Abstract: - This paper presents a study upon the design of 2D chaotic trajectories for an intelligent vehicle based on two points of interest that has to be monitored, in the case that the vehicle position in the field has to remain unpredictable. To reach this goal, we started from a classical 3D chaotic system – Lorenz system, and projected it into xOy plane. After that, using a rotation and a scale transformation we placed the two attractors exactly in the positions of the mentioned points of interest. By this, we obtained a trajectory that can monitor both points of interest with a single intelligent vehicle.

Key-Words: - trajectory planning, chaotic system, unpredictability.

1 Introduction
The problem of obtaining unpredictable trajectories for different types of unmanned intelligent vehicles or mobile robots represents one of the major challenges for researchers involved in the domain of artificial intelligence.

There are some important papers that must be mentioned as breakthroughs in this context.

One paper belongs to Y. Nakamura and A. Sekiguchi [1]. By considering that the area under investigation is specified by its boundaries, they designed a particular controller such that the total dynamics of a mobile robot is represented by the Arnold equation, which is recognized to present the chaotic behavior of noncompressive perfect fluid.

The paper of A. Jansri, K. Klomkarn, K. and P. Sooraksa [2] was inspired by the subject of seeking suitable chaotic patterns for the robot's trajectory. A set of twenty-five chaotic patterns are compared by computer simulation.

A significant research was done by L.S. Martins-Filho and E.E.N. Macau in [3] and [4]. In order to ensure a high level of unpredictability of robot path, they imposed a chaotic behavior through the exploitation of an area-preserving chaotic map as a trajectory planner.

Our methodology to develop highly opportunistic and unpredictable trajectories for external observers, in the case when two points of interest (POIs) can be specified inside the area under investigation, is based on the chaotic behavior of the well-known Lorenz system.

The rest of the paper is organized as follows: the second section presents the problem formulation and assumptions; section three describes in a step-by-step manner the design of chaotic trajectories. In the last two sections, implementation details and conclusions are offered.

2 Problem Formulation
The problem we have to solve in this paper can be described as follows:
- there is a geographical area of interest that has to be covered by an intelligent vehicle;
- there are two points of interest (POI) inside this area that are of the greatest importance;
- the trajectory of the vehicle has to be unpredictable so that an outsider cannot exactly estimate the future position of the vehicle.
In order to simplify the problem, we will assume that the area of interest is flat (2D surface) and that its boundaries are not relevant.

3 Chaos Trajectories Design

Our methodology that has to solve the mentioned problem is based on the design of a 2D chaotic trajectory inside the specified area. We chose to build chaotic trajectory basically for a specific feature called “sensitivity to initial conditions” that will assure the unpredictability.

In order to classify a dynamical system as chaotic, three important properties have to be met [5]:
1. it must be sensitive to initial conditions;
2. it must be topologically mixing; and
3. its periodic orbits must be dense.

Sensitivity to initial conditions (or butterfly effect) means that a very small change of the initial point of trajectory will produce a big change in its final trajectory point. Therefore, a randomly small perturbation of the existing trajectory may lead to considerably different future behavior. This property is considered to be a chief source of unpredictability.

Topologically mixing signifies that the system will progress over time so that any specified region of its trajectory will eventually partially cover any other given region.

The third important property of a chaotic system is that periodic orbits have to be dense. An orbit of a dynamical system is thought to be dense if it comes arbitrarily close to any point in the domain.

In the following paragraphs a step-by-step description of our methodology is provided.

3.1 Carefully picked Cartesian system of coordinates XOY

In order to specify the area of interest and its two POIs (denoted by A and B) we have to assume a coordinate system. For simplicity, we chose a Cartesian system that is established using the following steps:

a) locate the origin of the coordinate system to be the middle of the segment \( AB \);
b) set the X axis along the vector \( \overrightarrow{AB} \);
c) build the Y axis in order that xOy to be a Cartesian system;
d) by considering the \( \overrightarrow{AB} \) vector length to be \( 2 \cdot a \), we have the unit vector of the xOy coordinate system.

Fig.1. Establishment of the xoy coordinate system based on the two POIs (a=5)

3.2 Building the chaotic trajectory

Based on chaotic feature, we will force the two POIs (A and B) to be the equilibrium points of a chaotic attractor.

The performed steps are the following:

a) chose a 3D chaotic system;
b) decide which projection to be used;
c) move the equilibrium points of the chaotic system in the specified POI (A and B) using affine transformations.

There are a lot of alternatives in picking the chaotic system that will be used to generate the trajectory. We chose the well-known Lorenz chaotic system to describe our methodology, and more exactly its projection in xOy plane.

Lorenz attractor [6] is described by the following set of equations:

\[
\begin{align*}
\frac{dx}{dt} &= \sigma(-x + y) \\
\frac{dy}{dt} &= -rx - y - xz \\
\frac{dz}{dt} &= -bz + xy
\end{align*}
\] (1)

where the parameters have the following standard values: the Prandl number \( \sigma = 10 \), the geometric factor \( b = \frac{8}{3} \) and the Rayleigh number \( r = 28 \).

The equilibrium points of the Lorenz chaotic attractor [6] are:

\[
\begin{align*}
P_1 &= (0,0,0) \\
P_2 &= (\sqrt{b(r-1)}, \sqrt{b(r-1)}, r-1) = (6\sqrt{2},6\sqrt{2},27) \\
P_3 &= (-\sqrt{b(r-1)}, -\sqrt{b(r-1)}, r-1) = (-6\sqrt{2},-6\sqrt{2},27)
\end{align*}
\] (2)
Because we assumed that the vehicle has a 2D trajectory, we will consider only the x0y projection of the Lorenz attractor (Fig.3):

In order to place the two projections of unstable equilibrium points P2 and P3 symmetrically to the origin (0,0) and to move them in the required points \(A(-a,0), B(-a,0)\), we will have to compute the following two 3D affine transformations of coordinates of the Lorenz system:

a) First a rotation with an angle \(\theta = -\frac{\pi}{4}\) radians (\(\theta\) is the anticlockwise rotation angle) around Z axis. This transformation is described using the following equation:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\tag{3}
\]

b) Second, a scale transformation described by

\[
\dot{x} = \frac{a}{\sqrt{b(r-1)}} \tilde{x} \quad \dot{y} = \tilde{y} \quad \dot{z} = \tilde{z} \tag{4}
\]

Applying the transformation (3) and (4) we obtain the modified Lorenz attractor:

\[
\begin{align*}
\frac{dx}{dt} &= \left(\frac{r - \frac{1}{2}}{2}\right)\dot{x} + \frac{1}{\alpha}\left(\sigma - \frac{r}{2} - \frac{1}{2}\right)\dot{y} - \frac{1}{2}\dot{x}^2 + \frac{1}{2\alpha}\dot{y}^2 \\
\frac{dy}{dt} &= \alpha\left(\frac{r - \frac{1}{2}}{2}\right)\dot{x} - \left(\sigma + \frac{r}{2} + \frac{1}{2}\right)\dot{y} - \frac{\alpha}{2}\dot{x}^2 + \frac{\alpha}{2}\dot{y}^2 \\
\frac{dz}{dt} &= \dot{b}\tilde{z} + \frac{\alpha}{2}\dot{x}^2 - \frac{1}{2}\dot{y}^2
\end{align*}
\tag{5}
\]

where \(\alpha \equiv \frac{\sqrt{b(r-1)}}{a}\).

In the end, the equations (5) have to be suitably discretized to be used in trajectory generation of a real vehicle. The discretization process is not a simple one [7][8] because of two main reasons: i) it has to preserve the chaotic properties; and ii) it is inherently affected by errors. Due to error accumulation the discrete variant of the chaotic system (5) can become, after some time, non-chaotic. To avoid this, the discrete calculus is reset after a couple of minutes, the trajectory regaining its chaotic characteristic.

If the intelligent vehicle has enough computational power, a fourth-order Runge-Kutta integration scheme can be applied in order to obtain the discrete version of (5). If not, a similar procedure applied in [7] for Rossler chaotic system can be used to obtain an efficient discrete model.

**4 Implementation**

For implementing the vehicle motion on the designed chaotic trajectory, the following pseudocode is followed:
main()
{
    initialize b,r,a; /*parameter initialization
    (actual_x,actual_y)=obtain_coordinates(); /* starting point coordinates
    (x,y)=(actual_x, actual_y);
    for(;); /* infinite loop
    {
        if ((x,y)==(-a,0))OR((x,y)==(a,0)) /* verify if the vehicle is right in the chaotic
            /* system attractor
            then (x,y)=(x,y)+(0.01,0); /* the vehicle will move a little bit to avoid
            /* the so-called “fall into attractor problem”
            else (x,y)=compute_the_following_position(x,y); /* using the discretized version of (5),
                /* the next position of the vehicle is obtained
                go_to_position(x,y) /*the vehicle trajectory is controled until it
                reaches the point (x,y) */
    }
}

Fig.5: The pseudocode of the proposed methodology

After initializing some of the parameters already presented in section 3, we have to obtain the actual starting in-field position of the intelligent vehicle. Afterwards, the control system enters into an infinite loop where we verified if the position of the vehicle meets the exact coordinates of one of the two attractors. If the answer is affirmative, we have to face a “falling into attractor” case [9] and we have to “throw the system” in an unequilibrium point. If the answer is negative, a new point on the chaotic trajectory will be computed using a discretized form of equations (5), and the vehicle’s movement is controlled until it reaches the point. This procedures ends only if the vehicle is stopped.

If the vehicle encounters an obstacle, it will surpass this problem by already known techniques [10][11], the vehicle regaining its chaotic movement just after the obstacle avoidance procedure is finished. Obviously, the trajectory will be a brand new one due to the “sensitive to initial condition” feature of the chaotic systems.

5 Conclusion
In this paper we presented a novel approach, based on Lorenz system, in designing 2D chaotic trajectories for intelligent vehicles in an area under investigation, with two points of interest. This approach can be used even when the vehicle is confronted with the problem of obstacle avoidance.

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