Stock Portfolio Selection using Mathematical Programming: An Educational Approach

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Abstract: - In this paper we present an application of mathematical programming using real stock market data. Most of the students have some familiarity with the stock market and therefore can understand the problem easily. Here we explore the structure of the stock portfolio through mathematical programming (optimization) so as to achieve a performance higher from the one of the stock index.

Key-Words: mathematical programming, portfolio selection, stocks, optimization.

1 Introduction

The linear programming model has been applied in a large number of areas including finance, military applications, transportation scheduling, production and inventory management, telecommunications. Many problems simply lend themselves to a linear programming solution but in many cases some ingenuity is required for the modelling. Linear programming also has interesting theoretical applications in combinatorial optimization and complexity theory. The classical tool for solving the linear programming problem in practice is the class of simplex algorithms proposed and developed by George Dantzig [4]. Methods of nonlinear programming methods have also become practical tools for certain classes of linear programming problems, see for instead [1], [2], [3], [5], [8].

The problem of portfolio selection is not very old. In his work, Harry Markowitz [10], set the foundations of modern portfolio theory. From then on, many authors studied the problem of portfolio selection in many directions [6], [7], [14]. Within the modern economic reality, stock portfolio management is a problem faced, on the one hand, by many financial analysts and researchers and, on the other hand, by

practitioners who are called upon to make investment decisions for substantial investment funds. Obviously, stock portfolio selection is quite sensitive given the future commitment involved. The construction of a model for effective management in the sense of maximizing potential returns will be an interesting and quite useful issue.
In the last years, quite a few researchers have merged subjects from different disciplines into the learning process so as to achieve a better comprehension on the part of the students [11], [12]. In this paper we present a teaching model of mathematical programming through stock portfolio selection.

2 Problem Formulation

In Table 1, we present the projections for the performance of two stocks and the Stock Index of a stock market. These are data which arose from the implementation of simulation techniques, whose variability though does not differ much from the equivalent performance of such indexes in a real stock market. The projections in Table 1 refer to the year 2009 and are on a monthly basis.
Our task is to utilize these projections and to manage stock portfolios with two stocks having as our major competitor the Stock Index of the Stock Exchange for the year 2009. In essence we are tackling the portfolio problem from a game theory point of view [9], [13].

MONTHS	STOCK INDEX %	STOCK 1 %	STOCK 2 %
JANUARY	1.027	1.479	1.875
FEBRUARY	2.034	1.222	2.621
MARCH	2.703	2.058	4.492
APRIL	0.380	1.325	0.943
MAY	1.824	2.103	3.267
JUNE	-1.800	0.299	-1.728
JULY	0.953	0.789	1.988
AUGUST	-2.071	0.106	-2.096
SEPTEMBER	-1.981	0.184	-1.494
OCTOBER	-2.092	-0.157	-0.943
NOVEMBER	2.965	1.337	4.071
DECEMBER	3.597	3.523	4.821

Table 1**.** Projections for the Performance of Stocks of a stock market. Source: Computer-simulated Data

We could briefly suppose that the management of a stock portfolio includes the following phases:
Phase 1 Stock portfolio selection. From the data of the exercise, stock portfolio selection is a given fact. There are two stocks and the major competitor is the Stock Index of the Stock Exchange.

Phase 2 Optimum Stock portfolio Structure. The stock portfolio structure refers mainly to the percentages of each stock's participation in the stock portfolio. The optimum stock portfolio structure will arise from the selection of the percentage of each stock's participation in the stock portfolio.
In the case of the management of the two stocks in Table 1, the manager's strategy for the first three months of 2009 is presented in Table 2. His strategies are quoted in the percentages x_1 και x_2 with which each stock participates in the stock portfolio. It goes without saying that $x_1 + x_2 = 100\% = 1$.

	Stock Index	*January*	*February*	*March*
Manager	Stock 1	x_1	x_1	x_1
	Stock 2	x_2	x_2	x_2

Table 2. The Investment Strategy of the Manager. Percentages of the Stocks' Participation in the Stock portfolio

The percentages x_1 and x_2 with which each stock participates in the stock portfolio every trimester (3-month management) will arise from an investment strategy through which the manager will maximize his profits by minimizing the profits of the Stock Market. The profits from one stock in relation to the stock index are calculated through the function:
Profit =Perform. of Stock – Stock Index Perform.

$$K_{jt} = MET_{jt} - GEN_t \quad j = 1,2 \quad t = 1,2,3 \qquad (1)$$

where:
$MET_{jt} =$ Performance of stock j in period t
$GEN_t =$ Stock index performance in period t

	Stock Index	*January*	*February*	*March*
Manager	Stock 1	K_{11}	K_{21}	K_{31}
	Stock 2	K_{12}	K_{22}	K_{32}

Table 3. Table of Manager's Profits

If the manager implements the strategy of Table 2, then every month, based on Table 3, his profits from the percentages x_1 and x_2 with which each stock participates in the stock portfolio, will be:

$x_1K_{11} + x_2K_{12} = V_1$ in time period t=1.
$x_1K_{21} + x_2K_{22} = V_2$ in time period t=2.
$x_1K_{31} + x_2K_{32} = V_3$ in time period t=3.

If V is the profits from the Manager's investment

strategy for the trimester, then the percentages x_1 and x_2 with which each stock participates in the stock portfolio will arise from the Linear Programming problem: $\max V$
subject to the following restrictions:

$$\sum_{i=1}^{3} x_i K_{it} \geq V \text{ , } t = 1,2$$

$$\sum_{i=1}^{2} x_i = 100\% = 1$$

$$x_j \geq 0$$

$$j = 1,2$$

In the case of ν time periods and for μ stocks, the linear programming problem will be: $\max V$
subject to the following restrictions:

$$\sum_{i=1}^{\mu} x_i K_{it} \geq V \text{ , } t = 1,2,....,\nu$$

$$\sum_{i=1}^{\mu} x_i = 100\% = 1$$

$$x_j \geq 0$$

$$j = 1,2,...\mu$$

2.1 Stock portfolio management with two stocks

The presentation of the investment strategy for the first trimester (for the months of January, February and March) is presented in Table 4.

Stock Index

		January	*February*	*March*
Manager	Stock 1	0,4526	-0,8129	-0,645
	Stock 2	0,8484	0,5863	1,7807

Table 4. Table of the Manager's Profits.
Source: Data from Table 1.

Based on the data of Table 4, the manager's problem in the form of a linear programming program is as follows: $\max V$ subject to the following restrictions:

$$0.4526x_1 + 0.8484x_2 \geq V$$

$$-0.8129x_1 + 0.5863x_2 \geq V$$

$$-0.6450x_1 + 1.7807x_2 \geq V$$

$$x_1 + x_2 = 1$$

$$x_1, x_2 \geq 0$$

Based on the above, the resulting optimum stock portfolio structure for the first three months of 2009 will be: $x_1 = 0$, $x_2 = 1$ with profit V=0.5863. The presentation of the investment strategy for the second trimester (for the months of April, May and June) is presented in Table 5.

Stock Index

		April	*May*	*June*
Manager	Stock 1	0.94554	0.2794	2.0991
	Stock 2	0.5632	1.4439	0.0721

Table 5. Table of Manager's Profits
Source: Data from Table 1

Based on the data from Table 5, the manager's problem in the form of a linear programming program is as follows: $\max V$ subject to the following restrictions:

$$0.9454x_1 + 0.5632x_2 \geq V$$

$$0.2794x_1 + 1.4439x_2 \geq V$$

$$2.0991x_1 + 0.0721x_2 \geq V$$

$$x_1 + x_2 = 1$$

$$x_1, x_2 \geq 0$$

Based on the above, the resulting optimum stock portfolio structure for the second trimester of 2009 will be: $x_1 = 0.5695$ and $x_2 = 1 - x_1 = 0.4307$.

2.2 The Outcomes of the management as a whole

We repeated the above procedure of the management problem for the next two trimesters as well. The outcomes of these maximizations are presented in Table 6. The first and second columns of Table 6 present the percentages of our stock portfolio with the two stocks. The third column of the Table presents the evolution of the performance of the stock portfolio through time as it arises based on the following function:

$d\_port = x_{1t}met\_1_t + x_{2t}met\_2_t$ (Performance of the Stock Portfolio)

$x_{1t} =$ Optimum Percentage of stock 1 in the portfolio

$x_{2t} =$ Optimum Percentage of stock 2 in the portfolio

$met\_1_t =$ Performance of Stock 1

$met\_2_t =$ Performance of Stock 2

MONTHS	x_{1t} = Perc of Stock 1	x_{2t} = Perc of Stock 2	d_port_t	s_port^*
JANUARY	0,00	1,00	1,88	1,88
FEBRUARY	0,00	1,00	2,62	4,50
MARCH	0,00	1,00	4,49	8,99
APRIL	0,56	0,44	1,16	10,14
MAY	0,56	0,44	2,62	12,76
JUNE	0,56	0,44	-0,59	12,17
JULY	0,31	0,69	1,62	13,78

AUGUST	0,31	0,69	-1,41	12,37
SEPTEMBER	0,31	0,69	-0,97	11,40
OCTOBER	0,00	1,00	-0,94	10,45
NOVEMBER	0,00	1,00	4,07	14,52
DECEMBER	0,00	1,00	4,82	19,35

Table 6. Management Outcomes, Percentages in the stock portfolio structure. Source: Table 1 Data Processed

*This column expresses the cumulative performance of the stock portfolio and is calculated based on the data in column 4 as follows:

$$s\_port_t = s\_port_{t-1} + d\_port_t \text{ , with}$$

$$s\_port_o = d\_port_{t=1}$$

In Table 7, we present the cumulative performance of the Stock Index, of the Stock portfolio and of stocks 1 and 2 respectively. The predominance of the potential of stock portfolio management in relation to both the Stock Index and the growth of the two particular stocks is obvious and confirmed by the graphic representation of Figure 1.

MONTHS	STOCK INDEX	STOCK PORTFOLIO	STOCK 1	STOCK 2
JANUARY	100,00	100,00	100,00	100,00
FEBRUARY	102,03	102,62	101,22	102,62
MARCH	104,79	107,23	103,31	107,23
APRIL	105,19	108,47	104,67	108,24
MAY	107,11	111,31	106,88	111,78
JUNE	105,18	110,65	107,19	109,85
JULY	106,18	112,44	108,04	112,03
AUGUST	103,98	110,85	108,16	109,68
SEPTEMBER	101,92	109,77	108,35	108,04
OCTOBER	99,79	108,73	108,18	107,02
NOVEMBER	102,75	113,16	109,63	111,38
DECEMBER	106,45	118,61	113,49	116,75

Table 7. Management Outcomes ,Cumulative Performance Source: Table 1 Data Processed

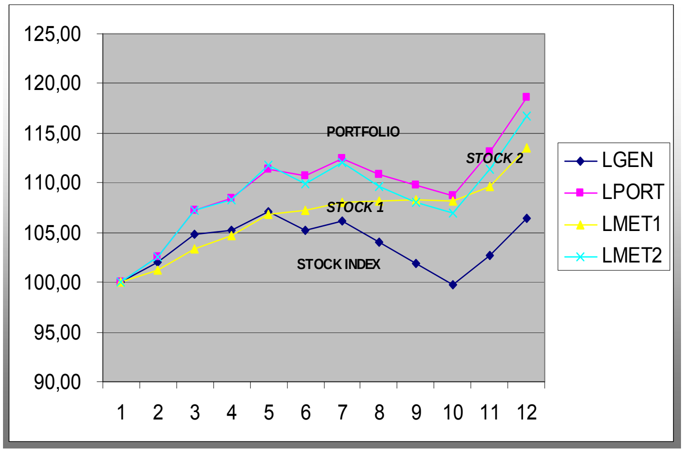

Fig. 1. Comparison of the Cumulative Performance of the Stock portfolio with the Performance of the Stock Index and the Cumulative Performance of the two stocks

3 Conclusion

Stock portfolio selection can be performed using tools such as mathematical programming which are easily digested and understood by the students. In this way they can not only appreciate the immediate use of mathematics to the practical but important problems, but also start to be exposed to the problems of modern finance.

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