Numerical simulation of the large power cables using the finite element method

DANIELA CĂRSTEA
High-School Industrial Group of Railways, Craiova
ROMANIA
E_mail: danacrst@yahoo.com

Abstract: – In this work we survey our research on domain decomposition and related algorithms for large power electric cables. The equations that describe the behaviour of the fields in electromagnetic devices are coupled because most of the material properties are temperature dependent and may depend on temperature in a nonlinear way. More, any power loss is transformed in local heating so that the heat source in the thermal model is the Joule-Lenz effect of the electrical current. Here we assume that the sources of the magnetic field have sinusoidal time dependence and we neglect the effects due to the displacement currents.

The target example is from the electric engineering. It is a large power cable in a free space. General symmetry of the device can be used for efficient software. The algorithms for analysis are presented in the context of the finite element method. The results can be extended for simulation of distributed-parameter systems described by elliptic and parabolic equations.

The domain decomposition at the level of the problem is used. In this way the coupled magneto-thermal problem can be analyzed by a reduction of the computational effort.

Key Words – Electrical cables; Coupled fields; Finite element method.

1 Introduction
The physical systems have the energy distributed spatially. An accurate mathematical model for a given physical system is an equation or a set of equations with partial derivatives [1]. In some assumptions we model the real systems by ordinary differential equations where the theory and numerical algorithms are well developed. Generally, we speak about distributed parameter systems (DPS) where the mathematical models are partial derivative equations. The variety of the actual DPS and their mathematical models results in significant difficulties in analytical solutions so that the numerical models represent the basis of computer modelling [2].

Our work will concentrate on a large class of DPS from the engineering, called elliptic and/or parabolic systems. This class includes a lot of systems from the electrical engineering and mechanics described by elliptic and/or parabolic partial derivatives equations. Such systems are the electromagnetic devices where the mathematical models are very complex because of the natural interaction between more physical fields. Although the field quantities are vectorial variables, a scalar model is obtained for each component of the vectorial equation, or, by the potential formulation the field equations can be simplified.

Many physical systems with the energy distributed spatially in a domain \( \Omega \), are described by scalar or vectorial equation in the form ([4], [5]):

\[
\frac{\partial u(t,x)}{\partial t} = \sum_{i,j=1}^{2} \frac{\partial}{\partial x_j} (a_{ij} \frac{\partial u}{\partial x_i}) + f(u)
\]

\((x,t) \in \Omega \times (0,t_f)\)

where \( \Omega \) is a bounded domain in \( \mathbb{R}^2 \) with boundary \( C \), and \( u \) is an unknown function. In Eqn. (1) variable \( x \) represents the spatial co-ordinates of a point from analysis domain, \( t \) is the time, and \( t_f \) is the final time of the time interval for analysis or control. Eqn. (1) is solved with a given initial condition and specified boundary conditions.

The time dependent electromagnetic field problems are usually solved using differential models of diffusion type. Many practical problems of great interest in electromagnetics involve time-harmonic fields and this case will be considered in this work.

In electrical power transmission and distribution, insulated power cables are widely used. The performance on power carrying capacity is determined by the heat dissipation.

Our target example is a large power cable where more physical fields interact so that we can not ignore the natural coupling of these fields [2]. Practically, an electromagnetic device is the house of electromagnetic, thermal and mechanical fields. An accurate model involves a coupled model and the development of the numerical algorithms for coupled problems. In some engineering application of the electrical cables, these operate with high power and...
high frequency currents. Under these conditions the current density is not unvarying in a cross section of the cables, but increases powerfully towards the periphery (skin effect). These aspects must be considered in an accurate analysis of the cables, especially in a dangerous environment without sufficient cooling. From a practical viewpoint of view the skin effect does not cause overheating locally, but it is important in the selection of the conductor material and the solution for the cooling.

2. A coupled magneto-thermal model

In the alternating current, the skin effect appears but in the most practical systems the conductor is stranded (that is made up several tightly wound strands of conductor insulated from each other) so as to force the currents to flow through the entire cross section of the conductor and thereby utilize the material better. Hence the validity of assuming uniform density as in direct current systems can simplify the computation.

![Fig.1 – A cross-section of the cable](image)

In large power cables as in Fig. 1, the proximity and skin effects can not be neglected so that in our analysis we consider them [10]. This high-voltage tetra-core cable has three triangle sectors with phase conductors and round neutral conductor in the lesser area of the cross-section above. All the conductors are made of copper. Each conductor is insulated and the cable as a whole has a three-layered insulation. The cable insulation consists of inner and outer insulators and a protective braiding (steel tape). The sharp corners of the phase conductors are chamfered to reduce the field crown. The corners of the conductors are rounded. Empty space between conductors is filled with some insulator (air, oil etc.).

Let us consider the coupled problem of the magnetic and thermal fields. The magnetic field in A-formulation is described by the equation [1]:

\[
\nabla \times (\nu \nabla \times A) + \sigma \frac{\partial A}{\partial t} = J_s
\]  

(2)

where \(\sigma\) is the electric conductivity, \(\nu\) is the magnetic reluctivity and \(J_s\) is the excitation current density.

This is the case of many practical engineering problems with geometric shape and size invariant in one direction. Let \(z\) denote the Cartesian co-ordinate direction in which the structure is invariant in size and shape. In this case the magnetic vector potential \(A\) has one component, which is independent of the \(z\) co-ordinate. In such a case both the magnetic vector potential and the source current \(J_s\) reduce to a single component oriented entirely in the axial direction and vary only with the co-ordinates \(x\) and \(y\). Consequently, the component \(A_z\) (for simplicity we give up the subscript \(z\)) satisfies the diffusion equation [2]:

\[
\nabla (\nu \nabla A_z) - \frac{\partial A_z}{\partial t} = -J_s
\]  

(3)

The thermal field is described by the heat conduction equation ([7], [8]):

\[
\frac{\partial}{\partial t} [(c\gamma)(T) \cdot T] + \nabla [-k(T) \cdot \nabla T] = q
\]  

(4)

\[
T(x,0) = T_0(x) \quad x \in \Omega
\]  

(5)

where: \(T(x, t)\) is the temperature in the spatial point \(x\) at the time \(t\); point \(k\) is the tensor of thermal conductivity; \(\gamma\) is mass density; \(c\) is the specific heat that depends on \(T\); \(q\) is the density of the heat sources that depends on \(T\); \(T_0(x)\) is the initial temperature. In Eqn. 3 the heat source \(q\) represents the ohmic losses of the electrical current, that is:

\[
q = \sigma(T) \cdot (\frac{\partial A_z}{\partial t})^2
\]  

(6)

with \(\sigma\) the electrical conductivity of the material.

In 2D for a spatial domain with the boundary \(C\) as a reunion of disjoint parts \(C1, C2, C3\) and \(C4\), the boundary conditions for Eqn. 3 are [7]:

**Dirichlet’s condition:**

\[
T(x, y, t) |_{C1} = T_D(x, y, t)
\]  

(7)

**Neumann’s condition:**

\[
[k \frac{\partial T}{\partial n} + q_n] |_{C2} = 0
\]  

(8)

**Convection:**

\[
[k \frac{\partial T}{\partial n} + h(T \cdot T_{\infty})] |_{C3} = 0
\]  

(9)

**Radiation:**

\[
[k \frac{\partial T}{\partial n} + e\sigma_B (T^4 \cdot T_{\infty}^4)] |_{C4} = 0
\]  

(10)

In these boundary conditions the significances of the parameters and quantities are: \(T_D\) is a known
function; \( q_n \) is an imposed flux; \( T_\infty \) is the environment temperature or the cooling medium temperature; \( h \) is the convection heat transfer; \( \varepsilon \) is the emissivity and \( \sigma_\text{B} \) is Stefan-Boltzmann constant.

The equations of the electromagnetic fields and heat dissipation in electrical engineering are strongly and non-linear, since the magnetic material properties are temperature dependent and the heat sources depend on the electrical current density [1].

3. Domain decomposition method

One of the motivations for coupled problems are:
- Two or more physical systems interact
- Two or more physical fields co-exist in the same electromagnetic device
- The physical properties of the materials are strongly dependent on the temperature, especially the following characteristics: electric conductivity, magnetic permeability, specific heat and thermal conductivity
- The heat sources in thermal systems represent the Joule effect of the electric currents (driven currents or induced currents)

In the area of the coupled fields we define two levels of decomposition, that is, we define a hierarchy of the decompositions:
- One at the level of the problem
- The other at the level of the field

In our target example, we have a magneto-thermal system. The two physical fields interact strongly. For a numerical simulation, we decompose the coupled problem in two sub-problems: a magnetic problem and a thermal problem, each of them with disjoint or overlapping spatial domains. This is the first level of decomposition. At the next level, we decompose each field domain in two or more subdomains [3]. The decomposition is guided both by the different physical properties of the materials, and the difference of the mathematical models. Practically we have an elliptic-parabolic problem for the magnetic system [9]. In conducting parts of the cable system, the mathematical model of the magnetic system is a parabolic equation. In insulation the magnetic field is described by an elliptic equation [9].

It is obviously that a fully coupled model involves a simultaneous solution of the field equations. But we can use the physical considerations in numerical solution of the coupled problem. Thus, the time constants for the magnetic field and thermal field are different so that it is not necessary to update the material electrical properties for each temperature value. Consequently the steady-state of the magnetic field is computed for a large time step or the material electrical properties are updated at imposed temperature values. The temperature is computed at small time steps. A curve for the dependence of the electrical properties with the temperature can be included in a software product.

The analysis domain is not the same for the two fields. For the magnetic field we can consider the whole domain presented in Fig. 1. For the thermal field we limit the analysis domain to the conducting parts.

4 Numerical results

We shall present the results of the numerical simulation for the cable using the finite element method [10]. It is obviously that the current density is not uniform. The non-uniformity of the current density generates non-uniformity of the temperature distribution and local heating that destroys different parts of the system and finally, the whole system. Fig. 2 shows the temperature map of the cable using the Quickfield program [10].

The load of the conductors are currents of amplitude equal to 250 A at the frequency of 50 Hz. In post-processing stage of the FEM program, a lot of physical quantities can be obtained. They are of great importance for the electrical engineers in the evaluation of the device performance [2]. These derived quantities are presented in user’s manual of any software CAD. The voltage amplitude is 7000 V.

The non-uniformity of the temperature is due to the non-uniformity of the current density in system. In computation of the total current in the cable, the skin effect and proximity effect of the cable cores were not ignored.

Stress analysis problem is very important in a global analysis of the cable performance. A program
for the stress analysis imports the temperature field from the heat transfer problem and the magnetic forces from the time-harmonic magnetic problem. The global effect is the deformation of the cable components. The displacement map and the vectors of displacements are shown in Fig. 3.

5 Conclusions
The analysis of distributed parameter systems is a complex problem so that the analytical solutions can not be obtained. Many practical engineering problems involve geometric shape and size invariant in one direction. In the case of the electric cables we considered the axis Oz as the co-ordinate direction in which the structure is invariant in size and shape. This is the case of a plane-parallel field or translational field problem, where A has one component.

In this work we tried to present some numerical algorithms in area of DPS in engineering. Although we limited the presentation from the programmer's viewpoint of a conventional computer, the results can be extended to parallel computing.

Domain decomposition offers an efficient approach for large-scale problems or complex geometrical configurations. This method in the context of the finite element programs leads to a substantial reduction of the computing resources as the time of the processor.

Although we limited the presentation to the domain decomposition considering physical properties of the field problem, the partitioning of the domain can be performed according to the mathematical models of the field problem (operator decomposition).

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