Study of capacitive and inductive characteristics of nanoellipsoidal

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Abstract : - When the metallic and nonmetallic nanoellipsoidal interacts with the optical wave at a special frequency, they show some interesting nanocapacitorial characteristics. As a result, this system behaves as nanocircuit. The first part of this article concerns the calculation of effective impedance. Then the parameter of capacitor and inductor is specified by impedance. we study dependence of parameter of capacitor and inductor to direction of optical wave and dimensions of nanoellipsoidal.

Keywords: - optoelectronic, nanoellipsoid, nanosphere, capacitor, inductor, circuit.

1.Introduction

The capacitive and inductive characteristics of nanosphere have been studied for the first time by researchers of university of Pensylvania. Interactions between these nanoparticles and optical wave have been investigated. Nanosphere can be made of metallic or nonmetallic material. So based on their type, they can behave such as nanoinductor or nanocapacitor. The use of this approach on nanoantenna fabrication and high speed computers is expected.

2. theoretical concepts

To begin, we consider a nanosphere of radius R, made of a homogenious material with dielectric function $\varepsilon(\omega)$, that is a complex quantity. Suppose that a uniform monochromatic electromagnetic field illuminate on nanosphere with sphere adius much smaller than the wavelength. So the scattered electromagnetic fields in the vicinity of the sphere and the total fields inside it may be obtained with very good approximation using the well known time harmonic quasistatic approach. So fields inside and outside the sphere is [5]:

$$\vec{E}_{in} = \frac{3\varepsilon_0}{\varepsilon + 2\varepsilon_0}\vec{E}_0 ,$$

$$\vec{E}_{out} = \vec{E}_0 + \vec{E}_{dip} = \vec{E}_0 + \frac{3\vec{u}(\vec{p}.\vec{u}) - \vec{p}}{4\pi\varepsilon_0 r^3} \qquad (1)$$

that

$$\vec{p} = 4\pi\varepsilon_0 R^3 \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0}\vec{E}_0 , \quad \vec{u} = \frac{\vec{r}}{r}$$

ε_0 is the permittivity of the outside region and $\vec{r}$ is the position vector from the center of sphere to observation point on the surface of the sphere. the normal component of displacement current, $-i\omega D_n$, is continuous. So

$$-i\omega(\varepsilon - \varepsilon_0)\vec{E}_0.\hat{n} = -i\omega\varepsilon_0\vec{E}_{dip}.\hat{n} + i\omega\varepsilon\vec{E}_{res}.\hat{n} \qquad (2)$$

that $\hat{n}$ is outward normal unit vector and $\vec{E}_{res} = \vec{E}_{\text{int}} - \vec{E}_0$ represents residual field internal to the nanosphere. Integration of (2) over the upper hemispherical surface represents the total displacement current [5]:

$$-i\omega(\varepsilon - \varepsilon_0)\pi R^2 \left|\vec{E}_0\right| = -i\varepsilon\omega\pi R^2 \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0}\left|\vec{E}_0\right| -$$

$$-i\omega\varepsilon_0 2\pi R^2 \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0}\left|\vec{E}_0\right| \qquad (3)$$

that

$$-i\omega(\varepsilon - \varepsilon_0)\pi R^2 \left|\vec{E}_0\right| = I_{imp}$$

$$-i\varepsilon\omega\pi R^2 \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0}\left|\vec{E}_0\right| = I_{sph} \qquad (4)$$

$$-i\varepsilon\omega\pi R^2 \frac{\varepsilon-\varepsilon_0}{\varepsilon+2\varepsilon_0}\left|\vec{E}_0\right| = I_{sph}$$

that I_{imp}, I_{sph} and I_{fringe} represent displacement current source, displacement current of residual field in the nanoparticle and displacement current of dipole field out of nanoparticle or fringe current, respectively. from above relationship, it can be found that the total current source at a node is split in two current branches such that these current branches obey the kirchhoff law. The effective potential difference between upper and lower hemispherical surfaces of the sphere is [5]:

$$\langle V\rangle_{sph} = \langle V\rangle_{fringe} = R\frac{\varepsilon-\varepsilon_0}{\varepsilon+2\varepsilon_0}\left|\vec{E}_0\right| \tag{5}$$

Effective current is evaluated in (4), so the impedance is:

$$Z_{sph} = (-i\omega\varepsilon\pi R)^{-1},$$

$$Z_{fringe} = (-i\omega 2\pi R\varepsilon_0)^{-1} \tag{6}$$

By above equation, behavior of nanoparticle can change by the sign of dielectric function.

In case of nonmetallic nanosphere, the real part of dielectric function is positive, thus the real part of Z_{sph} is equal to the impedance of capacitor and the imaginary part of it is equal to the impedance of resistor. In that vacuum dielectric constant, ε_0, is always positive, so Z_{fringe} is equal to the inductor impedance.

For metallic nanopartcles, the investigation of conditions in where surface plasmon can be excited, is important. For gold or silver nanosphere, surface plasmon can be excited in IR and visible band and as a result, dielectric function can be negative. Therefore, the equivalent impedance of the nanosphere can be negatively capacitor, at any frequency for which $\mathrm{Re}[\varepsilon] < 0$. This can be interpreted as inductance.

At this article, we are going to investigate the capacitive and inductive behavior of nanoellipsoid under irradiation. To begin, we consider a nanoellipsoid of axes a, b and c. we suppose that linearly polarized optical wave illuminate on nanoellipsoid such that polarization axes is in line of the smallest axes of nanoellipsoid, c. suppose that light wavelength is much bigger than nanoelliosoid. Thus quasistatic approximation can be used.

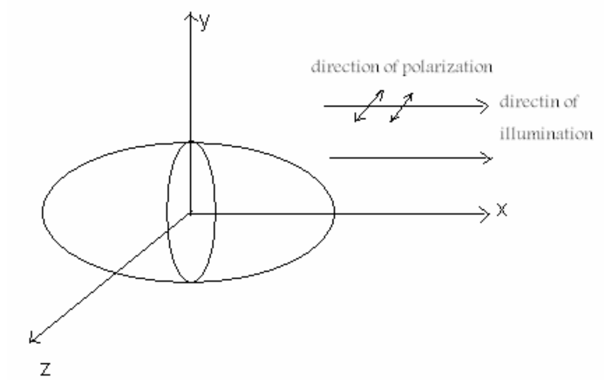

Fig.1. nanoellipsoid orientation and light direction

To begin, the relation between cartezian and ellipsoidal coordinates is[7]:

$$\frac{x^2}{a^2+\rho}+\frac{y^2}{b^2+\rho}+\frac{z^2}{c^2+\rho}=1 \rightarrow -c^2<\rho<\infty$$

$$\frac{x^2}{a^2+\mu}+\frac{y^2}{b^2+\mu}+\frac{z^2}{c^2+\mu}=1 \rightarrow -b^2<\mu<-c^2$$

$$\frac{x^2}{a^2+\nu}+\frac{y^2}{b^2+\nu}+\frac{z^2}{c^2+\nu}=1 \rightarrow -a^2<\nu<-b^2$$

that ν, μ and ρ are spatial coordinates of ellipsoidal system.

For potential evaluation, Φ_1 and Φ_2 represent internal and external potentials. External potential is originated from potential of external field and dipole potential on ellipsoid. For ellipsoidal coordinate [1]:

$$\Phi_0 = -E_0\left[\frac{(c^2+\rho)(c^2+\mu)(c^2+\nu)}{(a^2-c^2)(b^2-c^2)}\right]^{\frac{1}{2}} \quad , \Phi_1 = \frac{\Phi_0}{1+\frac{L_3(\varepsilon_i-\varepsilon_0)}{\varepsilon_0}}$$

$$\Phi_p = \Phi_0\frac{\frac{abc}{2}\left(\frac{\varepsilon_0-\varepsilon_i}{\varepsilon_0}\right)\int_\rho^\infty \frac{dq}{(c^2+q)f(q)}}{1+\frac{L_3(\varepsilon_i-\varepsilon_0)}{\varepsilon_0}}$$

that ε_i and ε_0 are interior and exterior dielectric function of ellipsoid, respectively. By using of boundary condition:

$$-i\omega(\varepsilon_i-\varepsilon_0)\left|\frac{\partial\Phi_0}{h_\rho\partial\rho}\ _{\rho=0}\right| = -i\omega\varepsilon_0\left|\frac{\partial\Phi_p}{h_\rho\partial\rho}\ _{\rho=0}\right| +$$

$$-i\omega\varepsilon_i\left|\frac{\partial}{h_\rho\partial\rho}(\Phi_1-\Phi_0)\ _{\rho=0}\right|$$

the electric field E_{res} is

$$E_{res} = -\nabla(\Phi_1-\Phi_0)$$

and surface element for surface of ellipsoid is [3]:

$$d\sigma_{\rho=0}=\frac{1}{4}\times$$
$$\times\left[\frac{(\mu-\nu)^2\mu\nu}{(a^2+\mu)(b^2+\mu)(c^2+\mu)(a^2+\nu)(\nu+b^2)(\nu+c^2)}\right]^{\frac{1}{2}}d\mu d\nu$$

Displacement current can be evaluated by integrating of $-i\omega D_n$ over the whole surface of ellipsoid, so

$$-i\omega\int_{-a^2}^{-b^2}\int_{-b^2}^{-c^2}(\varepsilon_i-\varepsilon_0)\left|\frac{\partial\Phi_0}{h_\rho\partial\rho}\right._{\rho=0}\left|d\sigma\right.$$
$$=-i\omega\int_{-a^2}^{-b^2}\int_{-b^2}^{-c^2}\varepsilon_0\left|\frac{\partial\Phi_p}{h_\rho\partial\rho}\right._{\rho=0}\left|d\sigma-i\omega\int_{-a^2}^{-b^2}\int_{-b^2}^{-c^2}\varepsilon_i\left|\frac{\partial}{h_\rho\partial\rho}(\Phi_1-\Phi_0)\right._{\rho=0}\right|d\sigma$$

left side of the above relationship represents the displacement current of source. On the right side of this equation, the first term is originated from electric dipole and the second term is originated from residual field.

$$I_s=-i\omega E_0\,a\,b\frac{(\varepsilon_i-\varepsilon_0)}{2}\left(\frac{1}{(a^2-c^2)(b^2-c^2)}\right)^{\frac{1}{2}}\times$$
$$\times\int_{-a^2}^{-b^2}d\nu\left[\frac{1}{(a^2+\nu)(\nu+b^2)}\right]^{\frac{1}{2}}\int_{-b^2}^{-c^2}d\mu\left[\frac{(\mu-\nu)^2}{(a^2+\mu)(b^2+\mu)}\right]^{\frac{1}{2}}$$

also for displacement current generated by dipole moment:

$$I_{sph}=-i\omega\varepsilon_0a^2\,b^2c\,\frac{E_0\left(\frac{\varepsilon_0-\varepsilon_i}{\varepsilon_0}\right)}{1+\frac{L_3(\varepsilon_i-\varepsilon_0)}{\varepsilon_0}}\times$$
$$\times\left\{\frac{1}{2}\left[\int_\rho^\infty\frac{dq}{(c^2+q)\,f(q)}\right]_{\rho=0}+c^2\left[\frac{\partial}{\partial\rho}\int_\rho^\infty\frac{dq}{(c^2+q)\,f(q)}\right]_{\rho=0}\right\}$$
$$\times\int_{A'}\frac{1}{\sqrt{\mu\nu}}\sqrt{\frac{(c^2+\mu)(c^2+\nu)}{(a^2-c^2)(b^2-c^2)}}\quad d\sigma$$

We integrate over half surface of nanoellipsoid. The displacement current generated by residual field in nanoellipsoid is:

$$I_{fringe}=-i\omega\varepsilon_i\,a\,bE_0\,\frac{\frac{L_3(\varepsilon_i-\varepsilon_0)}{\varepsilon_0}}{1+\frac{L_3(\varepsilon_i-\varepsilon_0)}{\varepsilon_0}}\times$$
$$\times\int_{A'}\frac{1}{\sqrt{\mu\nu}}\sqrt{\frac{(c^2+\mu)(c^2+\nu)}{(a^2-c^2)(b^2-c^2)}}\quad d\sigma$$

Now we need to evaluate the average potential difference between two hemiellipsoid. If polarization direction of field is on z diretion, the nanoellipsoid will orient such that the boundary between two hemiellipsoid is normal to the polarization direction of internal electric field. Average potential difference originated from E_{res} is:

$$\langle V\rangle=\frac{\iint(\Phi_1-\Phi_0)ds}{\iint ds}$$

$$=\frac{\dfrac{4L_3\left(\frac{\varepsilon_i-\varepsilon_0}{\varepsilon_0}\right)E_0}{1+4L_3\left(\frac{\varepsilon_i-\varepsilon_0}{\varepsilon_0}\right)}\int z\left(2\pi\sqrt{\frac{a^2+b^2}{2}\left(1-\frac{z^2}{c^2}\right)}\right)dz}{\times\int_{-a^2}^{-b^2}d\nu\left[\frac{\nu}{(a^2+\nu)(\nu+b^2)(\nu+c^2)}\right]^{\frac{1}{2}}\int_{-b^2}^{-c^2}d\mu\left[\frac{(\mu-\nu)^2\mu}{(a^2+\mu)(b^2+\mu)(\mu+c^2)}\right]^{\frac{1}{2}}}\qquad(16)$$

Z_{sph} and Z_{fringe} for silver and Au_2S nanoellipsoid for different quantities a,b and c are presented in tables 1 and 2, respectively.

Table 1. impedances on silver nanoellipsoid illuminated by wavelength 633nm for different sizes ($(\varepsilon_i=(-19+0.53i)\varepsilon_0)$)

a (nm)	b (nm)	c (nm)	Z_{sph}	Z_{fringe}
80	50	40	0.8618-30.393i	2.9371e+2 i
100	50	40	0.6749-24.1946i	2.3003e+2 i
100	50	20	0.5127-18.3787i	1.7473e+2 i
100	70	20	0.4746-17.0157i	1.6177e+2 i
100	20	20	0.7202-25.8176i	2.4546e+2 i
20	20	20	4.1852-1.5003e+2 i	1.4264e+3 i

Table 2. impedances on Au_2S nanoellipsoid illuminated by wavelength 633nm for different sizes ($(\varepsilon_i=5.44\varepsilon_0)$)

a (nm)	b (nm)	c (nm)	Z_{sph}	Z_{fringe}
80	50	40	1.0798e+2 i	2.9371e+2 i

100	50	40	84.5689 i	2.3003e+2 i
100	50	20	64.240 i	1.7473e+2 i
100	70	20	59.4760 i	1.6177e+2 i
100	20	20	90.2420 i	2.4546e+2 i
20	20	20	5.2443e+2 i	1.4264e+3 i

For silver nanoellipsoid, Z_{sph} can be interpreted as impedance of inductor and resistor. Electronic circuit equivalent to silver nanoellipsoid is shown in fig. 2, also Z_{fringe} can be interpreted as capacitor.

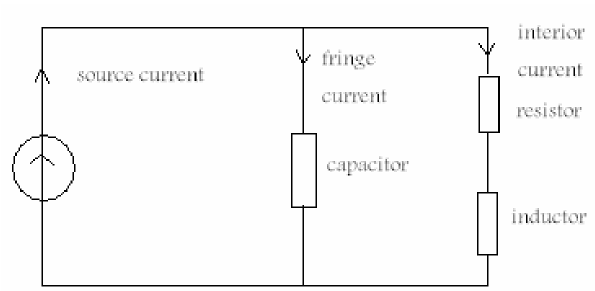

Fig.2. electrooptical behavior of exposed metallic nanoellipsoid

For Au_2S nanoellipsoid, the both of Z_{fringe} and Z_{sph} are equal to capacitor. electronic circuit equivalent to the nanoellipsoid is shown in fig. 3. the value of impedance for metallic and nonmetallic nanosphere and nanorod are given in tables 1 and 2.

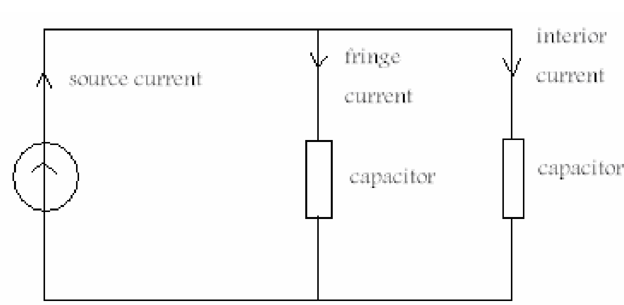

Fig.3. electrooptical behavior of exposed nonmetallic nanoellipsoid

as a result, the nanoellipsoid size and its orientation toward optical wave impress on value of impedances.

3.Conclusion

in this article, at first, the equivalent impedances of nanoellipsoid by different orientation were evaluated and equivalent electronic circuit for both type of nanoellipsoid was presented. it was found that different behavior for equivalent electronic circuit was attained by orientation or light polarization changing . Therefore nanoellipsoid can be used in nanooptical switching by coupling of nanoellipsoids or nanorod usage, analyzing of nanoantenna is possible.

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