Gamma(F,B,\theta)(t)| \leq f_\gamma(t) \\ \Theta(F,B,\theta)(t) \geq f_\theta(t) \\ A(F,r,\theta)(t) \geq f_A(t) \\ G(t,F(t),Z(t),\theta(t)) = 0 \\ L(t,F(t),Z(t),\theta(t)) \leq 0 \end{array} \right\}. \tag{3}$$

The control sets $U(F,t)$ are assumed to be nonempty closed convex sets of $\mathbb{R}^N$ [9]. We define the set $K = \{\theta(t) \in \mathbb{R}^N : \theta(t) \in U(F,t), m_{[0,T]} \otimes \mathcal{P} - \text{almost surely}\}$[10].

[7] $F_x$, $F_{xx}$ denote respectively the vector of first and second partial derivative of $F$ with respect to the scalar $x$

[8] Throughout this paper we calculate the portfolio gamma and delta as derivatives with respect to the bond price rather than with respect to the underlying short rate. We find that more intuitive appealing since a bond is a traded instrument. But there are situations where portfolio sensitivities defined with respect to a benchmark rate (e.g. LIBOR) are more appropriate.
Also in a multifactor interest rate model the portfolio delta is a matrix since in that case we have more than one bond that spans the market, and the portfolio gamma is a tensor.

[9] This is assumed in order that the theoretical existence of the solution of the problem to be guaranteed. More details exist in Kiriakopoulos [11], Chapter 3.

[10] $m_{[0,T]}$ denotes the Lebesgue measure in the interval $[0,T]$

We seek portfolio strategies that maximizes the expected utility of terminal wealth **and** belong to the $U(F,t)$[11]. Despite the fact that the unrestricted optimal portfolio strategies are not unique, we restrict ourselves to the optimal portfolio strategies that belong to the set $U(F,t)$. Provided that the functions $f_\delta(t), f_\gamma(t), f_\theta(t), f_A(t)$ are not chosen such that the set $U(F,t) \bigcap \widehat{\mathcal{A}}(T,v)$ is empty, we can have optimal portfolio strategies that reflect the risk management objectives and constraints set by the bank, the bank regulators and the market, over the whole horizon of the portfolio life.

# 3 Problem Formulation and Algorithm Presentation

This algorithm maximizes the expected terminal wealth subject to constraints on the portfolio sensitivities and to other constraints imposed by trading desks or by risk ranagement. One of the $N$ derivatives may be the cash account itself. The reason that we have incorporated the cash account into the vector of derivatives is that this gives us greater flexibility. One might question the necessity of a cash account given that the algorithm is intended for use in a trading environment, where usually there is not an explicit cash account as such, but instead short term money market instruments (like deposits or Treasury Bills). It is clearly the case that for the cash account the derivative that represents will have zero volatility term. We assume that the trader has a utility of terminal wealth $U(V)$.
We have the maximization program:

$$\max_{\theta \in \mathcal{A}'(T,v) \bigcap K} E[V^{(v,\theta}(T))]$$
$$\text{subject to}$$
$$E[\xi(T)V^{(v,\theta)}(T)] \leq v$$
$$\text{where}$$
$$\mathcal{A}'(T,v) = \{\theta \in \mathcal{A}(T,v), E[(V^{(v,\theta)}(T)] < \infty\}$$

where $\xi(t)$ is the state price deflator[12]

Also we assume that the linear equation 2 has a unique, strong solution for every initial condition $v > 0$[13].

[11] Since it has been implicitly assumed that the number of derivative securities is greater than the number of Brownian motions that spans the interest rate market, the unrestricted problem of utility maximization of terminal portfolio wealth has more than one solution ([11], Chapter 1). The set of the optimal portfolio strategies for the unrestricted problem is denoted by $\widehat{\mathcal{A}}(T,v)$.

[12] $\xi(t) = \beta(t)Z(t)$. If $a(t)$ is the usual market price of risk ([21])then $Z(t) = \exp\left(-\int_0^t a(s)dw(s) - \frac{1}{2}\int_0^t a^2(s)ds\right)$.

[13] In this paper we do not discriminate between original and equivalent martingale measure,and consequently between original $w(t)$ Brownian motion and equivalent "risk-neutral" one. This is something which is very common in the financial literature for valuing derivative securities.We make the optimization in the real

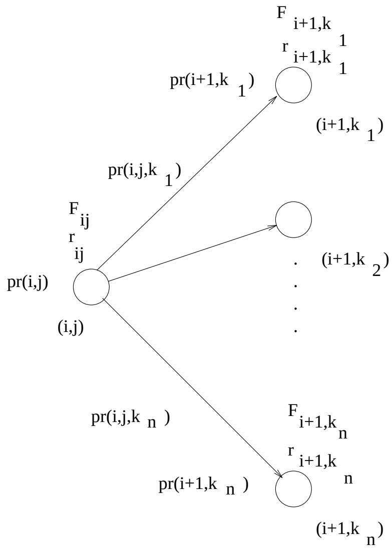


Figure 1: The branching process

We assume that we have a $n$-nomial non-recombining tree for the discretization of the rate [14]. So the node $(i, j)$ represents the ordered pair $(i\Delta t, r_{ij})$, where $\Delta t > 0$ is the time approximation step, and $r_{ij}$ is the approximating value of the rate. The value of the derivative vector at this node is denoted by $F_{ij}$ [15]. The probabilities emanating from this node to the $n$ nodes $(i + 1, k_1), \cdots, (i + 1, k_n)$ at time $(i + 1)\Delta t$ are denoted respectively by $pr(i, j, k_1), \cdots, pr(i, j, k_n)$ and the probability of **being** at node $(i, j)$ is denoted by $pr(i, j)$ (Figure 1). As usual, $(0, 0)$ is the starting node (and so $pr(0, 0) = 1$) with initial rate $r_{00} = r_0$ and portfolio budget $v$. Also the portfolio horizon is assumed to be $T = M\Delta t$. At each node of the tree we introduce an $\mathbb{R}^N$-valued vector $\theta_{ij}$ that represents the position in the derivatives at that particular node. So

$$\theta_{ij} = \begin{bmatrix} \theta_{1,ij} \\ \vdots \\ \theta_{N,ij} \end{bmatrix}$$

where $\theta_{k,ij}$ is the position in the $k$-derivative at node $(i, j)$. Let us assume that from the node $(i, j)$ the $n$ nodes that emanate have $y$ coordinates belonging to the set $K_{ij}$. We use $K_i$ to denote the set of all the $y$ coordinates of the nodes at time $i\Delta t$.

The expected terminal wealth is

$$\sum_{k \in K_M} pr(M, k) V_{Mk}, \tag{4}$$

world since we do not primarily interested in valuing securities but in finding optimal portfolios

[14]In general if our state variable is $k$-dimensional, then it can be approximated by an $k + 1$-dimensional Markov process. So for interest rate models with $k$ bonds that span the interest rate market, the tree that we use is $k + 1$-nomial, [1, 2, 6, 7].

[15]In general, if we have a variable $x$ its value at the node $(i, j)$ is denoted by $x_{ij}$.

and our objective is to maximize it. Since the solution of the portfolio value S.D.E. [2] is a Markov process, the local consistency condition [14], page 71, for the mean of the portfolio equation $V$ becomes:

$$\sum_{k \in K_{ij}} pr(i, j, k) \left[V_{(i+1)k} - V_{ij}\right] = \mu(i\Delta t, V_{ij}, \theta_{ij})\Delta t, \quad \forall (i, j), i = 0, \cdots, M - 1,\ j \in K_i. \tag{5}$$

The local consistency equation for the portfolio variance becomes

$$\begin{aligned} \textstyle\sum_{k \in K_{ij}} pr(i, j, k) \left[V_{(i+1)k} - V_{ij}\right]^2 - \{\mu(i\Delta t, V_{ij}, \theta_{ij})\Delta t\}^2 \\ = \sigma^2(i\Delta t, V_{ij}, \theta_{ij})\Delta t, \\ \forall (i, j), i = 0, \cdots, M - 1,\ j \in K_i. \end{aligned} \tag{6}$$

Also at each node $(i, j)$ the portfolio value is equal to the portfolio position times the price of the appropriate interest rate derivative plus the payoff of the derivative at that node. So, we have

$$V_{ij} = \theta_{ij}^{\top} \left[F_{ij} + g_{ij}\right], \ \forall (i, j) : i = 0, \cdots, M,\ j \in K_i. \tag{7}$$

The budget constraint is $V_{00} = v > 0$, where $v$ is any positive number. Thus, from equations [4, 5, 6, 7] we have the maximization program on the discretization tree The result of this non-linear optimization program is the portfolio positions $\theta_{ij},\ i = 0, \cdots, M,\ j \in K_i$ at each node of the tree, so that the portfolio value is maximised in a way consistent with our risk preferences (expressed through control sets $U(F_{ij}, i\Delta t)$). We note that the portfolio position is chosen so that the risk limits imposed by risk management hold almost surely, that is to say, at every node of our tree. We also observe that when the utility function has a linear form, then our problem reduces to the maximization of terminal wealth of our portfolio. Finally we note that the above is a non-convex optimization problem[16]. In most practical applications, where we are interested only in the maximization of portfolio value, convexification is possible due to the quadratic nature of the local variance consistency constraints.

# 4 Numerical Results

In the implementation of the above algorithm, the interest rate model that we have used is the generalised model of Hull and White [9]. A feature of this model is that that it fits the initial term structure and the initial volatility structure of interest rates. The volatility of the short rate is assumed to have the functional form be $\sigma_r(t, r(t)) = \sigma_r(0, r(0))r(t)^{\beta}$. The model characteristics are displayed in Table 1.

The trinomial Hull and White model has been used for valuing all the derivatives and for calculating all the hedging parameters in the portfolio. We include in the portfolio the following twelve types of interest rate derivative

Declining Initial Term and Volatility Structure

| | |
|---|---|
| Time Horizon $(M)$ | 20 |
| Time Step $(\Delta t)$ | 1 |
| $\beta$ | 1 |
| Rate at time $i\Delta t$ | $0.1(1-(i\Delta t)/300)$ |
| Volality at time $i\Delta t$ | $0.14(1-(i\Delta t)/300)$ |

Table 1: Yield curve characteristics

securities: bonds, swaps, European and American calls on a bond, European and American puts on a bond,European and American payer's swaoptions, European and American receiver's swaoptions, caps and floors [16]. We assume that the utility function is logarithmic i.e. $U(x) = \ln(x)$. For simplicity we have assumed that the portfolio sensitivity bounds are constant. They are given in Table 2. Also we have not specified minimum local appreciation constraints.

Risk Limits

| | |
|---|---|
| Delta limit $(\Delta)$ | 0.513483 |
| Gamma limit $(\Gamma)$ | 0.315038 |
| Theta limit $(\Theta)$ | 5.420731 |

Table 2: Portfolio risk profile

The portfolio positions[17] in the derivative securities are assumed to be in the interval $[-10, 10]$.
Our portfolio horizon is 7 periods and the number of nodes used for the optimization is 57. The total number of variables introduced is 684. The portfolio budget $v$ is equal $\$103.5$. Results are shown in figures 2,3,4,5

From the figures above we can see that the optimal portfolio greeks are within the boundaries set in table 2.

# 5 Conclusion

In this paper we have designed a tree-based algorithm for the problem of maximizing the expected value of a derivatives portfolio subject to constraints on the portfolio sensitivities. Its main advantage is that it does not require another discretization axis for the portfolio value. Instead, using the local consistency conditions, it discretizes the portfolio S.D.E. on the existing tree structure. The algorithm is flexible enough so that various interest rate models can be used so that the interest derivative claims can be valued. The various LP's is very easy to solve and the optimal portfolios can have the required risk/return characteristics set by the trading desks or the risk management of the bank.

[16] The derivatives characteristics are available from the authors upon request

[17] in numbers

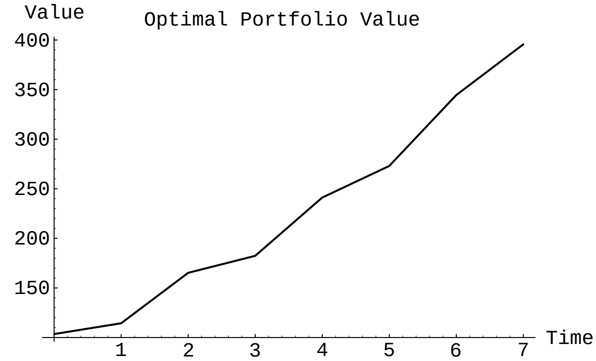


Figure 2: Optimal Portfolio Value

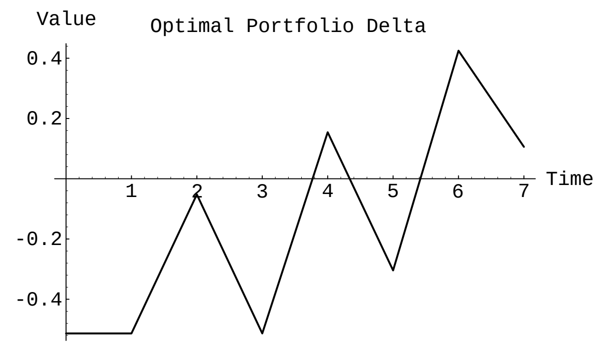


Figure 3: Optimal Portfolio Delta

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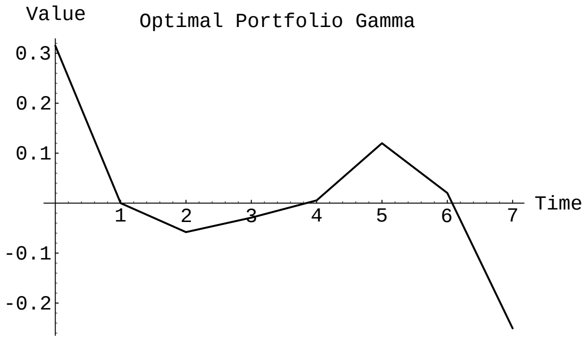


Figure 4: Optimal Portfolio Gamma

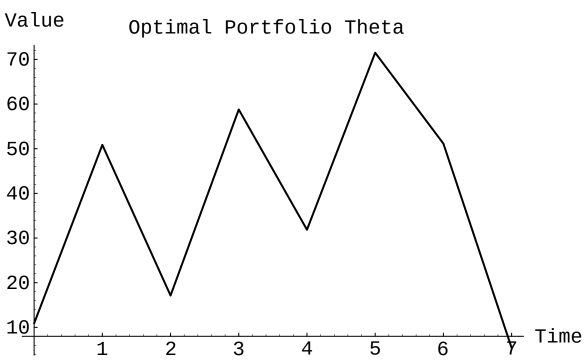


Figure 5: Optimal Portfolio Theta

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