Model-based velocity control of electrohydraulic servo systems

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Abstract: It is proposed a strategy which eliminates some nonlinearities of electrohydraulic systems. Using some properties of these systems more simple models are provided with good approximations on the whole operational area. Considering the velocity control problem of nonlinear hydraulic servo systems a model-based controller is designed subjected to known and time-varying parameters. Numerical results were provided to show the effectiveness of proposed approach.

Key-Words: electrohydraulic, velocity control, modeling, reduced model.

1 Introduction
Hydraulic cylinder actuators are widely used in many fluid power systems, such as robots, aircraft, construction machinery, and agricultural machinery. The hydraulic system typically includes a pump for supplying pressurized hydraulic fluid and a directional valve for controlling the flow of hydraulic fluid to a hydraulically actuated device such as an actuator, cylinder, or motor. One of the fundamental tasks in designing hydraulic actuating systems is the development of effective velocity control of the actuator. Supply pressure variations and varying loads affect the flow rate and therefore, cause the control system to produce undesirable behavior. The conventional approach to the controller design of these systems may not assure satisfactory control performance. Many researchers attempted to develop adequate controls for accurate and efficient control of nonlinear systems.

The traditional and widely used approach to the control of hydraulic systems is based on the local linearization of the nonlinear dynamics about a nominal operating point [1], [2]. Reference [3] proposed the use of pressure feedback to improve the performance of classical PD controller. The last years demonstrated that nonlinear control techniques, can deliver high performance to hydraulic systems: a backstepping approach [4] was used to improve the motion accuracy for a hydraulic actuator with a three-way valve; load uncertainty parameters were taken into account to result in a precise motion of a single-rod hydraulic actuator [5]. These control strategies use nonlinear models to design the controllers.

Other papers proposed adaptive controllers [6] based on linearized models. Fuzzy logic control is another capable control technology for handling the high nonlinearity inherent in hydraulic actuating systems [7]. Articles [6], [8] demonstrated that the application of nonlinear robust control techniques is a necessity for successful operation of electro-hydraulic systems.

The dynamics of these systems are highly nonlinear and their models inevitable contain parametric uncertainties and unmodeled dynamics. Here, it is proposed a strategy which eliminates some nonlinearities of these systems. Using mathematical transformations more simple models are provided with good approximations on the whole area of operation of these systems. Then a class of models will be used in order to design a velocity controller.

The paper is organized as follows. In section 2 the dynamic equations of the system under study are presented. The reduced model is developed in section 3. Control strategy and numerical results are presented in section 4 and finally conclusions are drawn in section 5.

2 System dynamics
The hydraulic servo system (Fig.1) is made up of a proportional valve MOOG DDV633, a pump unit with constant flow rate, a pressure relief valve, a linear motor and sensors in order to determine the state parameters of system.

Fig. 1. The hydraulic servo system.
The global model of hydraulic servo system is obtained by combining physical model equations [4, 5] which describe the actuator, valve and pressure dynamics. By defining $y = x_1$, $y' = x_2$, $p_1 = x_3$, $p_2 = x_4$, $x_5 = x_3$ the state space representation of model is:

$$\dot{x}_1 = y = x_2$$

$$\dot{x}_2 = \frac{1}{m}[p_A - p_S - \text{sign}(y)(F_c - c|x_2|) - F_R]$$

$$\dot{x}_3 = \frac{e}{V_{oi} + A_x} [Q_{c1} - A_x x_2 - k_1(x_3 - x_4)]$$

$$\dot{x}_4 = \frac{e}{V_{oi} + A_x (s - x_3)} [-Q_{c2} + A_x x_2 + k_1(x_3 - x_4)]$$

$$\dot{x}_5 = -x_4 + k U$$

where, parameters of system are described in Table 1, and $Q_{c1}$ and $Q_{c2}$ are defined by (6) and (7)

$$Q_{c1} = Q_s = C_s \sqrt{\frac{2}{\rho}} w x_s \sqrt{p_3 - p_1}$$

$$Q_{c2} = Q_s = C_s \sqrt{\frac{2}{\rho}} w x_s \sqrt{p_2}$$

$$Q_{c1} = Q_s = C_s \sqrt{\frac{2}{\rho}} w x_s \sqrt{p_3 - p_1}$$

$$Q_{c2} = Q_s = C_s \sqrt{\frac{2}{\rho}} w x_s \sqrt{p_2} - p_2$$

These linear models can be used then to design a velocity controller.

In order to reduce the nonlinearities of system, a new state variable ($x_5$) is introduced through (8). Also the effect of Coulombian friction, hydraulic leakages and dead volumes $V_{oi}$, $V_{oi}$ are ignored. Two important relations were identified: (9) for the advance stroke and (10) for the return stroke. These two relations can be approximated with zero, if and only if, the supply pressure of system $p_s$ is constant. A similar procedure is described in [9] but only for symmetric motors.

$$x_p = p_1 - \alpha x_4 = x_3 - \alpha x_4$$

$$E_a(p_1, p_2) = p_1 + \frac{1}{\alpha^2} p_2$$

(8)

$$E_a(p_1, p_2) = p_1 + \frac{1}{\alpha^2} p_2 = 0$$

(9)

$$E_r(p_1, p_2) = \alpha^2 p_1 + p_2 = 0$$

(10)

Now, we will evaluate the value of $E_a(p_1, p_2)$:

$$\alpha^2 \frac{\partial}{\partial t} p_1 + \frac{1}{\alpha^2} \frac{\partial}{\partial t} p_2 = \frac{\partial}{\partial t} \left( p_1 \frac{\alpha^2}{1 + \alpha^3} x_3 \right)$$

$$\frac{1}{\alpha^2} \frac{\partial}{\partial t} \left( p_2 \alpha^2 + x_3 \right) + \frac{1}{\alpha^2} \frac{\partial}{\partial t} \left( p_5 - x_3 \right) = \frac{\alpha^2}{1 + \alpha^3}$$

(11)

Now, we will evaluate the value of $E_r(p_1, p_2)$:

$$p_5 = \alpha^2 p_1 + p_2$$

(12)

Based on (8) and (11), $p_1$ and $p_2$ pressures are computed:

$$p_1 = \frac{p_s + x_3}{1 + \alpha^3}$$

$$p_2 = (p_5 - x_3) \frac{\alpha^2}{1 + \alpha^3}$$

(13)

Based on (8) and (13), $p_1$ and $p_2$ are:

$$p_1 = \frac{p_s}{1 + \alpha^3} x_3$$

$$p_2 = (p_5 - x_3) \frac{\alpha^2}{1 + \alpha^3}$$

(14)

Now, we can evaluate the value of $E_a(p_1, p_2)$:

$$\alpha^2 \frac{\partial}{\partial t} p_1 + \frac{\partial}{\partial t} p_2 = \frac{\alpha^2}{1 + \alpha^3} \frac{\partial}{\partial t} (p_5 + x_3) + \frac{1}{1 + \alpha^3} \frac{\partial}{\partial t} (p_5 - x_3) = 0$$

(15)

3 Reduced model of servo system

Once we have the nonlinear model, the objective is to approximate initial system by means of a linear model, or a family of linear models.

Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{oi}$</td>
<td>80 $10^{-4}$ m³</td>
<td>lines volume (include dead volume)</td>
</tr>
<tr>
<td>$A_x/A_y$</td>
<td>0.39</td>
<td>area coefficient</td>
</tr>
<tr>
<td>$s$</td>
<td>200 mm</td>
<td>piston stroke</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.5 $10^3$ N/m²</td>
<td>fluid bulk modulus</td>
</tr>
<tr>
<td>$k_1$</td>
<td>$c_0$</td>
<td>leakage coefficient</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0.61</td>
<td>valve discharge coefficient</td>
</tr>
<tr>
<td>$x_s$</td>
<td>2 mm</td>
<td>valve spool displacement</td>
</tr>
<tr>
<td>$w$</td>
<td>1.6 mm</td>
<td>valve area gradient</td>
</tr>
<tr>
<td>$F_c$</td>
<td>4 N</td>
<td>Coulombian friction</td>
</tr>
<tr>
<td>$c$</td>
<td>50 s⁻¹/m</td>
<td>viscous friction coefficient</td>
</tr>
<tr>
<td>$\rho$</td>
<td>890 kg/m³</td>
<td>mass density</td>
</tr>
<tr>
<td>$U$</td>
<td>+10V</td>
<td>control signal</td>
</tr>
<tr>
<td>$k_1$</td>
<td>100</td>
<td>valve gain</td>
</tr>
<tr>
<td>$\tau$</td>
<td>10 ms</td>
<td>time constant</td>
</tr>
<tr>
<td>$p_1$, $p_2$</td>
<td>-</td>
<td>pressure in cylinder chambers</td>
</tr>
<tr>
<td>$Q_{c1}$, $Q_{c2}$</td>
<td>-</td>
<td>flow through the valve resistances</td>
</tr>
<tr>
<td>$y$, $y'$</td>
<td>-</td>
<td>piston displacement, velocity</td>
</tr>
<tr>
<td>$F_s$, $F_f$</td>
<td>-</td>
<td>load, friction force</td>
</tr>
</tbody>
</table>
Based on property (9), equation (3) and (4), it’s easy to obtain (18) (for the advance stroke), using the next transformation:

\[ \dot{p}_1 = K x_5 \sqrt{p_5 - x_p} \frac{1}{\sqrt{1 + \alpha^2}} - A x_2 \]

\[ \dot{p}_1 = \frac{-A x_2}{\varepsilon} \alpha \dot{p}_2 + \frac{A s}{\varepsilon} \phi \dot{p}_2 = K x_5 \sqrt{p_5 - x_p} \frac{1}{\sqrt{1 + \alpha^2}} + \alpha A x_2 \]

Using again (9) yields,

\[ \dot{p}_2 = -\frac{\varepsilon}{A s} \left( \frac{\alpha^2 + 1}{\alpha^2} \right) \left( K x_5 \sqrt{p_5 - x_p} \frac{1}{\sqrt{1 + \alpha^2}} - A x_2 \right) \]

Using (15)-(16)* \( \alpha^3 \)

\[ \dot{x}_p = \dot{p}_1 - \alpha \dot{p}_2 = K x_5 p_5 - x_p - K_2 x_2 \]

where:

\[ K_1 = K x_5 \frac{\varepsilon K (\alpha^2 + 1)(\alpha^3 + 1)}{\sqrt{1 + \alpha^2} A s} \]

\[ K_2 = \frac{A x_2}{\varepsilon} \frac{\alpha^2 + 1}{\alpha^2} \frac{1}{\sqrt{1 + \alpha^2}} \]

\[ K = C_s \frac{2}{\rho} \]

Using a similar procedure for the return stroke yields

\[ \dot{x}_o = \dot{p}_1 - \alpha \dot{p}_2 = K x_5 x_p \sqrt{\alpha p_s + x_p} - K_4 x_2 \]

where:

\[ K_1 = \frac{\varepsilon K (\alpha^2 + 1)(\alpha^3 + 1)}{\sqrt{1 + \alpha^2} A s} \]

\[ K_4 = \frac{\varepsilon (\alpha^2 + 1)(\alpha^3 + 1)}{\alpha^2} \]

So, special properties (9) and (10), the assumption of equal dead volumes and negligible, and also some mathematical transformations lead to a reduced model with two components, (27) and (28), first for the advance and second for the return stroke, where \( x_2 \) is the actuator velocity.

For tracking processes the variation of \( x_p \) has small values around of zero. So, the nonlinear terms \((\sqrt{p_5 - x_p}, \sqrt{\alpha p_s + x_p})\) can be well approximated by means of binomial expansions.

\[ \dot{x}_2 = -\frac{c}{m} x_2 + \frac{A}{m} x_p \]

\[ \dot{x}_2 = \frac{c}{m} x_2 + \frac{A}{m} x_p \]

\[ \dot{x}_3 = -\frac{1}{\tau} x_3 + \frac{k}{\tau} U \]

Substituting old states \( x \) with \( z \) yields a reduced model with one time-varying coefficient: (29) for the advance and (30) for the return stroke.

\[ \dot{z}_2 = -\frac{c}{m} z_2 + \frac{A}{m} z_2 \]

\[ \dot{z}_2 = \frac{c}{m} z_2 + \frac{A}{m} z_2 \]

\[ \dot{z}_3 = -\frac{1}{\tau} z_3 + \frac{k}{\tau} U \]

\[ \dot{z}_3 = \frac{1}{\tau} z_3 + \frac{k}{\tau} U \]

where: \( x_2 = z_i, x_p = z_2, x_3 = z_3 \), \( K_i(z_i) = \frac{K_i}{2\sqrt{p_s}}, K_i(z_3) = \frac{K_i}{2\sqrt{\alpha p_s}} \)

The numerical results (Fig. 2.) prove that the reduced model can approximate the nonlinear model and also all state variables of system.
4 Control strategy. Numerical results

Considering the reduced model of system (29),(30), it’s easy to observe that time-varying coefficients $K_a$ and $K_r$ depend only by spool movement $z_3$. So, if we have a small family of linear systems which can approximate the nonlinear system, then it’s not difficult to build a state feedback controller with variable gain. In order to design the velocity controller the reduced model will be modified through discretization of $K_a$ and $K_r$ at specified intervals. The interval around which time-varying coefficients are quantized is based on approximation performances. Fig 3 shows the approximation results for different levels of discretization: 2, 5 and 10 intervals.

So, the proposed reduced model will be used to design the controller using only 2 or 3 discretization intervals, and therefore, 4 or 6 linear models. The feedback variables were then linear interpolated. Fig.4 shows the velocity controller performances (MBVC) versus a model-based predictive controller (MPC) which use a linearized model around central position of spool valve.

5 Conclusion

The generic nonlinear model of an electrohydraulic system was reduced to a family of linear systems which can approximate the behavior of these systems on the whole operational area. They were identified two terms which reduce the nonlinearities of hydraulic servo systems. A model-based controller was designed for velocity control in these systems. The numerical results show the effectiveness of proposed control approach. Future researches will be addressed to improve the control strategy using a model-based adaptive controller which includes also the load influence.

References: