The influence of the way of modelling the radiative heat transfer on the temperature distribution in a charge heated inductively

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Abstract: The paper presents an analysis how the way of modelling radiative heat transfer influences on temperature distribution in a charge heated by induction. The calculations were provided as coupled allowed coupling of electromagnetic and temperature fields. Two-dimensional calculation model was used. The results were obtained from the experiment consisting of several calculation cases. They differ each other in the intensity of heating and the way in which the heat transfer was taken into consideration. There were modeled cases where multiple reflections phenomena was taken into account, effective emissivity was applied, free heat radiation to half space was analyzed and finally radiation heat transfer was completely neglected. Depending on the adopted models the maximum temperature deviation in the heated charge was determined as 296 °C.

Key-Words: Radiative heat transfer, induction heating.

1 Introduction
Induction heating seems to be more and more popular heating method. The design of induction heating systems takes advantage of computer simulations. A simulation model of an induction heating process must take into consideration an analysis of electromagnetic and temperature fields, and in some cases also the fields of phase transition or stress / displacement. Apart from the number of the physical fields calculated, corresponding models differ also in the applied simplifications.

The paper analyses the influence that those various models of radiative heat transfer have on the obtained results. In the experiment temperature distribution was monitored on two cross-sections \( p_1 \) and \( p_2 \) (see Fig. 4). Calculations were made by using professional software Flux 2D and the own single purpose program Trad prepared by the authors, which allows the calculation of radiative heat transfer with taking into account multiple reflections.

1 Calculation model
A heater model chosen for the experiment was one for flat charges with the geometry shown in Fig. 1. The basic dimensions of the model are as follows: height of the charge \( h_c = 0.1 \text{m} \), half of the width of the charge \( w_c = 0.02 \text{m} \), air gap between the charge and thermal insulation \( a_c = 0.007 \text{m} \), thickness of thermal insulation \( w_i = 0.005 \text{m} \), height of thermal insulation \( h_i = h_{iw} = 0.005 \text{m} \). The dimensions of the inductor are the following: height of inductor profile \( h_w = 0.012 \text{m} \), its width \( w_w = 0.01 \text{m} \), thickness of inductor’s wall \( t_w = 0.002 \text{m} \). The dimension of the model in the direction on axis \( z \) is 0.5m.

Fig. 1. Calculation model geometry
Looking at the design, it is a typical heating system because the general guidelines suggest that the minimal ratio of inductor length to charge length should be between 1.2 and 1.

The charge was made from non-magnetic steel (ASTM 321), thermal insulation was a plate made of ceramic fibre, and the inductor was made from copper (Fig.1, Fig.2).

![Fig. 2. Boundary conditions location](image)

The analysis of electromagnetic field was performed on the basis of Equation (1)[1], [2]:

$$\mathbf{j}\omega \gamma \mathbf{A} + \nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A}\right) = \gamma \nabla \mathbf{V}$$

where:
- \(A\) - magnetic vector potential, Vs/m,
- \(\omega\) - angular frequency, rad/s,
- \(\gamma\) - conductivity, S/m,
- \(\mu\) - magnetic permeability, Vs/(Am),
- \(V\) - electric potential of source, V.

With appropriate boundary conditions (2).

\[\mathbf{ae}, \mathbf{ef}, \mathbf{fd}, \mathbf{da} : \mathbf{A} = 0\]  

Thermal calculations were conducted on the basis of Fourier-Kirchhoff's equation (3) [1], [3]:

$$\rho c \frac{\partial T}{\partial t} + \nabla (\lambda \nabla T) = q$$

where:
- \(\rho\) - density, kg/m³,
- \(c\) - specific heat, J/(kg K),
- \(T\) - temperature, K,
- \(t\) - time, s,
- \(\lambda\) - thermal conductivity, W/(mK),
- \(q\) - volumetric density of heat sources, W/m³.

With appropriate boundary conditions. For all calculation variants the following boundary conditions (Fig.2) were identical:

\[\bar{a}, \bar{d}, \bar{a}, \bar{e} : - \lambda \frac{dT}{dn} = 0\]  

\[\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{h}, \bar{i}, \bar{j}, \bar{k}, \bar{l}, \bar{m}, \bar{n}, \bar{o}, \bar{p} : - \lambda \frac{dT}{dn} = \alpha (T - T_a) + \varepsilon \sigma (T^4 - T_a^4)\]

\(\alpha = 10, T_a = 20\)

for edges: \(\bar{a}, \bar{b}, \bar{c}, \bar{d} : \varepsilon = 0.6; \bar{m}, \bar{n}, \bar{o} : \varepsilon = 0.4; \bar{g}, \bar{h}, \bar{i}, \bar{j}, \bar{k}, \bar{l}, \bar{m} : \varepsilon = 0.5\)

\[\bar{b}, \bar{c}, \bar{g} : - \lambda \frac{dT}{dn} = \alpha (T - T_a)\]

\(\alpha = 10, T_a = 20\)

\[\bar{q}, \bar{r}, \bar{s}, \bar{t}, \bar{q}, \bar{r}, \bar{s}, \bar{t} : T = \text{const}\]

for edges: \(\bar{q}, \bar{r}, \bar{s}, \bar{t}, \bar{q} : T = 30 ^\circ C; \bar{e} : T = 20 ^\circ C\).

Depending on the variant, the boundary conditions describing heat transfer through radiation for edges \(\bar{b}, \bar{c}, \bar{g}\) were changed. They are discussed in detail in section 2.1.
2.1 Models of radiative heat transfer

The induction heating is often used as an introductory stage in the metal thermal treatment and forging. The temperatures in the charge, depending on the kind of the heated metal, may be different from 500 to 1200 °C. It is widely believed that for so high temperatures radiative heat transfer must not be neglected [4], [5], [6].

Some of the commercial programs that are used for a coupled analysis of electromagnetic and temperature fields, like Opera, do not allow for radiative heat transfer in any way. Others, like Flux, can model a boundary condition as in Equation (15), which represents radiative heat transfer to half-space. As can be seen in Fig.1, the charge is surrounded by a thermal insulation layer, which should be taken into account while considering radiative heat transfer.

There are programs which allow the assumption of a boundary condition for radiative heat transfer between two surfaces (which is sufficient for the adopted calculation model (Fig.1)), for example Fluent, but it cannot carry out a full analysis of electromagnetic field. It can be concluded from the specifications, that program Ansys allows for radiative heat transfer by the method of multiple reflections, however, the authors have not as yet come to a possibility of using this program.

Calculation model presented in Fig.1 suggests that radiative heat transfer in this system should be modeled taking into account multiple reflections.

A mathematical model presented below is written for a cylindrical system (as more complex one because it takes into consideration self-irradiance). A computer program Trad prepared on the basis of this model makes possible calculations for both flat and cylindrical systems.

The following simplifying assumptions were adopted in the experiment:

- The whole system is in thermal steady state (quasi-steady state);
- Both primary and reflected radiation are of diffusive (Lambertian) type;
- Radiating surfaces are curved (for cylindrical system);
- Radiating surfaces emit energy evenly through the whole spectral range (grey bodies).

A simple formula for radiation energy balance, after taking into account Lambert and Kirchhoff laws, yields a differential equation system of (8) and (9) is obtained [3], [7]. The solution of the equations system (8), (9) describes total radiant exitance emitted from the surfaces \( S_1 \) and \( S_2 \) (Fig. 3). Total radiant exitance \( U \) describes effective radiation of the surface, which is a sum of primary and reflected radiations. Equations (10) and (11) describe the resulting irradiance \( I \) and heat transfer \( Q \) for the surfaces \( S_1 \) and \( S_2 \), respectively.

It must be underlined that for curved surfaces systems the reflected radiation is the result both mutual- and self-irradiance. That is why Equation (8) contains integration over all surfaces. In the case of flat surfaces self-irradiance does not occur, thus in Equation (9) domain of integration was reduced to the opposite surface.

\[
U_1(x_1) = a_1 \sigma T_i^4 + (1 - a_1) \int_{S_1} k(x_1, \tilde{x}_1) U_1(\tilde{x}_1) d\tilde{S}_1 + \int_{S_2} k(x_1, x_2) U_2(x_2) dS_2
\]

\[ (8) \]

\[
U_2(x_2) = a_2 \sigma T_i^4 + (1 - a_2) \int_{S_1} k(x_2, x_1) U_1(x_1) dS_1
\]

\[ (9) \]

\[
I_i(x_i) = \frac{U_i(x_i) - a_i \sigma T_i^4}{1 - a_i}
\]

\[ (10) \]
\( Q_i(x_j) = a_i \left[ I_i(x_j) - \alpha T_i^4 \right] \) \hspace{1cm} (11)

\[ U_{ip} = a_i \sigma T_{ip}^4 + \left( 1 - a_i \right) \]

\[ \cdot \sum_{j=1}^{2n} k_{iqj} U_j \Delta S_j \]

\[ \Delta S_{ip} = \frac{1}{\Delta S_p \Delta S_{jq}} \int \left[ k(x_i, x_j) \right] dS_i \quad dS_j \] \hspace{1cm} (13)

where:

- \( U_i \) - total radiant exitance from \( i \)-th surface, W/m²,
- \( a \) - absorption coefficient \( a = \varepsilon \), -
- \( k(x_i, x_j) \) - optical coupling function (integral kernels), -
- \( I_i \) - irradiance on the \( i \)-th surface, W/m²,
- \( Q_i \) - radiative heat exchange, W/m²,
- \( \Delta S_{ip} \) - discrete element of surface \( i \)-th, -
- \( n_j \) - number of \( j \)-th surface elements, -
- \( i = 1,2 \) - index of surface, -

In a general case such a system of Fredholm integral equations of the second kind that was obtained cannot be solved by analytical methods. In this paper the solution was obtained with the use of numerical iterative method with a proper discretization of surface, owing to which equations (8) and (9) were reduced to linear equations (12). In order to minimize numerical errors resulting from a discrete division of the surface into elements the kernel \( k(x_i, x_j) \) was averaged by pairs of elements according to formula (13), where, depending on the density of surface division and the system geometry, the integral may be reduced do sum of a few terms.

\[ \varepsilon = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \] \hspace{1cm} (14)

\[ -\lambda \frac{dT}{dn} = \varepsilon \sigma (T^4 - T_a^4) \] \hspace{1cm} (15)

where:

- \( \varepsilon_1 \) - emissivity of \( bc \) edge, -
- \( \varepsilon_2 \) - emissivity of \( gl \) edge, -

Another way of modelling of the radiative heat transfer may be using of the effective emissivity formulation expressed by Equation (14) [3]. It is possible to use it because edges \( bc, gl \) are parallel and usually located close to each other. Emissivity determined in this way may be used in Equation (15) describing a flux of energy emitted by the heated surface, but still the problem of determining the ambient temperature \( T_a \) remains unsolved.

Another possible simplification is an assumption that the charge is not surrounded by a thermal insulation layer and emits energy freely to half-space.

The influence of such simplifications on the obtained results is presented below in the section 3.

3 Calculation experiment

Calculation variants differ in the way the radiative heat transfer is modelled on the edges \( bc, gl \), in assumed ambient temperature, and in the intensity of heating. The monitored quantity was temperature distribution obtained on cross-sections \( p_1 \) and \( p_2 \) (Fig. 4).

![Fig. 4. Cross-sections on which the temperature was monitored](image)

The calculations were conducted until the temperature in the heated charge was around 1000 °C for the variant in which radiative heat transfer was modelled using program \( Trad \), that is for 36 seconds for intensive heating and 420 seconds for non-intensive heating.

For intensive heating (inductor current 3000A) the following variants were considered:

- **MI1** - radiative heat transfer modelled with the use of program \( Trad \);
- **MI2** - radiative heat transfer modelled with the use of effective emissivity (14) and boundary condition (15). Ambient temperature \( T_a = 20 \, ^\circ \text{C} \);
MI3 - radiative heat transfer modelled as free emission to half-space, $T_a = 20 \degree C$

MI4 – radiative heat transfer neglected.

Similar calculation variants were carried out for non-intensive heating (inductor current 1000A):

- LI1 - radiative heat transfer modelled with the use of program Trad;
- LI2 - radiative heat transfer modelled with the use of effective emissivity (14) and boundary condition (15). Ambient temperature $T_a = 20 \degree C$;
- LI2a - as in LI2 but assumed ambient temperature $T_a = 1000\degree C$;
- LI2b - as in LI2 but assumed ambient temperature $T_a = 500 \degree C$;
- LI3 - radiative heat transfer modelled as free emission to half-space, $T_a = 20 \degree C$;
- LI4 - radiative heat transfer neglected.

Figure 5 presents an example temperature distribution for the whole heating system for variant MI1. In Figure 6 an analogous distribution is presented for variant MI2.

Comparing the two figures it can be seen that only taking into account multiple reflections allows modeling of temperature distribution in the whole system (charge, thermal insulation, inductor). A similar conclusion can be reached on the basis of Figure 7, where the distribution of temperature on section $p_1$ is presented for all variants of intensive heating.

Maximum temperature deviation in the charge on the cross-section is 29 \degree C. In relation to reference variant MI1 maximum temperature deviation is 19 \degree C (between variants MI1 and MI3). The temperature distribution closest to the reference variant was obtained for variant MI2, that is for the model using the effective emissivity.

The influence of the model of radiative heat transfer on the temperature distribution on the surface of the charge (section $p_2$) (Fig.9) is similar. Maximum temperature deviation in comparison with the reference variant in this case is 16 \degree C. From the diagram it can also be concluded that modeling of radiative heat transfer using effective emissivity produces the results closest to the ones obtained with the use of the model allowing for multiple reflections.

Figures 10 and 11 present temperatures distributions obtained for non-intensive heating. As it could be
expected, the differences between temperatures are much more significant in this case.

Fig. 8. Temperatures distributions on section p₁ limited to the area of the charge for intensive heating after 36 seconds

Maximum temperature deviation in the area of the charge on cross-section p₁ in relation to reference variant is 162 °C. The smallest deviation can be observed in the case where the modeling of the radiative heat transfer was neglected. A similar dependency can be seen in Fig. 11 presenting temperatures distributions on section p₂ for non-intensive heating.

The presented experiment does not allow any general conclusions. Yet as it can be seen in Figures 9 to 11, the sole change in intensity of heating causes that different simplified models of radiative heat transfer produce results closest to the referential results.

Fig. 9. Temperatures distributions on section p₂ for intensive heating after 36 seconds

Fig. 10. Temperatures distributions on section p₁ limited to the area of the charge for non-intensive heating after 420 seconds

For intensive heating the best simplification is to use effective emissivity (variant MI₂), for non-intensive heating the results that are closest to the referential ones are obtained in the model in which radiative heat transfer is neglected (LI₄). It can be thus concluded that it is very difficult to select a simplified model of radiative heat transfer without adversely affecting the accuracy of the calculations.

Fig. 11. Temperatures distributions on section p₂ for non-intensive heating after 420 seconds

4 Conclusion
The paper analyses the influence of different models of radiative heat transfer on the temperature distribution in the induction-heated charge. Ten different models were analysed. The model taking into account multiple reflections was adopted as a referential variant (the authors’ program Trad was used).

In the case of intensive heating, the temperature distributions that were closest to the referential distributions were obtained for variant MI₂, in which the heat transfer was modeled with effective emissivity. However, in the case of non-intensive heating the best results were obtained in the variant where the transfer was neglected.

Therefore, it can be concluded that simplified models of radiative heat transfer cannot be used if the accuracy of calculations is to be maintained. Additionally, only the model allowing for multiple reflections makes it possible to carry out an analysis of temperature field in the whole heating system consisting of the charge, thermal insulation and inductor.
Still, using Trad is not flawless. First of all, the calculation time is significantly longer, for example in the case under consideration, one time step without Trad lasted 2 minutes, while the same step with Trad took around 30 minutes. Also, in the case when the heat flux exchanged between surfaces is substantial a question of the calculation stability occurs. This problem could be partly solved by shortening of the time step.

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