Abstract— The paper purpose is to dignify some aspects regarding the calculus model on guided self-supporting gyroplane used to local observations and scientific equipment carrying. The calculus methodology consists in numerical simulation of different gyroplane evolution. The gyroplane model which will be presented has six DOF and autonomous control system. The analyzed results will be the flight parameters and performances. The discussions will be focused around the possibility to realize this innovative gyroplane using air jet control for the movement around the center of mass, including the compensation for the torque of the rotor.

Keywords— gyroplane, simulation, mathematical model, self-supporting, guidance;

NOMENCLATURE

- $\alpha$ - Attack angle (tangent definition);
- $\beta$ - Sideslip angle (tangent definition);
- $\delta_x$ - Axial command;
- $\delta_r$ - Roll command;
- $\delta_m$ - Pitch command;
- $\delta_y$ - Yaw command;
- $\psi$ - Azimuth angle;
- $\theta$ - Inclination angle;
- $\phi$ - Bank angle;
- $\rho$ - Air density;
- $\Psi$ - Propeller thrust gain;
- $\Lambda$ - Propeller pitch;
- $\Omega$ - Propeller rated speed;
- $\Omega$ - Body angular velocity;
- $A, B, C, E$ - Inertia moments;
- $C_a, C_p, C_e$ - Aerodynamic coefficients of force in the mobile frame;
- $C_x, C_y, C_z$ - Aerodynamic coefficients of momentum in the mobile frame;
- $F_0 = \rho \frac{V^2}{2} S$ - Reference aerodynamic force;
- $H_o = F_0 l$ - Reference aerodynamic couple;
- $T_0$ - Reference thrust force;
- $H_o = T_0 l$ - Reference couple thrust;
- $l$ - Reference length;
- $m$ - Mass;
- $p, q, r$ - Angular velocity components along the axes of mobile frame;
- $S$ - Reference area;
- $T$ - Thrust;
- $t$ - Time;
- $V$ - Velocity;
- $u, v, w$ - Gyroplane velocity components in a mobile frame;
- $V_{sp}, V_{zp}, V_{zp}$ - Velocity components in Earth frame;
- $OXYZ$ - Normal Earth-fixed frame;
- $xyz$ - Body frame (mobile frame);
- $x_p, y_p, z_p$ - Coordinates in Earth-fixed frame.

I. INTRODUCTION

The paper aims to evaluate the modeling and simulation of the performance of an aircraft with an original design, which combines the features of a specific self-supporting balloon with the evolution and response specific to helicopter, the gyroplane, shown in Fig. 1. It shall be composed of an anchor ring inflatable jacket filled with Helium (He), which realize a part of ascension force, which has in the middle a ducted propeller with oriental air jet that provides the remaining ascension force and control command. A gondola containing electro-optical equipment, computer equipment and a communication system, necessary for carrying out specific activity on the bottom is mounted.
As the gyroplane has a central axis of symmetry the model of computation will be similar to the other aircraft with axial symmetry (rockets, balloons, etc.), thus reducing the problem to two spatial dimensions, a dimension along the symmetry axis and one in the side. As a consequence of axial symmetry in the moment equations for these types of aircraft, products of inertia are zero \( E = 0; D = 0; F = 0 \), leaving only the axial moments of inertia \( A, B, C \), which last two are practically equal \( B = C \). Starting from this idea the equations of computation will be constructed similar to those described in the paper [3] the commands number is four due to the possibility of the control of the thrust vector, as shown in [4]. Unlike the case with flight at high speed and very long trajectory (the case of ballistic missiles [5]), because the development is carried out on small distances and low speeds we do not consider the shape and rotation of the Earth, assuming from the outset an inertial frame linked to ground with the axis \( y_p \) orientated vertically upwards.

II. GENERAL MOVEMENT EQUATIONS

A. Kinematical equations

Unlike paper [4], which covers the regular airplane case, where the kinematical equation used Euler angles, in our case, similarly with launching rocket for the satellite, with the vertical initial launching direction, the papers [3], [6], [7] recommend modified Euler angles, notated with asterisk, which have first rotation in vertical plane. The complete rotation matrix indicated in [3] is:

\[
A_p = A_y^* \cdot A_y \cdot A_\phi^* = \begin{bmatrix} cp^* & 0 & -sp^* \\ -cf^* \cdot st^* & sp^* & cf^* \cdot st^* \\ sf^* \cdot ct^* & -cf^* \cdot st^* & sp^* \cdot ct^* \end{bmatrix},
\]

B. Dynamical equations

Developing cross products from paper [3], we can obtain matrix representation of the dynamical equations:

- Force equations in the Earth frame

\[
\begin{bmatrix} \ddot{V}_{sp} \\ \dot{V}_{sp} \\ \dot{V}_{sp} \end{bmatrix} = \frac{1}{m} \left[ B_p \begin{bmatrix} C^d_s \\ C^d_p \\ C^d_\phi \end{bmatrix} + T_0 \begin{bmatrix} C^f_s \\ C^f_p \\ C^f_\phi \end{bmatrix} \right] + \begin{bmatrix} 0 \\ 0 \\ M \end{bmatrix} - \begin{bmatrix} \dot{g} \end{bmatrix} + \begin{bmatrix} \frac{1}{m} F_a \end{bmatrix};
\]

where \( F_a \) is ascension force.

- Moment equations in the body frame

\[
\begin{bmatrix} \dot{\phi} \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} = J^{-1} \left[ \begin{bmatrix} C^d_s \\ C^d_p \\ C^d_\phi \end{bmatrix} + \begin{bmatrix} C^f_s \\ C^f_p \\ C^f_\phi \end{bmatrix} \right] + J^{-1} \left[ \begin{bmatrix} (B-C)pq \\ (C-A)pq \end{bmatrix} \right] + \begin{bmatrix} \dot{a}_k L A_p F_a \end{bmatrix}.
\]

where we denoted:
- Connection matrix

\[ K_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \] (14)

-Inertial moments inverse matrix:

\[ J^{-1} = \begin{bmatrix} 1/A & 0 & 0 \\ 0 & 1/B & 0 \\ 0 & 0 & 1/C \end{bmatrix} \] (15)

where the inertial moments are given by:

\[ A = \int (y^2 + z^2) \, dm; \quad B = \int (x^2 + z^2) \, dm; \quad C = \int (x^2 + y^2) \, dm. \] (16)

and the matrix \( B_p = A_p^T \) is the transpose matrix given by (2)

For the aerodynamic coefficients calculus we used the method indicated in [4] and [8], based on polynomial series expanding:

\[ C_i^a = a_i + a_i (x^2 + \beta^2) + a_i (\tilde{z}_p - \tilde{z}_n); \]
\[ C_i^a = -b_i \alpha + b_i \alpha^3; \quad C_i^i = c_i \tilde{\phi}; \quad C_i^i = d_i \beta + d_i \beta^3; \]
\[ C_i^i = d_i \alpha + d_i \alpha^3, \] (17)

where, by definition [8]:

\[ \alpha = -\arctan(v/u), \quad \beta = \arctan(w/u). \] (18)

Through the thrust coefficients we insert the commands:

\[ C_i^T = C_i \delta_a; \quad C_i^T = C_i \delta_a; \quad C_i^T = C_i \delta_l; \quad C_i^T = C_i \delta_n; \] (19)

where we assumed axial command \( \delta_a \) is ensured by increasing or decreasing the rotor speed, and the command in roll \( \delta_r \), pitch \( \delta_p \), yaw \( \delta_n \) achieved by stator tilting.

III. GUIDANCE COMMAND

Resuming [3], the guidance commands for the gyroplane are simple relation:

\[ u = K_i A_p \left[ u_x, u_y, u_z, \int \right] + U \left[ u_{x}, u_{y}, u_{z}, \int \right], \] (20)

where the main control signals are:

\[ u_x = k_{u_x} \dot{h}_x - k_{u_x} V_{sp}; \quad u_y = k_{u_y} \dot{h}_y - k_{u_y} V_{sp}; \quad u_z = k_{u_z} \dot{h}_z - k_{u_z} V_{sp}; \]
\[ u_{x'} = k_{u_{x'}} \tilde{\theta} - k_{u_{x'}} \tilde{\phi}; \quad u_{y'} = k_{u_{y'}} \tilde{\phi} - k_{u_{y'}} \tilde{\psi}; \quad u_{z'} = k_{u_{z'}} \tilde{\psi} - k_{u_{z'}} \tilde{\psi}, \] (21)

and the matrix \( A_p, U \) and \( K_i \) have the previously presented signification. The relative parameters \( \tilde{\theta}; \tilde{\phi}; \tilde{\psi}; \dot{\phi}; \dot{\psi}; \dot{h}; \dot{h}_x, \dot{h}_z \) are given by:

\[ \tilde{\theta} = 0_{\theta} - \theta^*; \quad \tilde{\phi} = 0_{\phi} - \phi^*; \quad \tilde{\psi} = 0_{\psi} - \psi^*; \]
\[ h_x = x_{pd} - x_{pd}; \quad h_y = y_{pd} - y_{pd}; \quad h_z = z_{pd} - z_{pd} \] (23)

where

\[ 0_{\theta}, 0_{\phi}, 0_{\psi}, p_{x_{pd}}, p_{y_{pd}}, p_{z_{pd}} \] (24)

are the input reference values.

The guidance commands are applied through the actuators which are approximated in the paper [3] by form:

\[ \dot{\delta}_x = -\frac{\delta_x}{\tau_{\delta x}} + k_{\delta x} u_x; \quad \dot{\delta}_y = -\frac{\delta_y}{\tau_{\delta y}} + k_{\delta y} u_y; \]
\[ \dot{\delta}_z = -\frac{\delta_z}{\tau_{\delta z}} + k_{\delta z} u_z, \] (25)

where \( \tau_{\delta x}, \tau_{\delta y}, \tau_{\delta z} \) are the time constants and \( k_{\delta x}, k_{\delta y}, k_{\delta z} \) are the gain constants.

IV. ASCENSION FORCE

For calculation of the ascension force, standard atmosphere elements in troposphere are considered known:

\[ \Theta(h) = \frac{T(h)}{T_0}; \quad \delta(h) = \frac{\rho(h)}{\rho_0}; \quad \alpha(h) = \frac{\rho(h)}{\rho_0}, \] (26)

where \( \Theta, \delta, \sigma \) represent the variation with height \( h \) of the main parameters of the air, reported to the value of the sea level, in normal conditions. Next, we make some technical considerations regarding the achievement system.

As first fact that must be taken into account is that the helium introduced in balloon is not in pure form but it is a mixture of helium with air. If you assume that the mixture is a percentage of 95% helium and 5% air, then the density of the mixture in standard conditions at the ground that will continue to work will be:

\[ \rho_{H_2O} = 0.95 \times 0.1785 + 0.05 \times 1.225 \pm 0.231 [kg/m^3], \] (27)

where it held that the pure helium density in standard conditions, at the ground is \( \rho_{H_2O} = 0.1785 [kg/m^3]. \)

Further, for ease of expression we call it blend "helium", although in reality it is a mixture of helium with air. On the other hand, inside the balloon we consider the total volume fixed, un deformable. To maintain the desired shape during movement it is necessary that inside there is a constant pressure excess in relation to

\[ \Delta p = 125 + 0.427 \times V_{max}^3 \] (Pa),

where the maximum velocity of flight is given in [m/s], the value of the supra-pressure being obtained in Pascal. If we admit that the maximum vehicle velocity will be \( V_{max} = 5m/s \) then, the ratio between supra-pressure and normal pressure on ground level is:

\[ \xi = \Delta p / \rho_0 \equiv 136/101325 = 1.342 \times 10^{-3} \] (29)

For maintaining the lifting volume constant, we will build two separate compartments, one filled with helium and the other filled with air. During the ascend the volume occupied by air is going to drop by being expelled from the compartment and the volume occupied by helium at the beginning \( V_{H2O} \) and the total volume of the lifting enclosure \( V_p \)

\[ k = V_{H2O}/V_p \] (30)

then we can find the volume of helium at a given altitude:
\[ V_H = V_{nr} \frac{T}{T_0} \frac{p_0 + \Delta p}{p_0 \delta + \Delta p} = \frac{T}{T_0} \frac{p_0 + \Delta p}{p_0 \delta + \Delta p} V_{r} = k \frac{\Theta}{\delta + \xi} V_{r} = k \frac{\sigma}{\delta} V_{r} = k V_{r} \]

where we noted:
\[ \delta^* = (\delta + \xi)(1 + \xi) \]  
(31)

and:
\[ \sigma^* = \delta^*/\Theta = \frac{(\sigma + \xi \Theta^{-1})(1 + \xi)}{1 + \xi} \]  
(32)

\[ \sigma^* = \frac{(\sigma + \xi \Theta^{-1})(1 + \xi)}{1 + \xi} \]  
(33)

In the previous relations we have considered that the temperature of helium and air found in the enclosure are equal to the outside temperature, and that the necessary suprressure for maintaining the enclosure shape is founds in a density growth. The hypothesis leads to the fact that for the inside gases we are working with modified values of the pressure ratio (\(\delta^*\)), and also modified densities (\(\sigma^*\)) which are close to the ratios used in standard atmosphere. Further more, at ground level, these ratios are also unitary because they are based on the standard values of the air on the ground. Taking into account that \(\xi\) is small, for the modified ratios we have the approximate relation:
\[ \delta^* = (\delta + \xi)(1 + \xi) \approx (\delta + \xi)(1 - \xi) \approx \delta + \xi - \delta = \delta^* \]  
(34)

\[ \sigma^* = \frac{(\sigma + \xi \Theta^{-1})(1 + \xi)}{1 + \xi} \approx \sigma + \xi \Theta^{-1} \sigma^* \]  
(35)

In this case the volume filled with air becomes:
\[ V_a = (1 - k/\sigma^*) V_p \]  
(36)

If we take into account that the volume filled with air drops while the machine is lifting, the equation (36) can give us a connection between the maximum altitude at which the lifting force exists and the value of the ratio of helium filling. Therefore, if we consider that the air is completely evacuated \(V_a = 0\), then, from the equations (35), (36) we obtain:
\[ k = \sigma^* \bigg|_{h_{\text{max}}} = \frac{(\sigma + \xi \Theta^{-1})(1 + \xi)}{1 + \xi} \bigg|_{h_{\text{max}}} \]  
(37)

Considering that from (26) we can express \(\Theta\) and \(\sigma\) as a function of \(h\), we can impose a certain value for altitude and we can determine the filling ratio \(k\) up till which we will have a continuous emptying process of the air and therefore an expansion of helium. It is obvious that when the altitude is rising, the filling ratio must be smaller, which leads to a smaller ascension force. Next, we evaluate the ascension force. Because the mass of helium remains constant during its expansion, we must evaluate only the mass of air that decreases during the evacuation while the altitude is rising. Admitting that the temperature of the lifting enclosure is equal to the outside one, the masses of helium and air are:
\[ m_H = k V_p \rho_{H0} \]  
(38)

\[ m_a = V_p \rho_a = V_p (1 - k/\sigma^*) \rho_a = V_p \rho_a (\sigma^* - k) \]  
(39)

In this case the ascension force is:
\[ F_{\text{asc}} = g V_p \rho_a (\sigma^* - m_a - m_H) = g V_p \rho_a \left[ \sigma^* - \sigma + \left(1 - \frac{\rho_{H0}}{\rho_a} \right) \right] \]  
(40)

In figure 2 it is presented the value of the ascension force given by the relation (40) for a volume \(V_p = 30 \text{ m}^3\) by taking into account the height at different values of the filling ratio \(k\). The value of the ratio \(k\) has been determined with the help of the relation (37), by imposing different values to the maximum lifting height, up to which the air is completely evacuated from the enclosure and the helium expansion is completed.

![Fig. 2 The lifting force diagram](image)

![Fig. 3 Air volume variation diagram in the lifting enclosure](image)

V. THRUST

The propeller thrust is determined by the relation:
\[ T = 0.5 \rho_{H0} \pi R^2 \left( R \Omega \right)^2 \Psi = 0.5 \rho_{H0} \pi R^2 \left( V / \Lambda \right)^2 \Psi \]  
(41)

If we consider the nominal value at a fix point:
\[ T_0 = 0.5 \rho_{H0} \pi R^4 \Omega_{\omega}^2 \Psi \]  
(42)

the thrust value in different flight conditions is obtained from:
\[ \frac{T}{T_o} = \sigma \Omega / \Omega_{\omega}^2 \left( \Psi / \Psi_{\omega} \right) \]  
(43)

On the other hand the thrust command operates over the pulsation:
\[ \Omega = \Omega_{\omega} \delta_x \]  
(44)

from where:
\[ T = T_o C_{xT} \]  
(45)

where:
\[ C_{xT} = \sigma \delta_x^2 \Psi / \Psi_{\omega} \]  
(46)

The thrust coefficient is obtained from the propeller pitch...
where the pitch is also modified by the command through the number of revolutions

\[ \Lambda = V / (R \Omega_0 \delta_v) \]

VI. INPUT DATA, CALCULUS ALGORITHM AND RESULTS

A. Input data for the model

As input data we use the geometrical elements of the gyroplane from Fig. 1. Mass characteristics of the model are:

\[ m = 75 \text{ kg} \quad A = 282 \text{ kgm}^2 \quad B = 153 \text{ kgm}^2 \quad x_{cm} = 1 \text{.} \]

Geometrical characteristics for the model are:

\[ l = 2 \text{ m} \quad S = 24.6 \text{ m}^2 \quad d = 0.5 \text{ m} \quad V_{ol} = 30 \text{ m}^3 \]

Aerodynamic characteristics are:

\[ a_1 = -1 \quad a_2 = -1 \quad a_{13} = -1 \times 10^{-5} \quad b_1 = -1 
\quad b_2 = -3 \quad c_1 = -2 \quad d_1 = -3 \quad d_2 = 0.5 \]

The thrust force has the form:

\[ T = T_0 C_{xT} \]

where \( T_0 = 993 \text{ [N]} \) and the thrust coefficient is given by the relation (46). For determining the nominal thrust we have used the dimensions:

\[ \Omega_0 = 364 \text{ s}^{-1} \quad \psi_0 = 0.03 \quad R = 0.6 \]

Thrust command is limited to: \( 0.5 < \delta_v < 1 \). It is obvious that for the null velocity (fixed point flight) at ground level with maximum command the thrust takes the nominal value:

\[ T = T_0 = 101 \text{ [kgf]} \]

B. Calculus algorithm

The calculus algorithm consists in multi-step method Adams' predictor-corrector with variable step integration method: [1] [10]. Absolute numerical error was 1.e-12, and relative error was 1.e-10.

C. Calculus test case

We will consider as a calculus test the situation when the gyroplane wants to automatically ascend from 100 m to 1000 m by using the guiding commands described by the relation (20). The starting lifting speed is 0.5 m/s, and the final lifting speed must be 0. The propeller movement generates a constant torque of \(-100 \text{ Nm}\) (in the opposite direction of the \( x \) body axe) that must be canceled by commands:

\[ V_{yp} = 0.5 \quad y_p = 100 \]

The input guidance commands as entry data by the relation (24) are:

\[ \theta_d = 0 \quad \psi_d = 0 \quad \phi_d = 0 \quad x_{pd} = 0 \quad y_{pd} = 1000 \text{ [m]} \quad z_{pd} = 0 \]

D. Results

In Fig. 4 we showed the diagram of lifting speed that increases very rapidly up to 5 m/s (maximum ground lifting speed) after which it drops due to the effect of thrust diminution. After the desired height is reached, the speed drops and becomes null.

![Lifting speed diagram](image1)

Figure 5 presents a lifting diagram of the gyroplane that obviously synchronizes the speed diagram. We can observe that approximately after 200 seconds the gyroplane has reached the desired height, and afterwards it continues to evolve at the desired altitude.

![Lifting diagram](image2)

In figure 6 it is presented the climb angle, which indicates the orientation of the velocity vector during the evolution. We can observe that it has approximately 90 degrees in its accessional period after which it becomes null at the horizontal flight.

Finally, in Figures 7 and 8 are presented thrust command (axial and roll)
VII. CONCLUSIONS

The conclusions are structured in three points as the following.

Guidance scheme: A first conclusion regarding the
guiding scheme consists in the fact that the gyroplane will have a pretty sophisticated command structure which will depend on the type of evolution. From this point of view we must define a number of evolutions that it will make several guiding structure that will be constructed in this purpose. On the other hand we can notice a transitory regime pretty badly adjusted (the climb angle diagram) which means that a part of the adopted coefficients are improper or that they can be improved as well as the entire command structure. This part of the command structure will be able to evolve in the same time with the development of the project when a series of used measurements will be better specified.

Thrust force: For vitiating the thrust force with height, the
velocity and the command there have been considered three
correction functions used for small airplanes with a fixed pitch propeller (UAVs). In order to approach the real model of
gyroplane this functions must be redefined.

Torque momentum. Besides the results presented there have been tested with the help of the calculus model previously described several values of the torque momentums that can be compensated by using commands, the maximum value being somewhere around 100-150 Nm. It is obvious that this value can be modified depending on the available force on the commands, but it is believed that the size order will remain the same. For higher values of the torque momentum it is recommended to use the anti-torque propeller.

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