# Graphing Polar Equations Using Spreadsheets

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*Abstract: -* Microsoft Excel is a helpful software tool for the teaching of calculus. One topic that arises in the traditional calculus sequence is the polar coordinate system, and the affiliated task of graphing a polar equation. A dynamic spreadsheet presentation can help the student appreciate how a curve is generated from the relationship between the polar coordinates.

*Key-Words*: - mathematics education technology polar equations Microsoft Excel spreadsheet

## 1 Introduction

Students who are non-math majors taking a standard calculus I-II-III sequence, sometimes find it difficult to make the transition from Cartesian coordinates to polar coordinates. In particular, the relationship between a curve and a polar equation $r = f(\theta)$ has aspects that may elude the neophyte. For instance the importance identifying values of $\theta$ corresponding to $r = 0$ or to where $r$ attains an extreme value may not be sufficiently appreciated. An additional subtlety that can be missed is the range of $\theta$-values that produces one copy of the polar curve.

Prior to age of the graphing calculator and the computer algebra system, the student's introduction to the sketching of polar curves typically involved a sequence of cases:

a) The simple graphs $r = \text{constant}$ (circle) , $\theta = \text{constant}$ (line)

b) A graph such as $r = 2\cos\theta$ (circle), produced by the student laboriously computing a table of $(r,\theta)$ pairs by 'hand' and then plotting the points that arise

c) Sketching a curve such as $r = 1 - 2\sin\theta$ (Limacon with a loop) by: drawing the graph of $r = 1 - 2\sin\theta$ in Cartesian coordinates, and then using this to read off the $r$-values that correspond to increasing values of $\theta$. Plenty of opportunities for the unwary to err.

The advent of sophisticated hand-held and desktop educational technology has greatly facilitated the exploration of polar graphing: taking the tedium out of the time intensive generation of $(r,\theta)$ tables; allowing the student to rapidly produce graphs of stimulating and sometimes mesmerizing complexity. That having been said however, there are many schools and colleges whose instructors and students do not have access to state of the art math software tools such as Mathematica, or to high powered

graphing calculators like the Texas Instruments Ti-89. Nonetheless, there are numerous such institutions and individuals who do have access to a PC loaded with a version of Microsoft Office. The Excel component of this suite is a useful instructional aid for the teaching of higher level mathematics. [1], [2]. In this paper the authors highlight the ability of Excel to effectively and rapidly produce curves of polar equations.

## 2 Excel as a Polar Graphing Tool

In this section of the paper the authors describe an in-house developed Excel spreadsheet that can serve as a graph generator for polar equations. The spreadsheet is a cost-effective user friendly teaching tool.

The authors wish to emphasize that the development of the spreadsheet did not require any software programming skills (in Visual Basic for example). They set out to develop a tool that is easy to construct and utilize. Only certain basic features of the Excel package were needed, such as cell referencing and table calculations. Key cells for data entry were designed to incorporate pop-up comments / instructions to aid the user.

The spreadsheet features a list of pre-set polar functions of the form $r = f(\theta)$, typical of cases found in most standard calculus sequence textbooks. These are accessible via a pull-down menu, and examples are shown below.

FIGURE 1 shows the four leaf polar curve corresponding to the selection

$$r = 3a\cos(2t)$$

where the variable $t$ is used in the Excel spreadsheet in place of the polar angle $\theta$, and the coefficient $a$ is an additional parameter appearing in the equation. The angle $\theta$ and the parameter $a$ can be individually varied by the user by means of Excel 'slider' control bars.



As the value of the parameter $t$ is altered, the point corresponding to the $t$-value moves along the curve.
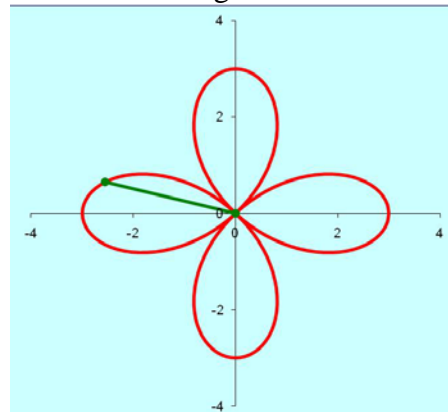


Fig. 1

The slider bar is a useful feature since it readily allows the student to explore functional aspects such as: the particular values of the polar angle that correspond to a zero value for the radius, and the ranges of the angle that trace the polar curve.

The spreadsheet can deal with more complicated polar graphs, as illustrated in FIGURE 2. This displays the graphical output corresponding to the case
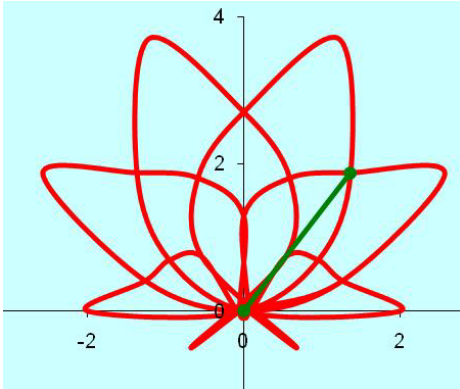
$$r = a\sin(t) + a\sin^3\left(\frac{5t}{2}\right)$$

Fig. 2

When the user selects the polar equation

$$r = e^{\sin t} - 2\cos(4t)$$

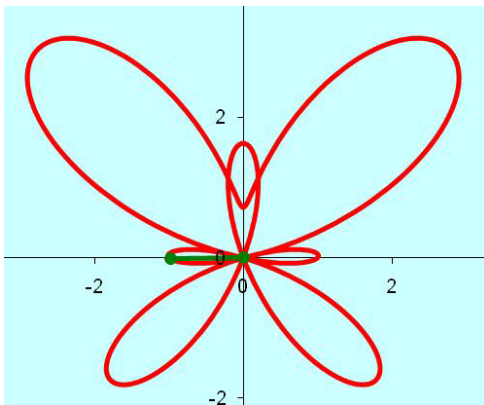the rather striking image of a butterfly is created (FIGURE 3).



Fig. 3

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[1] B.S. Gottfried, *Spreadsheet Tools for Engineers Using Excel,* New York: McGraw-Hill Science Engineering, 2005.
[2] S.C. Bloch, *Excel for Engineers and Scientists, 2e,* New York: John Wiley & Sons, 2003.
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[4] R.T. Smith and R.B. Minton, *Calculus*, New York: McGraw-Hill Science Engineering, 2002.