Length Determination of Flexible Toothed Wheel of a Double Harmonic Drive, Depending on the Deformation Elastic Force

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Abstract: - This paper presents a variant of analytical determination of flexible toothed wheel length, component of double toothed harmonic transmission, depending on elastic force of radial deformation, the geometrical parameters having constant values, aiming to reduce the tension status in flexible wheel and stress status of component elements of the wave generator.

Key-Words: - Flexible toothed wheel, double harmonic drive, deformation elastic force.

1 Introduction
Recent tendencies in machine building have led to apparition and improvement of modern transmissions. Thus, a special attention has been given to study and research of toothed harmonic drives. Toothed harmonic drive is based on the principle of transformation of rotation movement parameters due to harmonic deformation of a flexible wheel, which is a component of the mechanism.

This principle was proposed by A.I. Moskvitin for friction transmissions [1] which have electromagnetic wave generator. In the year 1959, C.W. Musser patented toothed harmonic drive [2], which incorporates a flexible toothed wheel and a mechanic wave generator.

Toothed harmonic drive turned of interest for research teams both in the Polytechnical Institute of Timisoara, where a toothed harmonic drive was realized in 1970, and in the Polytechnical Institute Cluj-Napoca where the prototype of a harmonic reducer which has toothed wheels with involute profile was designed and realized.

After the year 1975, research developed and results have materialized in realizing a stand with closed and open mechanical circuits for testing these harmonic drives at “Eftimie Murgu” University of Resita [5].
The research was resulted in a dividing head, a flexion-extension module and a pronation-supination one, used in construction of a didactic industrial robot.

Based on its construction and functional advantages, toothed harmonic drive knows a large construction diversification. A construction variant is the double toothed harmonic drive with wave generator dephased with \( \pi/2 \), proposed by the authors of this paper.

This toothed harmonic drive presents the advantage of reducing the state of tension from the flexible toothed wheel due to the disappearance of bending stress on the length of generatrix. Because in the rigid joining zone with output shaft of the drive the wheel wears out, the disadvantage posed by flexible toothed wheels with rigid joint is eliminated. At the same time, the stress state in the component elements of the wave generator is reduced due to self deformation of flexible toothed wheel in the second level. In this case, flexible toothed wheel appears like a thin cylindrical shell, which has the constant wall thickness \( h \) and over which a deformation elastic radial force \( F_e \) acts in gearing zones, as shown in Fig.1.

Another advantage is the reduction of tension state in flexible toothed wheel, due to disappearance of bending on flexible toothed wheel length and due to self deformation in the second level gearing. In the second level, radial deformations of increase or decrease versus undeformed neutral fibre diameter are rotated with \( \pi/2 \) against the first level, fact which leads to reducing the tension state in flexible wheel and the stress state of component elements of the wave generator.

The length of flexible toothed wheel with double gearing depends on geometrical elements, elastic force of radial deformation of flexible wheel, which appears like a tube with thin walls stressed by radial forces of deformation, ensuring the teeth radial displacement, necessary to realize the gearing.

2 Analytical Determination of Length of Flexible Toothed Wheel with Double Gearing

To determine the length of the flexible toothed wheel for the case of a toothed harmonic transmission with double gearing, it is necessary to establish the elastic displacements of a point from the median surface of the flexible toothed wheel, which depend on the flexible toothed wheel length and on the deformation elastic force \( F_e \).

In Fig.2, the functional scheme of the toothed harmonic transmission with double gearing, with wave generator dephased with \( \pi/2 \), is presented. The double flexible toothed wheel has the teeth number \( z_1 \) in the first level, respectively \( z_3 \) in the second level, and it shows itself as a thin cylindrical shell with inextensible median surface, to which it is necessary to establish the elastic displacements \( u \), \( v \) and \( w \) of a point from the median surface upon the coordinate axes of the chosen system.

From the theory of thin-walled tubes [3], the equations system for the displacements \( u \), \( v \) and \( w \) is:

\[
\begin{align*}
  u &= \frac{F_e r^3}{\pi D_\ell} \sum_{n=2,4,6}^{\infty} \frac{r c \cos \theta \cos t}{(n^2-1)^2} \left[ \frac{n^2 r^2}{3} + 2(1-v)r^2 \right] \\
  v &= \frac{F_e r^3}{\pi D_\ell} \sum_{n=2,4,6}^{\infty} \frac{r c \sin \theta \cos t}{(n^2-1)^2} \left[ \frac{n^2 r^2}{3} + 2(1-v)r^2 \right] \\
  w &= \frac{F_e r^3}{\pi D_\ell} \sum_{n=2,4,6}^{\infty} \frac{n^2 c \cos \theta \cos t}{(n^2-1)^2} \left[ \frac{n^2 r^2}{3} + 2(1-v)r^2 \right]
\end{align*}
\]
The radial displacement \( w = \delta \), which is the radial elastic deformation of the flexible toothed wheel from gearing geometrical conditions, is presented as follows:

\[
\delta = \frac{m k n_u}{2}
\]  

(2)

where:  
- \( m \) - teeth module;  
- \( n_u \) - the number of generator waves;  
- \( k = 1, 2, 3, … \)

For the waves generator with: \( n = 2, q = 2, \cos n \theta = \cos q \theta = 1, c = x = \ell, \theta = 0 \),

\[
w = \frac{F_e r^3}{\pi D} \left( n^2 - 1 \right)^{\frac{2}{3}} \left[ \frac{n^2 \ell^2}{3} + 2(1 - \nu) r^2 \right]^{\frac{2}{3}}
\]

(3)

resulting:

\[
[\pi D w (n^2 - 1)^2 \frac{n^2}{3}] \ell^2 - (F_e \cdot n^2 r^3) \ell + 2\pi D w (n^2 - 1)^2 (1 - \nu) r^2 = 0
\]

(4)

where: \( r = \frac{m z}{2} \);  
- \( z \) - the teeth number of the flexible toothed wheel;  
- \( D = \frac{E h^3}{12(1 - \nu^2)} \) - strip bending stiffness;  
- \( h \) - constant thickness of the flexible toothed wheel wall;  
- \( \nu \) - Poisson’s coefficient;  
- \( w = \delta = \frac{m k n_u}{2} \).

For \( k = 1; n_u = 2 \) results: \( w = \delta = m \).

3 Length Determination of Flexible Toothed Wheel Using the Solutions of Analytical Equation

From the second degree equation (4), the roots \( \ell_1 \) and \( \ell_2 \) are determined, depending on deformation elastic force \( F_e \).

The following notations are used:

\[
A = \pi D w (n^2 - 1)^2 \frac{n^2}{3} ;
\]

\[
B = -F_e \cdot n^2 r^3 ;
\]

\[
C = 2\pi D w (n^2 - 1)^2 (1 - \nu) r^2 ;
\]

\[
D = \frac{E h^3}{12(1 - \nu^2)} ,
\]

whom values are written in table 1.

By using Microsoft Excel software, \( \ell_1, \ell_2 \) mathematical solutions of equation (4) are obtained, which represent possible lengths of flexible toothed wheel with double gearing.

From the multitude of mathematical solutions \( \ell_1, \ell_2 \) of equation (4), only some of them have physical meaning.

Table 1 shows \( \ell_1, \ell_2 \) solutions – which have physical meaning – of the equations for different values of elastic deformation force. In fig. 3 is presented the graphical variation of length \( \ell \) for which the equation solutions have physical meaning, depending on longitudinal elasticity module for different values of elastic deformation force \( F_e \).
4 Conclusions
The paper presents the equation of length of flexible toothed wheel from double harmonic transmission with waves generator dephased with $\pi/2$, as well as the solutions of equations (4) which have physical meaning for different values of deformation elastic force $F_{e}$, with gear module $m$, radius of median fibre $r$, wall thickness of flexible toothed wheel $h$ and elasticity module $E$ considered constants.

The results presented support researchers and designers of double toothed harmonic transmissions in choosing the constructive solutions and the optimal length of double flexible toothed wheel.

References: