

# Methods and Systems for Identifying Parameters of AC Electrical Machines

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**Abstract:** - To analyze the operation of electric machines ordered through converters (inverter-corrector systems) must, mathematically, correction entry and exit inverter to be expressed in terms of alternatives sinus (a, b, c), as will be able to see this paper [1]. For an induction machine is preferable to use the stationary reference system (this system of reference is advantageous to use also for a variable reluctance machines or for the synchronous). It is also possible to represent mathematical equations corresponding electrical machine and its drive system in a rotational reference system. Ordered in an inverter for voltage harmonics from the machine terminals are produced mainly due process of switching the inverter, but can be influenced by the nature of load. Equations represented in a rotating reference system can be used to calculate the higher harmonics of the system using harmonic order reduction techniques. In this way you can create the necessary means for analyzing the dynamic behavior of systems that use all the corrector-inverter [2], [3], [4].

**Key-Words:** - coordinate system ( , ) coordinate system (d, q), voltage vector, current vector, harmonic analysis, operational amplifiers.

## 1 Introduction

### 1.1 Mathematical equations of A.C. machines in different coordinate systems

Logical extension of the system matrix and the generalized theory of electrical machines can read equations and power flux, the corresponding parameters of the machine, the expressions of electrical power and electromagnetic torque.

Stator voltage equation is formulated in a standstill.

Voltage space vector can be defined similar to a current space vector [1]:

$$u_s = \frac{2}{3}(u_A + au_B + a^2u_C) \quad (1)$$

Voltage equation of phase B is multiplied by „a”, the phase C is multiplied by „a<sup>2</sup>” and all three equations are multiplied by 2/3:

$$\left. \begin{aligned} u_A &= i_A R_s + \frac{d\Psi_A}{dt} \\ u_B &= \left( i_B R_s + \frac{d\Psi_B}{dt} \right) | \cdot a \\ u_C &= \left( i_C R_s + \frac{d\Psi_C}{dt} \right) | \cdot a^2 \end{aligned} \right| \cdot \frac{2}{3} \quad (2)$$

Bringing together the three equations is obtained:

$$\begin{aligned} \frac{2}{3}(u_A + au_B + a^2u_C) &= \\ &= \frac{2}{3}(i_A + ai_B + a^2i_C)R_s + \\ &+ \frac{d}{dt} \left[ \frac{2}{3}(\Psi_A + a\Psi_B + a^2\Psi_C) \right] \end{aligned} \quad (3)$$

Analogically, equation 1 defining voltages vector, similarly, the  $\Psi_s$  vector can be written as:

$$\Psi_s = \frac{2}{3}(\Psi_A + a\Psi_B + a^2\Psi_C) \quad (4)$$

So, the space vector of stator voltage is:

$$u_s = i_s R_s + \frac{d\Psi_s}{dt} \quad (5)$$

Analogically, for the rotor is obtained:

$$u_r = i_r R_r + \frac{d\Psi_r}{dt} \quad (6)$$

$u_r, i_r, \Psi_r$  have the same expressions as those of spatial vectors of the stator, the indices A, B, C being replaced by a, b, c.

The same equation can be expressed with Park vectors and can be obtained by designing the two phases  $\alpha, \beta$ . For stator and rotor voltage equations are [1]:

$$u_\alpha = i_\alpha R + \frac{d\Psi_\alpha}{dt}, u_\beta = i_\beta R + \frac{d\Psi_\beta}{dt} \quad (7)$$

Combining these two equations it obtained:

$$u_\alpha + ju_\beta = (i_\alpha + ji_\beta)R + \frac{d}{dt}(\Psi_\alpha + j\Psi_\beta) \quad (8)$$

which, given the transformation expressions from simplified complex equations into instantly, get global equivalent equation:

$$u = iR + \frac{d\Psi}{dt} \quad (9)$$

Similar to the derivation of voltage spatial vector equations and vector spatial flux equations can be derived from the three phases A, B, C or rectangular phases  $\alpha, \beta$ . The most convenient is to use reference system with  $\alpha, \beta$ , therefore, instead of depending on inductance matrices we will consider the following:

$$\begin{matrix} & A & B & C \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} l_{ss} & l_{ms} & l_{ms} \\ l_{ms} & l_{ss} & l_{ms} \\ l_{ms} & l_{ms} & l_{ss} \end{bmatrix} & \text{and} & \\ & a & b & c \end{matrix} \quad (10)$$

$$\begin{matrix} A & \cos\alpha & \cos(\alpha + 120^\circ) & \cos(\alpha + 240^\circ) \\ B & \cos(\alpha + 240^\circ) & \cos\alpha & \cos(\alpha + 120^\circ) \\ C & \cos(\alpha + 120^\circ) & \cos(\alpha + 240^\circ) & \cos\alpha \end{matrix}$$

where:

- $l_{ss}$  is own stator inductance;
- $l_{ms}$  is the mutual inductance of stator;
- $\alpha$  is the angle between the stator and rotor corresponding phases.

The inductances matrix is:

$$\begin{matrix} & 0 & \alpha & \beta \\ \begin{matrix} 0 \\ \alpha \\ \beta \end{matrix} & \begin{bmatrix} l_{ss} + 2l_{ms} & & \\ & l_{ss} - l_{ms} & \\ & & l_{ss} - l_{ms} \end{bmatrix} & \end{matrix} \quad (11)$$

The corresponding inductances of  $\alpha$  and  $\beta$  phases are equals and the spatial current vector is:  $i = i_\alpha + ji_\beta$ . So, for an arbitrary space distribution, the stator inductance is:

$$L_s = l_{ss} - l_{ms} \quad (12)$$

For sinusoidal spatial distribution:

$$l_{ms} = l_{AB} = L_{ms} \cos 120^\circ = -\frac{L_{ms}}{2} \quad (13)$$

$$l_{ms} = l_{AC} = L_{ms} \cos 240^\circ = -\frac{L_{ms}}{2}$$

$l_{ss}$  inductance is bigger than mutual inductance  $L_{ms}$ :

$$l_{ss} - l_{ms} = (L_{sl} + L_{ms}) - \left(-\frac{L_{ms}}{2}\right) = L_{sl} + \frac{3}{2}L_{ms} \quad (14)$$

Hence, the stator three-phase inductance:

$$L_s = L_{sl} + \frac{3}{2}L_{ms} \quad (15)$$

By considering equations 10, 12 and 13, in case spatial sinusoidal distribution:

$$l_{ss} + 2l_{ms} = L_{sl} + L_{ms} - 2\frac{L_{ms}}{2} = L_{sl} \quad (16)$$

So:

$$L_{s0} = L_{sl} \quad (17)$$

However, this result above needs correction, while the inductance is determined even of space harmonics multiplied by 3 phases and the number of poles. Similarly, the total three-phase rotor own inductance is [5]:

$$L_r = L_{rl} + \frac{3}{2}L_{mr} \quad (18)$$

$$L_{r0} = L_{rl} \quad (19)$$

Considering expression stator-rotor mutual inductance matrix, and making necessary changes to the components  $i_\alpha, i_\beta$ , the equation of flux is:

$$\begin{matrix} & i_0 & i_\alpha & i_\beta \\ & 0 & \alpha & \beta \\ \begin{bmatrix} \Psi_0 \\ \Psi_A \\ \Psi_B \end{bmatrix} & = \frac{3}{2}L_{sr} & A & B \begin{bmatrix} 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \end{matrix} \quad (20)$$

Stator mutual flux losses caused by rotor current:

$$\begin{aligned} \Psi_{sm} &= \Psi_{Am} + j\Psi_{Bm} = \\ &= \frac{3}{2}L_{sr}[(\cos\alpha \cdot i_\alpha - \sin\alpha \cdot i_\beta) + j(\sin\alpha \cdot i_\alpha + \cos\alpha \cdot i_\beta)] = \\ &= \frac{3}{2}L_{sr}[i_\alpha(\cos\alpha + jsin\alpha) + ji_\beta(\cos\alpha + jsin\alpha)] = \\ &= \frac{3}{2}L_{sr}(i_\alpha + ji_\beta)e^{j\alpha} = \frac{3}{2}L_{sr}e^{j\alpha}i_r \end{aligned} \quad (21)$$

Hence, stator-rotor mutual inductance is:

$$L_m = \frac{3}{2} L_{sr} \quad (22)$$

and it must be multiplied by the  $e^j$  factor due to the relative rotation of two reference systems. Stator-rotor mutual inductance matrix is transposed stator-rotor mutual inductance matrix. The same expression is obtained for the mutual inductance  $L_m$  if deemed transposed matrix and instead  $e^j$ , we obtain  $e^{-j}$  multiplier.

Hence the vector equations of flux in the natural reference system are:

$$\Psi_s = L_s i_s + L_m e^{j\alpha} i_r \quad (23)$$

$$\Psi_r = L_m e^{-j\alpha} i_s + L_r i_r$$

where  $L_s, L_r, L_m$  are definite in (15), (18), (22).

The equations become:

$$\Psi_{s0} = L_{s0} i_{s0} \quad (24)$$

$$\Psi_{r0} = L_{r0} i_{r0}$$

## 1.2 Electric power and electromagnetic torque

For symmetrical three-phase systems, instantaneous power can be expressed according to the reference system ( , ) as follows [2], [3]:

$$p = \frac{3}{2} (u_\alpha i_\alpha + u_\beta i_\beta) + 3u_0 i_0 \quad (25)$$

If, for three-phase vectors, to complex power ( $ui^*$ ) it is applied the simplified complex transformation method, we obtain:

$$ui^* = (u_\alpha + ju_\beta) \cdot (i_\alpha - ji_\beta) = u_\alpha i_\alpha + u_\beta i_\beta + j(u_\beta i_\alpha - u_\alpha i_\beta) \quad (26)$$

Hence, the instantaneous power is obtained as 3/2 times the real part of this expression - if components are not zero:

$$p = \frac{3}{2} \text{Re}[ui^*] \quad (27)$$

$$\text{occasionally, power sequence 0: } p_0 = 3u_0 i_0 \quad (28)$$

If we add to the instant power of the stator and rotor and if they are calculated separately by applying equation (27):

$$p = p_s + p_r = \frac{3}{2} \text{Re}[u_s i_s^* + u_r i_r^*] \quad (29)$$

	1	2	3	Sum	PI		Total
U [V]	38.66	0.00	0.00	0.00	0.00		
I [A]	0.254	0.000	0.000	0.000	0.000		
P [kW]	0.009	0.000	0.000	0.000	0.000	0.00	
S [VA]	0.011	0.000	0.000	0.000	0.000	0.00	
Q [VAr]	0.002	0.000	0.000	0.000	0.000	0.00	
PI [W]	0.000	0.000	0.000	0.000	0.000	0.00	
QI [VAr]	0.000	0.000	0.000	0.000	0.000	0.00	
cos φ	0.94	0.00	0.00	0.00	0.00		
PF	0.76	0.00	0.00	0.00	0.00		
PS [W/h]	0.001	0.000	0.000	0.000	0.000		
AS [W/h]	0.001	0.000	0.000	0.000	0.000		
AQ [W/h]	-0.001	0.000	0.000	0.000	0.000		
AP [W/h]	0.000	0.000	0.000	0.000	0.000		
AQI [W/h]	0.000	0.000	0.000	0.000	0.000		
APR [W/h]	0.001	0.000	0.000	0.000	0.000		
ASR [W/h]	0.000	0.000	0.000	0.000	0.000		
AQR [W/h]	0.000	0.000	0.000	0.000	0.000		
AQR [W/h]	-0.001	0.000	0.000	0.000	0.000		

Fig.1. Determination of main parameters of A.C. induction machine

If equation defining voltage and current space vectors are replaced in equation (29) is obtained:

$$\frac{3}{2} \text{Re} \left[ \frac{2}{3} (u_a + au_b + a^2 u_c) \cdot \frac{2}{3} (i_a + ai_b + a^2 i_c) \right] = \quad (30)$$

$$= u_a i_a + u_b i_b + u_c i_c$$

The value obtained is equal to an instantaneous value of power. In Fig. 1 are highlighted possibilities for determining and measuring the electric powers for a low power three-phase A.C. machine.

To obtain the appropriate electromagnetic torque relationship can use in a similar fashion to those described above, coordinate system ( , ), but the most convenient way to get this relationship is using common reference for the stator and rotor (System Reference (d, q)).

Expression in the reference torque (d, q) for a synchronous machine is:

$$\tau = -i_D L_{dD} i_q + i_Q L_{qQ} i_d - i_d i_q (L_d - L_q) \quad (31)$$

Rotor flux and space current vectors can be expressed in the reference system (d, q) as follows:

$$\Psi_r = \Psi_d + j\Psi_q = L_d i_d + L_{dD} i_D + j(L_q i_q + L_{qQ} i_Q) \quad (32)$$

$$i_r = i_d + j i_q$$

If the complex conjugate vector space of flux rotor is multiplied by the current space vector of rotor get:

$$\Psi_r^* i_r = L_d i_d^2 + L_{dD} i_D i_d + L_q i_q^2 + L_{qQ} i_Q i_q + j(-L_q i_q i_d - L_{qQ} i_Q i_d + L_d i_d i_q + L_{dD} i_D i_q) \quad (33)$$

$$\text{and: } \tau = -\frac{3}{2} \text{Im}[\Psi_r^* i_r] \quad (34)$$

Apply the multiplier 3/2 and introduce the term derivative of torque, the expression becomes equivalent to equations with two-pole machine, and for a multi-pole machine, mathematical expression should be multiplied by the number of poles P.

Equation 34 can be written in the following form by considering the angle between the 2 vectors:

$$\tau = -\frac{3}{2} |\Psi_r| |i_r| \sin \beta \tag{35}$$

By considering the interpretation of a product of vectors, torque is described by a vector that is perpendicular to the plane defined by  $\Psi_r$  and  $i_r$  with a value equal to 3/2 times the area of the parallelogram in Fig. 2, as follows:

$$\tau = -\frac{3}{2} \Psi_r \times i_r = \frac{3}{2} \Psi_s \times i_s \tag{36}$$

Torque expression obtained has not only spatial distribution but also a sinusoidal temporal variation arbitrary. In a multi-pole machine, it must be multiplied by the number of pole pair's "p" [3], [4].

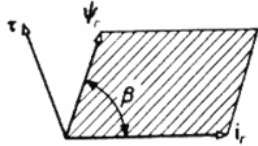


Fig.2. Torque vector result of flux and current vectors

## 2 Display and recording of three-phase vectors

Phase vectors can be displayed on an oscilloscope screen CRT or with a V: 2.4.2 06.1.2006, Lic: 39070171 V: 2.4.2 06.1.2006, Lic: 39,070,171, LabVIEW Data. Since the deflection plates acting in directions perpendicular to these tensions are applied rectangular system. To display will be products that will determine the appropriate tension. For the current vectors, appropriate voltages may be obtained by using shunting resistance. Place vector displayed on the oscilloscope can be photographed using special devices which make it possible to assess the size and elaboration results [6].

### 2.1 Display the voltage vectors

Display of voltage vectors is quite simple using the circuit of Fig. 3.a, where A, B and C terminals of the three phases and neutral of the star is 0 compared to the electric machine is tested. Point's neutral is not actually necessary in this situation so that the same configuration can be used for connection triangle. The common point of two resistors 0 must be connected to the common mass of amplifiers and the oscilloscope, while tensions  $u_x'$  and  $u_y'$  is applied to the inputs of horizontal and vertical amplifiers. [7]

Thus:

$$u_y' = \frac{1}{2} (u_b - u_c) \tag{37}$$

$$u_x' = u_y' - u_b + u_a = -\frac{1}{2} (u_b + u_c) + u_a$$

Comparing these expressions with real and imaginary parts of voltage vector:

$$\text{Re}[u] = \frac{2}{3} u_a - \frac{1}{3} (u_b + u_c) + u_x \tag{38}$$

$$\text{Im}[u] \frac{1}{\sqrt{3}} (u_b - u_c) = u_y$$

is obtained:

$$u_x' = \frac{3}{2} u_x; \quad u_y' = \frac{\sqrt{3}}{2} u_y \tag{39}$$

Input voltage is indeed proportional to the voltage vector coordinates considering a  $\sqrt{3}$  scale factor. This can be taken into account by setting the gain of amplifiers. Equations above are valid for the connection phase voltages and components containing zero star, but in this case, the projection vector on axis is not apparent in measurements of phase. Care should be taken not to apply excessive loads on voltage  $u_x'$ , because the task will contain  $u_y'$  component. If the load can not be reduced, the circuit configuration must be modified to eliminate unwanted interactions. The load is connected to terminals O'B (at  $u_y'$ ) and another identical load will restore the balance circuit terminals O'C.

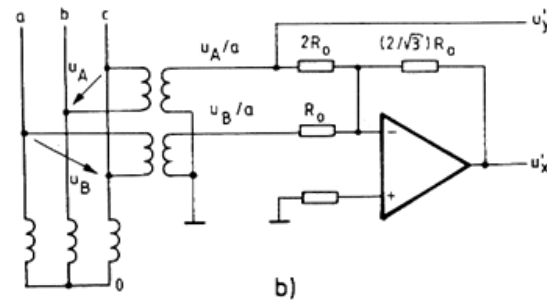
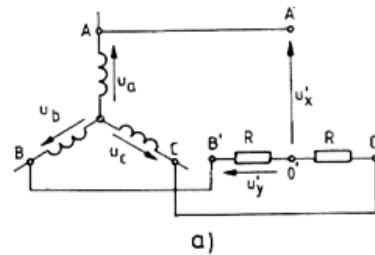


Fig.3 Determination of voltages at the terminals of a three-phase consumer  
a) Vector diagram. b) Practical realization

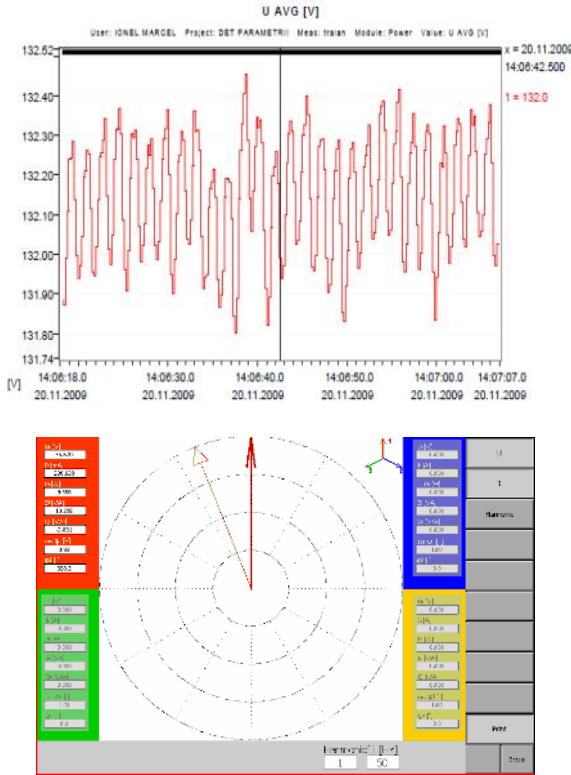


Fig.4. Displaying voltage vectors at the terminals of an induction motor at low frequency.

a) Sinusoidal diagram, b) Vector diagram

In practice, an indirect measurement scheme is usually used where the voltage transformer is connected to the line terminals of the test machine. If there is a null component,  $u_a + u_b + u_c = 0$ , equation 38 can be written in simpler form:

$$u_x = u_a - (u_b + u_c)$$

$$u_y = \frac{1}{\sqrt{3}}(u_b - u_c) \tag{40}$$

X and y components of the voltage vector u, can be obtained from the second voltage line using the following equation:

$$\sqrt{3}u_x = -\frac{1}{\sqrt{3}}u_A - \frac{2}{\sqrt{3}}u_B$$

$$\sqrt{3}u_y = u_A \tag{41}$$

As you can see, the scheme includes two power transformers and an operational amplifier. From this scheme follows signal proportional to the voltage vector components that can be displayed on the oscilloscope:

$$u'_x = \frac{\sqrt{3}}{a}u_x; \quad u'_y = \frac{\sqrt{3}}{a}u_y \tag{42}$$

These signals must be applied directly to horizontal and vertical oscilloscope inputs.

## 2.2 Display the current vectors

When we want to show current vectors on the oscilloscope screen, resulting tensions are proportional to the two components of the vector current and should be applied to the inputs of the oscilloscope deflection plates. Equations 38 or 40 can be used to calculate vector components.

When available only three terminals of winding machine are may be used current transformers. When we have 6 terminals, power transformers can be omitted from the measurements made. Three current transformers are needed if the current system is unbalanced, when there is zero current on the neutral line. In practice, however,  $i_0 = 0$  in most cases. When using current transformers, secondary currents  $i_b$  and  $i_c$  undergoing identical resistors (as in Fig. 5). Care should be taken to polarizatia each winding. For the sum of the three currents is 0, the voltage drop between points C and B is  $i_a R$ . Hence, three phase power system shown in points A, B and C will be proportional to the current system, so the current vectors can be displayed in the same way as the vectors of tension. Unfortunately, in most cases, the standard current transformers do not produce a sufficiently high voltage in the secondary for a small primary current [7].

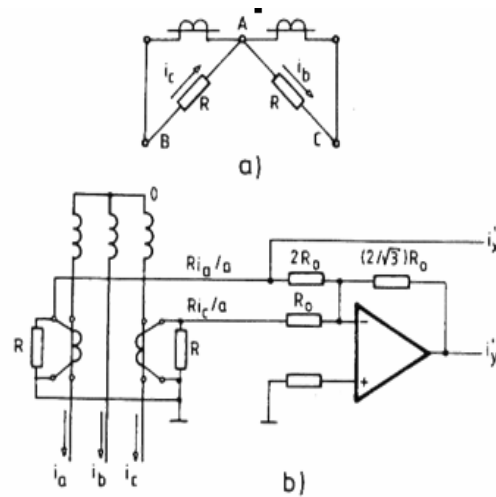


Fig.5 Determination of current at the terminals of a three-phase consumer

a) Vector diagram. b) Practical realization

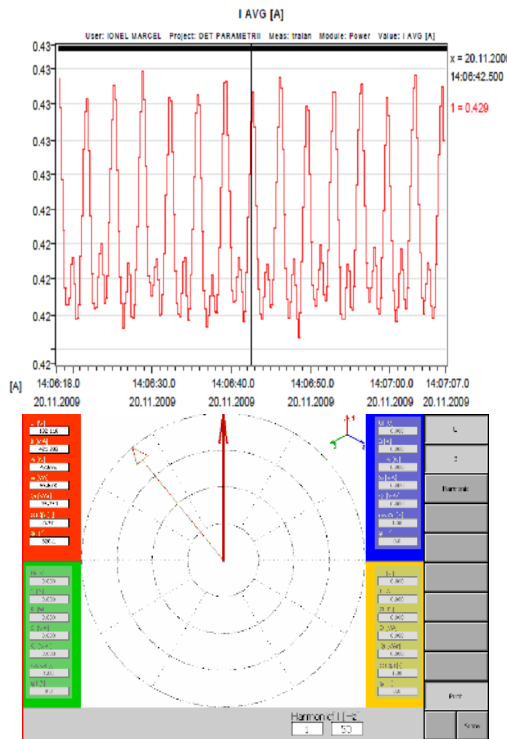


Fig.6. Displaying current vectors at the terminals of an induction motor at low frequency.  
a) Sinusoidal diagram, b) Vector diagram

The practice can be used scheme of Fig. 5.b with a general operational amplifier to display the current vectors. Since  $i_x = i_a$  take only problem is to generate  $i_y$ . From equation 38 can be shown, for example, that:

$$i_y = -\frac{1}{\sqrt{3}}i_a - \frac{2}{\sqrt{3}}i_c \quad (43)$$

Another way to generate y component current vector and is the analytical. This expression can be achieved by the circuit configuration shown in Fig. 5.b. Taking into account the report of transformers current transformers and voltage drop on the resistors of value R, the signals are proportional to the components x, y, the vector current:

$$i'_x = \frac{R}{a}i_x; \quad i'_y = \frac{R}{a}i_y \quad (44)$$

Current transformers can be replaced by other current sensors, Hall elements or optic-couplers. Current transformers can be used when accurate measurements of DC components transient phenomena are important for registration, or when the frequency is too low. In such cases must be installed other devices (resistors shunting).

#### 4 Conclusions

Nowadays due to the use of devices and microelectronic semiconductor electronic devices,

electromagnetic techniques were replaced with efficiency and high precision control. Electromagnetic converters have become often the simplest; allowing obtaining **any** desired mechanical characteristics of **any** type of motors. This comes from the need to identify stationary and dynamic parameters of electric machines controlled by static converters simultaneously with the identification of dynamic phenomena and deforming schemes introduced by them. However this does not mean that all types of engines are equal in terms of price, performance control, efficiency, optimal weight etc. This means that the electromagnetic structure and operating principle can be another crucial influence on the effectiveness and improving the energy system. That's why these, today more attention from the previous period, through better use of new materials and design of magnetic and electrical circuits [8]. The structure of such machines is often performed especially given electronic control and mechatronics to create specific converters. Electronic control system performance has improved only energy but also changed the general philosophy of design and complete development of new types of machinery. For further progress in this area is necessary to consider simultaneously supplying the electronic circuit and electromagnetic field structure [9]. A conventional electronic control (with pulse modulation - PWM), has some typical disadvantages include: high bandwidth of the fundamental harmonic voltage, pulses of torque, the influence of asymmetry in the structure, especially at low angular speed, the need for additional external cooling in low speeds, radio interference and energy with relatively low efficiency of machines.

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