The flow characteristics in a Sink-Swirl Flow within Two Disks

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Abstract: The flow characteristics in a sink-swirl flow within two disks are examined experimentally and numerically using the Reynolds stress model RSM. The predictions are compared against the experimental data in terms of the pressure drop and radial pressure profile. The overall agreement between the measurements and the predictions is reasonable. The experiments and the predictions have been carried out for three bulk Reynolds numbers $R_e$ of 5172, 3900 and 2598 and four contraction ratios $\beta$ of 24, 28, 34 and 48. The results show clearly that as the contraction ratio and the Reynolds number increase, the pressure coefficient $C_p$ increases. Stronger vortices will be produced resulting in a higher tangential velocity and hence a higher pressure drop. It is clear that the pressure decreases slowly from the inlet up to halfway along the disk and then decreases sharply as the flow approach the exit hole. This indicates that the flow moved towards a more favorable pressure gradient, hence the vortex flow accelerated according to the angular momentum conservation.

Keywords: Two disks; Sink-swirl flow; Reynolds stress turbulent model; Vortex generator

1 Introduction

The flow between two disks has received much attention due to its significant relevance to many practical applications in many areas, such as turbo-machinery, heat exchangers, double disc valves, bearings, squeeze film dampers, face seals, viscometer, and power transmission systems. In all confined vortex applications, it is important to understand adequately the overall flow field evolution as a function of both the geometrical and flow parameters. A good knowledge of these flows will improve the design and performance of a variety of vortex devices.

Little work has been done on inlet swirl flow between two stationary disks. DeSantis and Rakowsky [1] demonstrated experimentally the flow features in a sink-swirl flow within two discs, the radial and tangential velocity profiles were obtained using hot-wire anemometry. The swirling flow within a short vortex chamber with a small gap has been investigated analytically by Kwok et al [2]. Their results show that the values of apparent viscosity affect the velocity profiles within the vortex chamber. Savino and Keshock [3] investigated the swirling flow properties experimentally between a flat cylindrical chamber. They observed that the magnitude of radial and tangential velocities increases as the disc radius decreases. In addition, they showed a depression in radial profiles around the mid gap. Singh et al. [4] have studied the problem for inward flow between two stationary parallel disks. The experimental study involves the measurement of velocity field with laser Doppler anemometry and the mean governing equations have been solved numerically using $\kappa$-$\varepsilon$ model. They concluded that the velocity and pressure predicted numerically are close to experimental data. Murphy et al. [5] studied the inward flow numerically. They showed three distinguish regions, a region of strong viscous effect, a region where the viscous effect and inertia effect are equal and a region of strong inertial effect. Reynolds stress models have greater potential to represent turbulent flow phenomena more correctly than the two-equation models especially for swirling flow (Ferziger and Peric [6]). The main complexity in numerical modeling of complex turbulent swirling flows is the selection of suitable turbulence closure models. In simple flow cases the $\kappa$-$\varepsilon$ model performs well. However, it may be failed for strongly swirling flows that involves rigorous streamline bending. The last conclusion is clearly evident in a variety of studies; see for example the work of Nallasamy [7], Nejad et al. [8] and Weber [9]. A review of second-moment computations for engineering flows has been provided by Launder [10], and Leschziner [11]. The results of these computations demonstrate the power of RSM over eddy-viscosity models for curved flows, swirling flows and recirculation flows. German and Mahmud [12] have shown that the overall agreement between the measurements and the predictions obtained with both $\kappa$-$\varepsilon$ and Reynolds-stress turbulence models are reasonably good. However, some features of the isothermal and combusting flow fields are better predicted by the Reynolds-stress model. Jones and Pascau [13] and
Hockstra et al. [14] used the $\kappa$-$\epsilon$ turbulent model and a Reynolds stress transport equation model of a strong confined swirling flow. Once more, comparisons of the results with measurements show the superiority of the transport equation model, where $\kappa$-$\epsilon$ gave some discrepancies between the measured and predicted velocity fields.

The purpose of the present work is to study the pressure drop and the radial pressure in a sink-swirl flow within two disks with aspect ratio (defined here as the disk diameter / gap height ratio) equal to 60 experimentally and numerically (Fluent Inc., 2003) using the control-volume-based finite difference method.

## 2 Experimental Set-up

The present experiments have been conducted using a jet-driven swirl generator similar to the one utilized by Vatistas et al. [15]. The main difference between the two is that in the latest version, shown schematically in Fig. 1. It consists of two stationary parallel disks of radius $r_o = 381$ mm with a gap height $H = 12.7$ mm. The two disks were fabricated from Plexiglas. The upper disk with a concentric hole was connected to a suction compressor. The exit area is adjusted by replacing hole-plates with four different exit radius $r_e$ of 15.875, 13.335, 11.113 and 7.938 mm, which are corresponding to four contraction ratios $\beta = r_o/r_e$: 24, 28, 34, and 48 respectively. The swirl generator, see Fig. 2, with an inlet angle $\phi = 30^\circ$ was used for the present experiments. The aluminum vortex generator ring was used to introduce swirl through the two disks. It had an outer diameter of 762 mm, an inner diameter of 610 mm and a thickness of 12.7 mm. When air is sucked through the swirl generator, in addition to the radial velocity, it also develops a tangential component that depends on the value of the inlet angle. The generator had 48 inclined holes with diameter $D_{in} = 6.35$ mm, hence the total inlet area ($A_{in}$) is equal to $1.52 \times 10^3$ m$^2$. Measurements were made at three inlet air flow rates $Q_{in}$ of 0.0187, 0.0141, and 0.0094 (m$^3$/s) which are corresponding to three bulk Reynolds numbers ($R_e$=U$_{in}H$/v) of 5172, 3900, and 2598 respectively, where $U_{in}$ is the inlet radial velocity, and $\nu$ the kinematic viscosity. In order to obtain accurate measurements, a sensitive inclined manometer was used to measure the static pressure along the radius. The manometer contains Meriam oil with specific gravity equal to 1. The measurements of static pressure along the radius were obtained using 12 static pressure taps, in addition the measurements of the mean gage pressure difference $p_{out}$-$p_{in}$ were obtained, where $p_{out}$ and $p_{in}$ are the static pressure at the outlet and inlet respectively.

The estimated uncertainty is less than $\pm 8\%$ for the static pressure measurements. Air at standard temperature was the working fluid. The volumetric flow rate was recorded using a calibrated variable area rotameter which was located between the outlet of the experimental apparatus and the inlet (suction) of the compressor. This was carefully calibrated in standard conditions (1 atmosphere and $20\pm0.5^\circ$C). For the flow rate used, the uncertainty was estimated to be $\pm 2\%$.

### 3 Computational Details

The continuity and momentum equations can be written in tensor form as:

$$\frac{\partial}{\partial x_j}(\rho u_j) = 0 \tag{1}$$

$$\frac{\partial}{\partial x_j}((\rho u_i u_j)) = \frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_i}{\partial x_i} \right) \right]$$

$$+ \frac{\partial}{\partial x_j}(-\rho \overrightarrow{u_i} \overrightarrow{u_j}) \tag{2}$$

The Reynolds stress model (Launder et al., [16]) involves calculation of the individual Reynolds stresses, $\overrightarrow{u_i} \overrightarrow{u_j}$, using differential transport equations. The individual Reynolds stresses are then used to obtain
The transport equations for the transport of the Reynolds stresses, $-\rho u_i' u_j'$, can be written as follows:

\[
\frac{\partial}{\partial x_i} \left( \rho u_i u_j' \right) = \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i u_j'}{\partial x_j} \right) + \frac{\partial}{\partial x_k} \left[ \mu \frac{\partial}{\partial x_k} (u_i' u_j') \right] - \\
\rho \left( u_i' u_j' \frac{\partial u_i}{\partial x_j} + u_i' u_j' \frac{\partial u_j}{\partial x_i} \right) + \Phi_{ij} - \frac{2}{3} \rho \epsilon \delta_{ij}
\]

(3)

The term on the left-hand side of this equation represents the convection, the terms on the right-hand side represent the turbulent diffusion as proposed by Lien and Leschziner [17], molecular diffusion, stress production, pressure strain and the dissipation respectively. The pressure strain term $\Phi_{ij}$ is simplified according to the proposal by Gibson and Launder [18]. The dissipation rate, $\epsilon$, is computed with a model transport equation as

\[
\frac{\partial}{\partial x_i} \left( \nu \epsilon \right) = \frac{\partial}{\partial x_j} \left( \mu \epsilon \frac{\partial \epsilon}{\partial x_j} \right) - \frac{\epsilon}{k} \left[ C_{\epsilon 1} \frac{1}{2} P_{ij} \frac{\epsilon}{k} - C_{\epsilon 2} \rho \frac{\epsilon^2}{k} \right]
\]

(4)

where $\sigma_k = 1.0$, $C_{\epsilon 1} = 1.44$, and $C_{\epsilon 2} = 1.92$ are constants taken from Launder and Spalding [19]. Due to rigorous pressure gradients, the non-equilibrium wall functions were used near wall as proposed by Kim and Choudhury [20].

**Inlet boundaries for $k$ and $\epsilon$:** The turbulence intensity $I$ can be estimated from the following formula:

\[
I = \frac{u'}{U_{in}}
\]

(5)

The turbulence length scale $l$ is a physical quantity related to the size of the large eddies that contain the energy in turbulent flows. An approximate relationship between $l$ and the physical size of the disk diameter $D_o$ is

\[
l = 0.07 D_o
\]

(6)

The relationship between the turbulent kinetic energy $k$, and turbulence intensity $I$, is

\[
k = \frac{3}{2} \left( U_{in} I \right)^2
\]

(7)

where $U_{in}$ is the inlet radial velocity. The turbulence dissipation rate $\varepsilon$ can be determined as

\[
\varepsilon = C_{\mu} \frac{\rho^{3/4} k^{3/2}}{l}
\]

(8)

### 4 Solution Procedure

The governing differential equations for mass, momentum, turbulent kinetic energy and its dissipation rate were solved using the control-volume-based finite difference method. The problem here is considered to be incompressible, steady, axisymmetric, turbulent swirling flows. In this case, we can model the flow in 2D (i.e., solve the axisymmetric swirl problem) and incorporate the prediction of the swirl velocity, see Fig. 3. The difficulties associated with the solution of strongly swirling flows can be attributed to high degree of coupling in the momentum equations. High fluid rotation gives rise to large radial pressure gradient which drives the flow in the meridional plane. This, in turn, determines the distribution of the swirl in the field. Hence, segregated, implicit solver, which is well-suited for the sharp pressure and velocity gradients are more appropriate for the flow under consideration. The mesh is sufficiently refined in order to resolve the expected large flow parameter gradients. The under-relaxation parameters on the velocities were selected 0.3-0.5 for the radial and 0.9 for the swirl velocity components.

**Figure 3: Computational domain**

There is a significant amount of swirl between the two disks. The proper choice depends on the strength of the inlet flow components. To characterize the degree of swirling flow, the inlet swirl ratio $S$ is defined as:

\[
S = \frac{V_{in}}{U_{in}}
\]

(9)

where $V_{in}$ and $U_{in}$ are the inlet radial and tangential velocity components. Then, the simulations were performed for inlet swirl ratio equal to 1.73. Triangular mesh elements were used. A grid independent solution study was made by performing the simulations for three different grids consisting of 14300, 26000 and 32010 nodes. A mesh refinement study showed a grid of 26000 nodes to be fine enough to capture all the flow features. Boundary conditions have to be specified in order to solve the governing equations. At the inlet the values can be calculated from the given conditions at
the inlet boundary, see Fig. 2. The total inlet velocity vector \( \mathbf{q}_{\text{in}} \) has two components \( V_{\text{in}} \) and \( U_{\text{in}} \) and they are related to each other by:

\[
U_{\text{in}} = q_{\text{in}} \sin(\varphi), \quad V_{\text{in}} = q_{\text{in}} \cos(\varphi), \quad q_{\text{in}} = \frac{Q_{\text{in}}}{A_{\text{in}}}
\]

However, at the outlet boundary there is no information about the velocity and pressure field and some assumptions have to be made. The diffusion fluxes in the direction normal to the exit plane are assumed to be zero. The pressure at the outlet boundary is calculated from the assumption that radial velocity at the exit is neglected since it does not have the space to develop, so that the pressure gradient from \( r \)-momentum is given by

\[
\frac{\partial p}{\partial r} = \rho V^2_r r
\]

At the solids walls, the no-slip condition was applied where the velocities at the walls were specified to be zero. The center-line boundary was considered axis of symmetry. The pressure–velocity coupling is handled by using the SIMPLE-algorithm, the pressure staggering option scheme was used for the pressure interpolation, the first order upwind schemes were used for momentum, swirl velocity, turbulence kinetic energy, turbulence dissipation rate, and Reynolds stresses. Convergence was assumed when the residual of the equations dropped more than 3 orders of magnitude.

5 Results and Discussion

The pressure drop coefficient is defined as:

\[
C_p = \frac{2\Delta p}{\rho q_{\text{in}}^2}
\]

where \( \Delta p = p_{\text{out}} - p_{\text{in}} \) is the pressure drop between the outlet and inlet. The estimated uncertainty for the pressure drop coefficient \( C_p \) has appeared at the maximum of ±8%. Figure 4 compares the present experimental data with the RSM prediction of the pressure drop coefficient \( C_p \). It is clear that as the contraction ratio \( \beta \) and the Reynolds number \( R_e \) increase, the pressure coefficient \( C_p \) increases. Stronger vortices will be produced by increasing the contraction ratio and/or Reynolds number resulting in a higher tangential velocity and hence a higher pressure drop. It can be seen that the Reynolds stress model gives good agreement with the present experimental data and the percentage difference error between the predicted and experiments is less than 9%.

The following analysis illustrates the mean radial pressure distribution profile \( \Pi \), and the predicted radial pressure will be compared with present experimental data. The mean radial pressure distribution profile is defined according to the following equation:

\[
\Pi = \frac{2(p(\bar{r}) - p(\bar{r} = 1))}{\rho q_{\text{in}}^2}
\]

where the normalized radius \( \bar{r} \) is given by \( \bar{r} = \frac{r}{r_o} \).

Figure 5 compares the present experimental data with the RSM prediction of the radial pressure for Reynolds number \( R_e = 5172 \) and contraction ratio \( \beta = 24 \). It is clear that the pressure decreases slowly from the inlet (periphery) up to halfway along the disk and then decreases sharply as the flow approach the exit hole. Air enters through the peripheral gap between the two disks and converges to the center where it discharges axially through the exit hole in one of the disks. This indicates that the flow moved towards a more favorable pressure gradient, hence the vortex flow accelerated according to the angular momentum conservation. It can be seen that the Reynolds stress model gives good agreement with the present experimental data and the percentage difference error between the predicted and experiments is less than 10%. The estimated uncertainty for the radial pressure \( \Pi \) has appeared at the maximum of ±9%.
6 Conclusions

Two disks flow with inlet swirl was investigated both experimentally and numerically at different geometry parameter and Reynolds number. The Reynolds stress model is capable to predict the pressure drop and the radial pressure profiles. The results show clearly that as the contraction ratio and the Reynolds number increase, the pressure coefficient increases. It is clear that the radial pressure decreases slowly from the inlet and then decreases sharply as the flow approach the exit hole. A comparison of the results with measurement shows clearly the ability of the Reynolds-stress turbulence model in capturing the major features of a confined swirling flow.

References:


