Turbulence and Quantum Mechanics from Cosmic to Planck Scales

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Abstract: - A scale invariant model of statistical mechanics is applied to describe a modified statistical theory of turbulence and its quantum mechanical foundations. Hierarchies of statistical fields from cosmic to *Planck* scale are described. Energy spectrum of equilibrium isotropic turbulence is shown to follow *Planck* law. Predicted velocity profiles of turbulent boundary layer over a flat plate at four consecutive scales of LED, LCD, LMD, and LAD are shown to be in close agreement with the experimental observations in the literature. The physical and quantum nature of time is described and a scale-invariant definition of time is presented and its relativistic behavior is examined. New paradigms for physical foundations of quantum mechanics as well as derivation of *Dirac* relativistic wave equation are introduced.

Key-Words: - Statistical theory of turbulence; Quantum mechanics; Relativity; Dirac equation; TOE.

1 Introduction

It is well known that the laws of nature appear to reveal ever increasing similarities over a broad range of scales of space and time from the exceedingly large scale of cosmology to the minute scale of quantum optics (Fig.1). The similarities between stochastic quantum fields [1-17] and classical hydrodynamic fields [18-29] resulted in recent introduction of a scale-invariant model of statistical mechanics [30], and its application to thermodynamics [31] and fluid mechanics [32].

More recently, the implication of the model to the statistical theory of turbulence [33, 34] was investigated. In the present study the physical foundations of the problems of turbulence and quantum mechanics are further examined. Homogenous isotropic turbulence is identified as a spectrum of eddies (energy levels) with *Gaussian* velocity distribution, *Planck* energy distribution, and *Maxwell-Boltzmann* speed distribution. The nature of dissipation spectrum of isotropic turbulence is examined and a derivation of *Dirac* relativistic wave equation is presented.

2 A Scale Invariant Model of Statistical Mechanics

Following the classical methods [35-39] the invariant definitions of density ρ_β, and velocity of *atom* $\mathbf{u}_\beta$, *element* $\mathbf{v}_\beta$, and *system* $\mathbf{w}_\beta$ at the scale β are given as [31]

$$\rho_\beta = n_\beta m_\beta = m_\beta \int f_\beta du_\beta \quad , \quad \mathbf{u}_\beta = \mathbf{v}_{\beta-1} \qquad (1)$$

$$\mathbf{v}_\beta = \rho_\beta^{-1} m_\beta \int \mathbf{u}_\beta f_\beta d\mathbf{u}_\beta \quad , \quad \mathbf{w}_\beta = \mathbf{v}_{\beta+1} \qquad (2)$$

Similarly, the invariant definition of the peculiar and diffusion velocities are introduced as

$$\mathbf{V}'_\beta = \mathbf{u}_\beta - \mathbf{v}_\beta \quad , \quad \mathbf{V}_\beta = \mathbf{v}_\beta - \mathbf{w}_\beta \qquad (3)$$

such that

$$\mathbf{V}_\beta = \mathbf{V}'_{\beta+1} \qquad (4)$$

For each statistical field, one defines particles that form the background fluid and are viewed as point-mass or "*atom*" of the field. Next, the *elements* of the field are defined as finite-sized composite entities composed of an ensemble of "*atoms*" as shown in Fig.1.

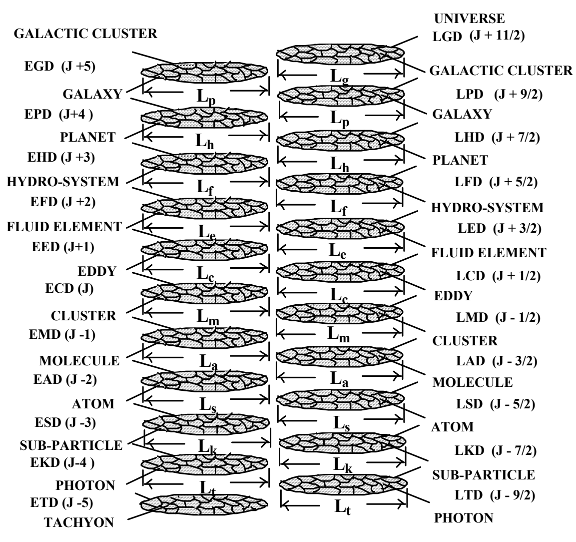


Fig.1 A scale invariant view of statistical mechanics from cosmic to tachyon scales.

Finally, the ensemble of a large number of "*elements*" is defined as the statistical "*system*" at that particular scale.

3 Scale Invariant Forms of the Conservation Equations for Chemically Reactive Fields

Following the classical methods [35-37], the scale-invariant forms of mass, thermal energy, linear and angular momentum conservation equations at scale β are given as [40]

$$\frac{\partial \rho_\beta}{\partial t} + \nabla.\left(\rho_\beta \mathbf{v}_\beta\right) = \Omega_\beta \qquad (5)$$

$$\frac{\partial \varepsilon_\beta}{\partial t} + \nabla.\left(\varepsilon_\beta \mathbf{v}_\beta\right) = 0 \qquad (6)$$

$$\frac{\partial \mathbf{p}_\beta}{\partial t} + \nabla.\left(\mathbf{p}_\beta \mathbf{v}_\beta\right) = -\nabla.\mathbf{P}_{ij\beta} \qquad (7)$$

$$\frac{\partial \boldsymbol{\pi}_\beta}{\partial t} + \nabla.\left(\boldsymbol{\pi}_\beta \mathbf{v}_\beta\right) = 0 \qquad (8)$$

that involve the *volumetric density* of thermal energy $\varepsilon_\beta = \rho_\beta \tilde{h}_\beta$, linear momentum $\mathbf{p}_\beta = \rho_\beta \mathbf{v}_\beta$, and angular momentum $\boldsymbol{\pi}_\beta = \rho_\beta \boldsymbol{\omega}_\beta$. Also, Ω_β is the chemical reaction rate, $\tilde{h}_\beta$ is the absolute enthalpy [40],

$$\tilde{h}_\beta = \int_0^T c_{p\beta} dT_\beta \qquad (9)$$

and $\mathbf{P}_{ij\beta}$ is the stress tensor [35]

$$\mathbf{P}_{ij\beta} = m_\beta \int (\mathbf{u}_{i\beta} - \mathbf{v}_{i\beta})(\mathbf{u}_{j\beta} - \mathbf{v}_{j\beta}) f_\beta du_\beta \qquad (10)$$

In the derivation of (7) we have used the definition of the peculiar velocity (3) along with the identity

$$\overline{\mathbf{V}'_{i\beta}\mathbf{V}'_{j\beta}} = \overline{(\mathbf{u}_{i\beta} - \mathbf{v}_{i\beta i})(\mathbf{u}_{j\beta} - \mathbf{v}_{j\beta})} = \overline{\mathbf{u}_{i\beta}\mathbf{u}_{j\beta}} - \mathbf{v}_{i\beta}\mathbf{v}_{j\beta} \qquad (11)$$

The classical definition of vorticity involves the curl of linear velocity $\nabla \times \mathbf{v}_\beta = \boldsymbol{\omega}_\beta$ thus giving rotational velocity a secondary status in that it depends on translational velocity $\mathbf{v}_\beta$. However, it is known that particle's rotation about its center of mass is independent of the translational motion of its center of mass. In other words, translational, rotational, and vibrational (pulsational) motions of particle are independent degrees of freedom that should not be necessarily coupled. To resolve this paradox, the iso-spin of particle at scale β is defined as the curl of the velocity at the next lower scale of β−1 [41]

$$\boldsymbol{\varpi}_\beta = \nabla \times \mathbf{v}_{\beta-1} = \nabla \times \mathbf{u}_\beta \qquad (12)$$

such that the rotational velocity, while having a connection to some type of translational motion at internal scale β−1, retains its independent degree of freedom at the external scale β as desired. A schematic description of iso-spin and vorticity fields is shown in Fig.2. The nature of galactic vortices in cosmology and the associated dissipation have been discussed [25, 42].

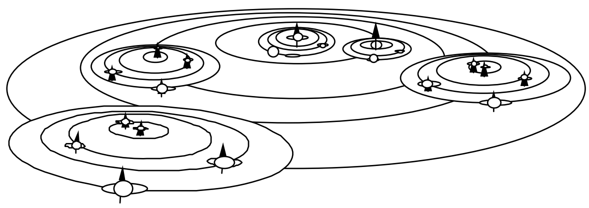


Fig.2 Description of internal (iso-spin) versus external vorticity fields in cosmology [41].

The local velocity $\mathbf{v}_\beta$ in (5)-(8) is expressed in terms of the convective $\mathbf{w}_\beta$ and the diffusive $\mathbf{V}_\beta$ velocities [40]

$$\mathbf{w}_\beta = \mathbf{v}_\beta - \mathbf{V}_{\beta g} \quad , \quad \mathbf{V}_{\beta g} = -D_\beta \nabla \ln(\rho_\beta) \qquad (13a)$$

$$\mathbf{w}_\beta = \mathbf{v}_\beta - \mathbf{V}_{\beta tg} \quad , \quad \mathbf{V}_{\beta tg} = -\alpha_\beta \nabla \ln(\varepsilon_\beta) \qquad (13b)$$

$$\mathbf{w}_\beta = \mathbf{v}_\beta - \mathbf{V}_{\beta hg} \quad , \quad \mathbf{V}_{\beta hg} = -\nu_\beta \nabla \ln(\mathbf{p}_\beta) \qquad (13c)$$

$$\mathbf{w}_\beta = \mathbf{v}_\beta - \mathbf{V}_{\beta rhg} \quad , \quad \mathbf{V}_{\beta rhg} = -\nu_\beta \nabla \ln(\boldsymbol{\pi}_\beta) \qquad (13d)$$

where ($\mathbf{V}_{\beta g}$, $\mathbf{V}_{\beta tg}$, $\mathbf{V}_{\beta hg}$, $\mathbf{V}_{\beta rhg}$) are respectively the diffusive, the thermo-diffusive, the translational and rotational hydro-diffusive velocities.

Because by definition fluids can only support compressive normal forces, no shear forces, the total stress tensor for fluids is expressed as [40]

$$\mathbf{P}_{ij} = p\delta_{ij} - \lambda \nabla.\mathbf{v}\delta_{ij} - 2\mu e_{lk}\delta_{lk}\delta_{ij}$$
$$= p\delta_{ij} - \lambda \nabla.\mathbf{v}\delta_{ij} - \mu \nabla.\mathbf{v}\delta_{ij} \qquad (14)$$

Making the conventional *Stokes* assumption, i.e. setting the bulk viscosity *b* to zero, the two *Lame* constants will be related by [43]

$$b = \lambda + \frac{2}{3}\mu = 0 \qquad (15)$$

and (14) reduces to [40]

$$\mathbf{P}_{ij\beta} = p_\beta \delta_{ij\beta} - \frac{1}{3}\mu_\beta \nabla.\mathbf{v}_\beta \delta_{ij\beta} = (p_{t\beta} + p_{h\beta})\delta_{ij\beta} \qquad (16)$$

that involves thermodynamic p_t and hydrodynamic p_h pressures [40]. Following the classical methods [35-37], by substituting from (13)-(16) into (5)-(8) and neglecting cross-diffusion terms the invariant forms of conservation equations are written as [40]

$$\frac{\partial \rho_\beta}{\partial t} + \mathbf{w}_\beta.\nabla\rho_\beta - D_\beta\nabla^2\rho_\beta = \Omega_\beta \qquad (17)$$

$$\frac{\partial T_\beta}{\partial t} + \mathbf{w}_\beta.\nabla T_\beta - \alpha_\beta\nabla^2 T_\beta = -\tilde{h}_\beta\Omega_\beta/(\rho_\beta c_{p\beta}) \qquad (18)$$

$$\frac{\partial \mathbf{v}_\beta}{\partial t} + \mathbf{w}_\beta.\nabla\mathbf{v}_\beta - \nu_\beta\nabla^2\mathbf{v}_\beta = -\frac{\nabla p_\beta}{\rho_\beta} + \frac{1}{3}\nu_\beta\nabla(\nabla.\mathbf{v}_\beta) - \frac{\mathbf{v}_\beta\Omega_\beta}{\rho_\beta} \qquad (19)$$

$$\frac{\partial \boldsymbol{\omega}_\beta}{\partial t} + \mathbf{w}_\beta.\nabla\boldsymbol{\omega}_\beta - \nu_\beta\nabla^2\boldsymbol{\omega}_\beta = -\boldsymbol{\omega}_\beta.\nabla\mathbf{w}_\beta - \frac{\boldsymbol{\omega}_\beta\Omega_\beta}{\rho_\beta} \qquad (20)$$

The main new feature of the modified form of the equation of motion (19) is its linearity due to the difference between the convective **w** versus the local velocity **v**. The linearity of (19) in harmony with *Carrier* equation [44] resolves the classical paradox of drag reciprocity [45, 46].

4 A Modified Statistical Theory of Turbulence

The invariant model of statistical mechanics (1)-(4) suggests that all statistical fields shown in Fig.1 are turbulent fields and governed by (5)-(8) [33, 34]. First, let us start with the field of laminar molecular dynamics LMD when molecules, clusters of molecules (cluster), and cluster of clusters of molecules (eddy) form the “atom”, the “element”, and the “system” with the velocities $(\mathbf{u}_m, \mathbf{v}_m, \mathbf{w}_m)$. Similarly, the fields of laminar cluster-dynamics LCD and eddy-dynamics LED will have the velocities $(\mathbf{u}_c, \mathbf{v}_c, \mathbf{w}_c)$, and $(\mathbf{u}_e, \mathbf{v}_e, \mathbf{w}_e)$ in accordance with (1)-(2). For the fields of LED, LCD, and LMD, typical characteristic “atom”, element, and system lengths are

$$\text{EED} \quad (\ell_e, \lambda_e, L_e) = (10^{-5}, 10^{-3}, 10^{-1})\ \text{m} \qquad (21a)$$

$$\text{ECD} \quad (\ell_c, \lambda_c, L_c) = (10^{-7}, 10^{-5}, 10^{-3})\ \text{m} \qquad (21b)$$

$$\text{EMD} \quad (\ell_m, \lambda_m, L_m) = (10^{-9}, 10^{-7}, 10^{-5})\ \text{m} \qquad (21c)$$

If one applies the same (atom, element, system) = $(\ell_\beta, \lambda_\beta, L_\beta)$ relative sizes in (21) to the entire spatial scale of Fig.1, then the resulting cascades or hierarchy of overlapping statistical fields will appear as schematically shown in Fig.3.

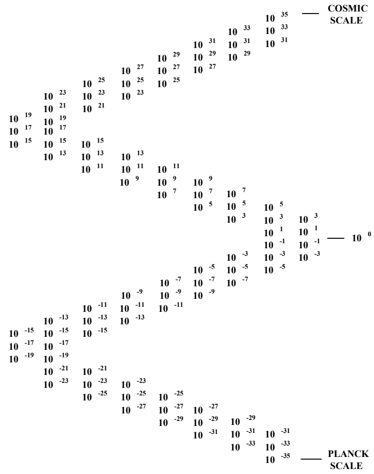


Fig.3 Hierarchy of statistical fields with $(\ell_\beta, \lambda_\beta, L_\beta)$ from cosmic to Planck scales [34].

According to Fig.3, starting from the hydrodynamic scale $(10^3, 10^1, 10^{-1}, 10^{-3})$ after seven generations of statistical fields one reaches the electro-dynamic scale with the element size 10^{-17}, and exactly after seven more generations one reaches the *Planck* length scale $(\hbar G/c^3)^{1/2} \simeq 10^{-35}\,\text{m}$, where G is the gravitational constant. Similarly, seven generations of statistical fields separate the hydrodynamic scale $(10^3, 10^1, 10^{-1}, 10^{-3})$ from the scale of planetary dynamics (astrophysics) 10^{17} and the latter from galactic-dynamics (cosmology) 10^{35} m.

The left hand side of Fig.1 corresponds to equilibrium statistical fields when the velocities of elements of the field are random since at thermodynamic equilibrium particles i.e. oscillators of such statistical fields will have normal or *Gaussian* velocity distribution. For example, for stationary homogeneous isotropic turbulence at EED scale, the experimental data of *Townsend* [47] confirms the *Gaussian* velocity distribution of eddies as shown in Fig.4.

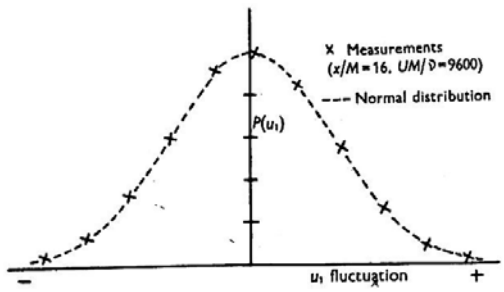


Fig.4 Measured velocity distribution in isotropic turbulent flow [47].

Because at thermodynamic equilibrium the mean velocity of each particle, *Heisenberg-Kramers* virtual oscillator [48], vanishes $<\mathbf{u}_\beta> = 0$ the energy of particle oscillating in two directions (x+, x-) is expressed as

$$\varepsilon_\beta = m_\beta \langle u^2_{\beta x+}\rangle / 2 + m_\beta \langle u^2_{\beta x-}\rangle / 2$$
$$= m_\beta \langle u^2_{\beta x+}\rangle = \langle p_\beta\rangle \langle \lambda^2_\beta\rangle^{1/2} \langle \nu^2_\beta\rangle^{1/2} \quad (22)$$

where $m_\beta <u^2_{\beta x+}>^{1/2} = <p_\beta>$ is the root-mean-square momentum of particle and $<u^2_{\beta x+}> = <u^2_{\beta x-}>$ by equipartition principle. At any scale β, the result (22) can be expressed in terms of either frequency or wavelength

$$\varepsilon_\beta = m_\beta \langle u^2_\beta\rangle = \langle p_\beta\rangle \langle \lambda^2_\beta\rangle^{1/2} \langle \nu^2_\beta\rangle^{1/2} = h_\beta \nu_\beta \quad (23)$$

$$\varepsilon_\beta = m_\beta \langle u^2_\beta\rangle = \langle p_\beta\rangle \langle \nu^2_\beta\rangle^{1/2} \langle \lambda^2_\beta\rangle^{1/2} = k_\beta \lambda_\beta \quad (24)$$

when the definition of stochastic *Planck* and *Boltzmann* factors are introduced as [33]

$$h_\beta = \langle p_\beta\rangle \langle \lambda^2_\beta\rangle^{1/2} \quad (25)$$

$$k_\beta = \langle p_\beta\rangle \langle \nu^2_\beta\rangle^{1/2} \quad (26)$$

At the important scale of EKD (Fig.1) corresponding to *Casimir* vacuum [49] composed of photon gas, the universal constants of *Planck* [50, 51] and *Boltzmann* [31] are obtained from (23)-(24) as

$$h = h_k = m_k c \langle \lambda^2_k\rangle^{1/2} = 6.626\times 10^{-34} \text{ J-s} \quad (27)$$

$$k = k_k = m_k c \langle \nu^2_k\rangle^{1/2} = 1.381\times 10^{-23} \text{ J/K} \quad (28)$$

Next, following *de Broglie* hypothesis for the wavelength of matter waves [2]

$$\lambda_\beta = h / p_\beta \quad (29)$$

the frequency of matter waves is defined as [31]

$$\nu_\beta = k / p_\beta \quad (30)$$

When matter and radiation are in the state of thermodynamic equilibrium (29) and (30) can be expressed as

$$h_\beta = h_k = h \quad , \quad k_\beta = k_k = k \quad (31)$$

The definitions (27) and (28) result in the gravitational mass of photon [31]

$$m_k = (hk / c^3)^{1/2} = 1.84278\times 10^{-41} \text{ kg} \quad (32)$$

that is much larger than the reported value of 4×10^{-51} kg [52]. The finite gravitational mass of photons was anticipated by *Newton* [53] and is in accordance with the *Einstein-de Broglie* theory of light [54-58]. *Avogardo-Loschmidt* number was predicted as [31]

$$N^o = 1/(m_k c^2) = 6.0376\times 10^{23} \quad (33)$$

leading to the modified value of the universal gas constant

$$R^o = N^o k = 8.338 \text{ kJ/(kmol-K)} \quad (34)$$

The classical definition of thermodynamic temperature that is based on two degrees of freedom

$$kT' = m < u^2_x > = 2m < u^2_{x+} > \quad (35)$$

was recently modified to a new definition based on single degree of freedom [59]

$$kT = m < u^2_{x+} > \quad (36)$$

such that

$$T' = 2T \quad (37)$$

The factor 2 in (37) results in the predicted speed of sound in air [60]

$$a = < u^2_{x+} >^{1/2} = \sqrt{p / (2\rho)} \simeq 357 \text{ m/s} \quad (38)$$

in close agreement with observations. Therefore, the square root of 2 in (38) resolves the classical problem of *Newton* concerning his prediction of velocity of sound as

$$a = \sqrt{p / \rho} \quad (39)$$

discussed by *Chandrasekhar* [61]

"Newton must have been baffled, not to say disappointed. Search as he might, he could find no flaw in his theoretical framework—neither could Euler, Lagrange, and Laplace; nor, indeed, anyone down to the present"

The factor of 2 in (37) also leads to the modified value of the mechanical equivalent of heat J [59]

$$J = 2J_c = 2\times 4.169 = 8338 \text{ J/(kcal)} \quad (40)$$

where the value $J_c = 4.169 \simeq 4.17$ [kJ/kcal] is the average of the two values J_c = (4.15, 4.19) reported by *Pauli* [62]. The number in (40) is thus identified as the universal gas constant (34) when expressed in appropriate MKS system of units

$$R^o = kN^o = 8338 \text{ J/(kmol-K)} \quad (41)$$

The modified value of the universal gas constant (41) was recently identified [63] as *De Pretto* number 8338 that appeared in the mass–energy equivalence equation of *De Pretto* [64]

$$E = mc^2 \quad \text{Joules} \quad = mc^2 / 8338 \quad \text{kcal} \qquad (42)$$

5 Energy Spectra of Isotropic Turbulence given by Planck Energy Distribution Law

The field of isotropic homogeneous turbulence is identified as equilibrium eddy dynamics EED, Fig.1, with turbulent eddies defined as clusters of molecular clusters constituting the elements of the field. In a recent investigation [33], it was shown that the energy spectrum of eddies in isotropic turbulence is governed by invariant *Planck* energy distribution law [33, 50]

$$\frac{\varepsilon_\beta dN_\beta}{V} = \frac{8\pi h}{u_\beta^3} \frac{\nu_\beta^3}{e^{h\nu_\beta / kT} - 1} d\nu_\beta \qquad (43)$$

schematically shown in Fig.5.

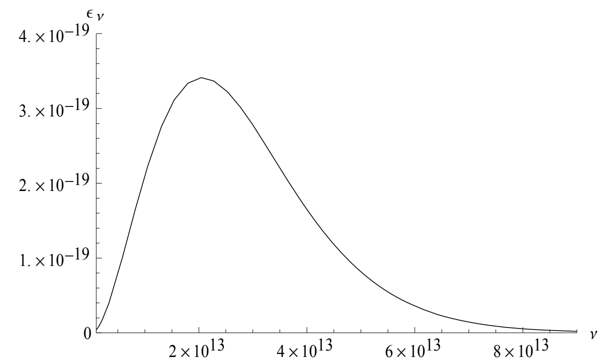


Fig.5 Planck energy distribution law governing the energy spectrum of eddies at the temperature T = 300 K.

From application of *Boltzmann* distribution of molecules in clusters and clusters in eddies it was found that [33]

$$N_\beta = \frac{g_\beta}{e^{h\nu_\beta / kT} - 1} \qquad (44)$$

that along with *Rayleigh-Jeans* number for degeneracy [51, 65, 66]

$$dg_\beta = \frac{8\pi V}{u_\beta^3} \nu_\beta^2 d\nu_\beta \qquad (45)$$

and $\varepsilon_\beta = h\nu_\beta$ give (43). It is interesting to examine a new interpretation of (45) that is directly relevant to number of particles rather than being derived from field quantization [67]. To this end the number of degeneracy of particles, *Heisenberg-Kramers* oscillators [48], in volume V_S is written as

$$g_\beta = 2V_S / \lambda_\beta^3 \qquad (46)$$

where λ_β^3 is the volume occupied by each oscillator and the factor 2 comes from allowing particles to have two modes either (up) or (down) iso-spin (polarization). Spherical volume V_S and rectangular volume V are related as

$$V_S = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} L^3 = \frac{4\pi}{3} V \qquad (47)$$

For systems in thermodynamic equilibrium temperature $3kT_\beta = m_\beta < u_\beta^2 >$ will be constant and hence $< u_\beta^2 >^{1/2} = < \lambda_\beta^2 >^{1/2} < \nu_\beta^2 >^{1/2}$ or

$$\lambda_\beta = u_\beta / \nu_\beta \qquad (48)$$

Substituting from (47) and (48) into (46) results in

$$g_\beta = \frac{8\pi V}{3u_\beta^3} \nu_\beta^3 \qquad (49)$$

that leads to the number of oscillators between frequencies ν_β and $\nu_\beta + d\nu_\beta$

$$dg_\beta = \frac{8\pi V}{u_\beta^3} \nu_\beta^2 d\nu_\beta \qquad (50)$$

in accordance with (45).

With *Gaussian* velocity distribution, Fig.4, the same chain of reasoning as employed in the classical kinetic theory of gas requires that the distribution of the speeds of oscillators (eddies) in stationary isotropic turbulence be given by the invariant *Maxwell-Boltzmann* distribution function

$$\frac{dN_{u\beta}}{N} = 4\pi \left(\frac{m_\beta}{2\pi k T_\beta}\right)^{3/2} u_\beta^2 \, e^{-m_\beta u_\beta^2 / 2kT_\beta} du_\beta \qquad (51)$$

By (51), one arrives at a hierarchy of embedded *Maxwell-Boltzmann* distribution functions for EED, ECD, and EMD scales shown in Fig.6.

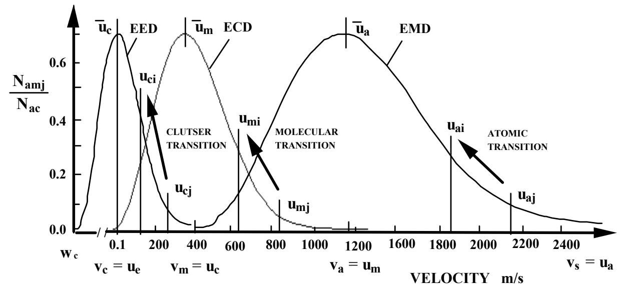


Fig.6 Maxwell-Boltzmann speed distribution viewed as stationary spectra of cluster sizes for EED, ECD, and EMD scales at 300 K [33].

To summarize, at equilibrium the statistical fields of any scale shown on the left-hand-side of Fig.1 will have elements with (a) *Gaussian* velocity distribution, (b) *Planck* energy distribution, and (c) *Maxwell-Boltzmann* speed distribution. For the conventional fields of ECD and EMD, it is well known that the conditions (a) and (c) are true and because of equilibrium between matter and radiation fields it is reasonable to expect that the condition (b) above will also apply to LCD and LMD. At the scale of LED, the conditions (a) shown by *Townsend's* data in Fig.4 and hence condition (c) are known to hold. Preliminary examination of the three-dimensional energy spectrum E(k) for isotropic turbulence measured by *van Atta* and *Chen* [68] in Fig.7

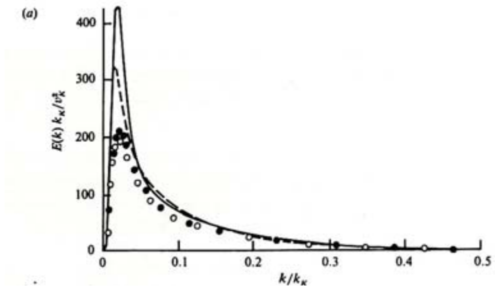


Fig.7 Normalized three-dimensional energy spectra for isotropic turbulence [68].

appear to support the validity of condition (b).

The schematic diagram given in Fig.8 from *Landahl* and *Mollo-Christensen* [69]

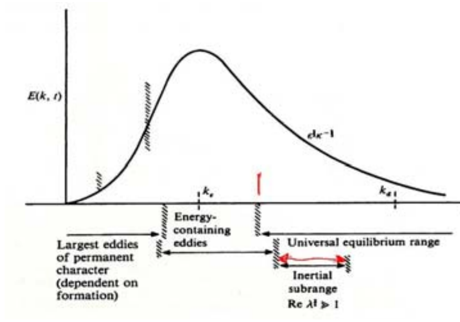


Fig.8 Behavior of three-dimensional spectrum E(k, t) in various wave number ranges [69].

also appears to support the validity of condition (b) namely that *Planck* law governs the energy spectrum of eddies in isotropic turbulence. Unfortunately, in spite of the large number of experimental studies, data that clearly and directly show the variation of three-dimensional energy spectrum E(k) with wave number k are still lacking.

According to Fig.8, the *Kolmogorov-Obukhov* $k^{-5/3}$ law [22, 23] is a local feature, valid only in the inertial subrange, of the more universal *Planck* law (43). For stationary isotropic turbulent fields, energy input into the system cascades down to smaller and smaller oscillators until it finally reaches the *Kolmogorov* dissipation length scale η_k at which point all of the added energy blends into background white noise and is removed from the system as heat. In view of Fig.6, η_k is naturally identified as the atomic length l_e of LED scale which is the same as the element λ_c of LCD scale

$$\eta_k = \ell_e = \lambda_c \tag{52}$$

that appears in *Boussinesq* eddy diffusivity [70]

$$\nu_e = \frac{1}{3}\ell_e u_e = \frac{1}{3}\lambda_c \mathrm{v}_c \tag{53}$$

On the other hand, the kinematic viscosity of LCD field is related to the "atomic" length l_c or the molecular mean free path λ_m that appears in *Maxwell's* formula for kinematic viscosity [30]

$$\nu_c = \frac{1}{3}\ell_c u_c = \frac{1}{3}\lambda_m \mathrm{v}_m \quad , \tag{54}$$

associated with viscous dissipation in fluid mechanics.

In stationary isotropic turbulent fields, energy flux occurs between randomly moving eddies of diverse size while leaving the system stochastically stationary in time. A schematic diagram of energy flux across hierarchies of eddies from large to small size is shown in Fig.9 from the study by *Lumley et al* [71]

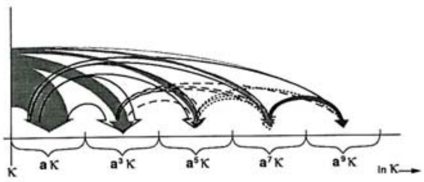


Fig. 9 A realistic view of spectral energy flux [71].

In the following section, it will be suggested that the exchange of particles, clusters, between various size eddies (energy levels) is governed by quantum mechanics through an invariant *Schrödinger* equation. Hence, the stochastically stationary state of various size eddies will be parallel to *Bohr's stationary states* in atomic theory [48].

For stationary isotropic turbulent fields the net energy flux across hierarchies of eddies should be a constant independent of viscosity such as is commonly assumed in the inertial subrange. Also, at equilibrium, the temperature will be constant $3k_B T = m < u^2 > = \varepsilon$. Therefore, if one expresses the cluster energy as $\varepsilon = mu^2$, and the eddy energy as $E = \sum \varepsilon = \alpha\varepsilon$, with the cluster velocity given as $u = \nu\lambda = 2\pi\nu/\kappa$ such that $du = d(2\pi\nu/\kappa) \propto \kappa^{-2}d\kappa$ where $\kappa = 2\pi/\lambda$ is the wave number, one arrives at

$$dE = Fd\kappa = 2\alpha mudu \propto \alpha\varepsilon^{2/3}(u\varepsilon^{-2/3})\kappa^{-2}d\kappa$$
$$= \alpha\varepsilon^{2/3}(\kappa^{-1}\kappa^{4/3})\kappa^{-2}d\kappa = \alpha\varepsilon^{2/3}\kappa^{-5/3}d\kappa \quad (55)$$

leading to the distribution function

$$F(\kappa) = \alpha\varepsilon^{2/3}\kappa^{-5/3} \quad (56)$$

that is the *Kolmogorov-Obukhov* $\kappa^{-5/3}$ law [22, 23].

The most central concept associated with turbulent dissipation is the spectral definition of turbulent viscosity introduced by *Heisenberg* [26]

$$\nu_k = \kappa\int_{\kappa}^{\infty}\left(\sqrt{\frac{F(\kappa'')}{\kappa''^3}}\right)d\kappa'' \quad (57)$$

This is because the spectral definition of kinematic viscosity (57) suggests that in stationary isotropic turbulence the dissipation spectrum should be closely related to the energy spectrum and thereby to *Planck* energy spectrum. Preliminary examination of dissipation spectrum shown in Fig. 10 from *McComb and Shanmugasundaram* [72] appears to support such a conjecture.

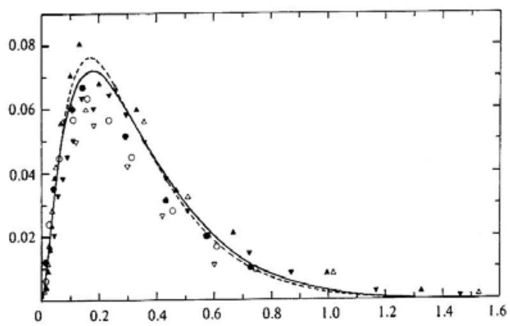


Fig.10 Comparison of scaled one-dimensional dissipation spectrum with experimental data [72].

The experimental data of Fig.10 along with *Planck* distribution function as well as this same distribution function shifted by a constant amount of energy are shown in Fig.11. One may view the experimental data in Fig.10 as dissipation spectrum of energy associated with the *Planck* energy spectrum plus a constant amount of energy added in order to maintain the turbulent field stationary.

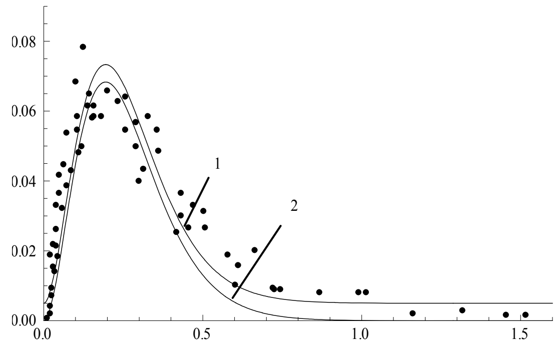


Fig.11 One-dimensional dissipation spectrum [72] compared with (1) Planck energy distribution (2) Planck energy distribution with constant displacement.

Similar comparison with *Planck* distribution as shown in Fig.11 is obtained with the more recent experimental data of one-dimensional dissipation spectrum of isotropic turbulence from study by *Saddoughi and Veeravalli* [73] given in Fig.12.

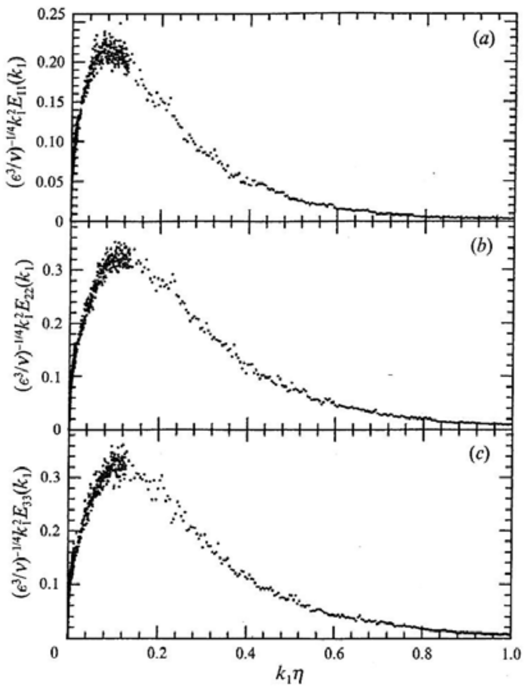


Fig.12 One-dimensional dissipation spectra measured at mid-layer for the low speed case (y = 515 mm, R_λ= 600) (a) u_1–spectrum (b) u_2-spectrum (c) u_3-spectrum [73].

Systematic comparisons of the three-dimensional energy spectrum in Fig.9 and the one-dimensional dissipation spectra in Figs.10-12 with *Planck* distribution (Fig.5) require further future investigations. In particular, the definitions used for the presentation of the experimental data in Figs.9-12 must be examined.

The normalized three-dimensional energy spectrum for homogeneous isotropic turbulent field was obtained from the transformation of one-dimensional spectrum of *Lin* [74] by *Ling and Huang* [75] as

$$E^* = \frac{\alpha^2}{3}(K^* + \alpha K^{*2}) \exp(-\alpha K^*) \qquad (58)$$

with the distribution shown in Fig.13.

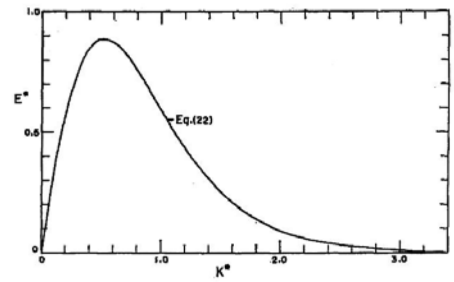


Fig.13 Normalized three-dimensional energy spectrum of weak turbulence [75].

The similarity between Fig.13 and the *Planck* distribution function shown in Fig.5 is apparent and with $h/kT = c_1$ one can express (43) as

$$\varepsilon_\nu dN = c_2\, \nu^3 \exp(-c_1 \nu) / [1 - \exp(-c_1 \nu)] \qquad (59)$$

to facilitate comparison with (58).

Because the velocity distribution of eddies in isotropic turbulence is known to be *Gaussian*, Fig.4, the distribution of the speed of eddies in isotropic turbulence must follow *Maxwell-Boltzmann* distribution function in accordance with the kinetic theory of gas. On the other hand, in a recent investigation [33] the invariant *Maxwell-Boltzmann* distribution function was directly derived from the invariant *Planck* energy distribution function. Therefore, it is expected that the energy spectrum of eddies in isotropic turbulence should follow the *Planck* law [33, 34].

In the scale invariant statistical theory of turbulence described above the statistical nature of the problem does not arise from the notion of fluid instabilities as in the classical theories of turbulence, but rather arises from similar considerations as those in the kinetic theory of gas in harmony with perceptions of *Heisenberg* [26]

"Turbulence is an essentially statistical problem of the same type as one meets in statistical mechanics, since it is the problem of distribution of energy among a very large number of degrees of freedom. Just as in Maxwell theory this problem can be solved without going into details of the mechanical motions, so it can be solved here by simple considerations of similarity."

It was also emphasized by *Heisenberg* that the study of flow instabilities cannot in principle lead to the understanding of turbulent phenomena itself as noted by *Chandrasekhar* [76]

"However, as Heisenberg has recently emphasized, investigation of stability along these lines, even if successful, cannot, in principle, lead to an understanding of the phenomena of turbulence itself; for the basic problem of turbulence is of an entirely different character. That this is the case becomes apparent when we ask ourselves the very elementary question, "What is the reason that a phenomena like turbulence can occur at all?". The answer must be that an ideal fluid is a mechanicals system with very large number of degrees of freedom and that, in consequence, it is theoretically capable of a very large number of different types of motions. Laminar flow is only one of the many possible motions that the system is capable of, and to expect that it will always be realized is as futile as to expect that in a gas we shall find all the molecules moving with the same velocity parallel to one another. It is far more likely that all forms of possible motions will be simultaneously present. The fundamental problem of turbulence would therefore appear to be a statistical one of specifying the probability with which the various types of motions may occur and are present. Stated in this way, it is clear that the problem of turbulence has an analogy with the problem of analyzing a continuous spectrum of radiation."

The significance of analogy between the energy spectra in turbulence and in optics, dry hydrodynamics [17], was further emphasized by *Chandrasekhar* [76]

"Now, returning to the optical analogy I referred to earlier, we know that under condition of equilibrium the distribution of energy in the continuous spectrum will be that given by Planck's law. We may ask whether a similar equilibrium spectrum exists for turbulence. In answering this question, we must keep in mind one important distinction between the optical analogue and turbulence. In optical case the equilibrium Planck spectrum will be reached, no matter what the initial distribution is. In contrast, turbulence can be maintained only by external energy, like continuous stirring, the energy available from thermal instability, or rotation in differentially rotating atmosphere. In other words, energy is required for the maintenance of turbulence; in the absence of such an agency, turbulence will decay and the spectrum will be a function of time."

The optical analogy discussed by *Chandrasekhar* in the above quotation becomes complete if one allows a constant energy input at small wave numbers (at system scale L_e of EED) that moves through the hierarchy of eddies (Fig.9) until it is dissipated into heat that is removed at *Kolmogorov* scale $\eta_k = l_e$.

According to the modified statistical theory of turbulence [33, 34], and in harmony with the perceptions of *Heisenberg* and *Chandrasekhar* discussed above, all flows that may appear laminar at scale β are actually bulk advection of turbulent flows at the scale of β−1. The hierarchies of embedded turbulent flows are most clearly seen in turbulent boundary layer over a flat plate when the solutions of (19) at scales β and β+1 were respectively found to be [34]

$$v^+_{\beta+1} = 5 + 8(2/\sqrt{\pi})^2 \ \mathrm{erf}(\ y^+_\beta / 32) \qquad (60)$$

and

$$v^+_\beta = 8\ (2/\sqrt{\pi})\ \mathrm{erf}(y^+_{\beta-1} / 8) \qquad (61)$$

For example, the solutions in (60) and (61) at β+1 = e and β = c correspond to LED and LCD and their comparisons with experimental data [36, 69, 77-79] are shown in Fig.14.

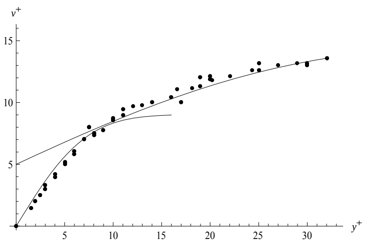


Fig.14 Comparison between the predicted velocity profiles from (60) and (61) at LED and LCD scales and experimental data in the literature [36, 69, 77-79].

Similarly, the solutions in (60) and (61) at β+1 = c and β = m correspond to LCD and LMD and at β+1 = m and β = a corresponding to LMD and LAD and their comparisons with experimental data of *Lancien et al.* [80] and *Meinhart et al.* [81] are shown in Figs.15 and 16, respectively.

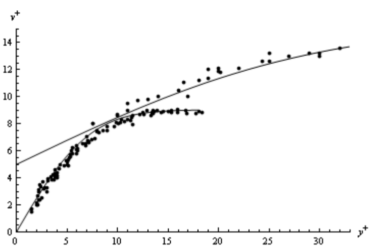


Fig.15 Comparisons between the predicted velocity profiles from (60) and (61) at LCD and LMD scales and experimental data in the literature [80].

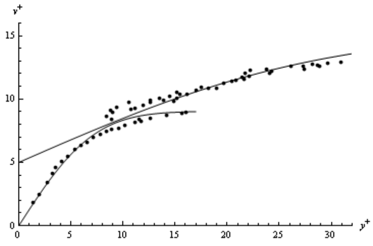


Fig.16 Comparisons between the predicted velocity profiles from (60) and (61) at LMD and LAD scales and experimental data in the literature [81]

The results in Figs.14-16 show close agreement between predictions and measurements for four consecutive statistical fields of LED, LCD, LMD, and LAD spanning a factor of 10^8 in spatial range.

6 Quantum Mechanical Foundations of Turbulence

The fact that the energy spectrum of equilibrium isotropic turbulence is given by *Planck* distribution (Figs.5, 9, 13) is a strong evidence for quantum mechanical foundation of turbulence [33, 34]. This is further supported by recent derivation of invariant *Schrödinger* equation from invariant *Bernoulli* equation [33]. For an incompressible potential flow the velocity potential Φ_β gives $\mathbf{v}_\beta = \nabla\Phi_\beta$ and the conservation equations (17) and (19) lead to the invariant *Bernoulli* equation [33]

$$\frac{\partial(\rho_\beta\Phi_\beta)}{\partial t} + \frac{(\nabla\rho_\beta\Phi_\beta)^2}{2\rho_\beta} + p_\beta = \mathrm{constant} = 0 \qquad (62)$$

Comparison of (62) with the *Hamilton-Jacobi* equation of classical mechanics [2]

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + U = 0 \qquad (63)$$

resulted in the introduction of the invariant action and quantum mechanic wave function as [33, 82]

$$S_\beta(\mathbf{x},t) = \rho_\beta\Phi_\beta \quad , \quad \Psi_\beta(\mathbf{x},t) = S'_\beta(\mathbf{x},t) = \rho_\beta\Phi'_\beta \qquad (64)$$

The wave function (64) was recently applied to derive [33] the invariant time dependent *Schrödinger* equation [83]

$$i\hbar_{o\beta}\frac{\partial\Psi_\beta}{\partial t} + \frac{\hbar^2_\beta}{2m_\beta}\nabla^2\Psi_\beta - \bar{U}_\beta\Psi_\beta = 0 \qquad (65)$$

It is now clear that the potential energy [33]

$$\tilde{U}_\beta = p_\beta = \rho_\beta V_\beta'^2 = n_\beta \bar{U}_\beta \qquad (66)$$

in (65) may be identified as *Poincaré* stress [84-86] that is responsible for the stability of "particles" or *de Broglie* wave packets.

It is emphasized that stability of particle, *Heisenberg-Kramers* virtual oscillator [48], is due to the external force it experiences because of the pressure (66) induced by the peculiar velocity of the "atoms" of the field itself. In other words, it is the peculiar motions of atoms themselves that stabilizes different size atomic clusters. For example, peculiar motions of clusters are responsible for stability of eddies, and peculiar motions of molecules are responsible for stability of clusters, and so on. Hence, at the scale of stochastic electrodynamics β = s ESD and chromo-dynamics β = k, EKD (see Figs. 1, 3) the model suggests that peculiar motions of photons and tachyons are respectively responsible for the stability of electrons and photons. In view of the invariant *Planck* (43) and *Schrödinger* (65) equations, the model also suggests that *Planck* spectrum in equilibrium radiation represents stationary sizes of photon clusters, *de Broglie* molecules of light [86] or light bundles. Therefore, different "colors" of light are now recognized to correspond to different size photon clusters, photon wave packets, in accordance with the perceptions of *Newton* [53] and *Einstein* [66].

The formal derivation [33] of *Schrödinger* equation (65) from *Bernoulli* equation (62) provides for a new paradigm of the physical foundation of quantum mechanics. Soon after the introduction of his equation, *Schrödinger* himself [88] tried to identify some type of ensemble average interpretation of the wave function. Clearly, according to (64) the statistical ensemble nature of Ψ_β naturally arises from the velocity potential Φ_β. Therefore, the dual and seemingly incompatible objective versus subjective natures of Ψ_β emphasized by *de Broglie* [1-3] can now be resolved. This is because the objective part of Ψ_β (64) is associated with the density ρ_β and accounts for the particle localization while the subjective part of Ψ_β is associated with the complex velocity potential Φ_β that accounts for the observed action-at-a-distance as well as renormalization [89] and thereby the success of *Born's* [90] probabilistic interpretation of Ψ_β.

The definition of wave function (64) also helps to clarify the nature of wave-particle duality and *de Broglie's* theory of double solutions [1-3, 91, 92]. This is because the velocity potential Φ_β acts as guidance wave that contains global information about the environment while remaining non-observable since it operates at the "hidden" "atomic" scale. Thus, the reason for success of separation of particle from wave according to *de Broglie* [1-3] and *Bohm and Vigier* [7, 9] becomes understood. Moreover, the reason for the failure of *von Neumann* no hidden variable theory [92-94] becomes clear since (65) for hierarchies of embedded statistical fields, Fig.1, could be subject to infinite hidden variables in harmony with *Gödel's* incompleteness theorem.

As one moves to smaller scales, one always finds a *continuum* because each element is composed of an entire statistical field (see Fig.1), such that one can again define a velocity potential $\Phi_{\beta-1}$ and thereby define a new wave function $\Psi_{\beta-1} = \rho_{\beta-1}\Phi'_{\beta-1}$. The cascade of particles as singularities embedded in *guidance waves* is in exact agreement with the perceptions of *de Broglie* concerning interactions between the particle and the "*hidden thermostat*" [3]

"Here is another important point. I have shown in my previous publications that, in order to justify the well-established fact that the expression $|\psi(x,y,z,t)|^2 d\tau$ gives, at least with Schrödinger's equation, the probability for the presence of the particle in the element of volume dτ at the instant t, it is necessary that the particle jump continually from one guidance trajectory to another, as a result of continual perturbation coming from subquantal milieu. The guidance trajectories would really be followed only if the particle were not undergoing continual perturbations due to its random heat exchanges with the hidden thermostat. In other words, a Brownian motion is superposed on the guidance movement. A simple comparison will make this clearer. A granule placed on the surface of a liquid is caught by the general movement of the latter. If the granule is heavy enough not to feel the action of individual shocks received from the invisible molecules of the fluid, it will follow one of the hydrodynamic streamlines. If the granule is a particle, the assembly of the molecules of the fluid is comparable with the hidden thermostat of our theory, and the streamline described by the particle is its guiding trajectory. But if the granule is sufficiently light, its movement will be continually perturbed by the individual random impacts of the molecules of the fluid. Thus, the granule will have, besides its regular movement along one of the streamlines of the global flow of the fluid, a Brownian movement which will make it pass from one streamline to another. One can represent Brownian movement approximately by diffusion equation of the form $\partial\rho / \partial t = D\Delta\rho$, and it is interesting to seek, as various authors have done recently, the value of the

coefficient *D* in the case of the Schrödinger equation corresponding to the Brownian movement.

I have recently studied (14) the same question starting from the idea that, even during the period of random perturbations, the internal phase of the particle remains equal to that of the wave. I have found the value $D = \hbar / (3m)$, which differs only by a numerical coefficient from the one found by other authors.

This concludes the account of my present ideas on the reinterpretation of wave mechanics with the help of images which guided me in my early work. My collaborators and I are working actively to develop these ideas in various directions. Today, I am convinced that the conceptions developed in the present article, when suitably developed and corrected at certain points, may in the future provide a real physical interpretation of present quantum mechanics."

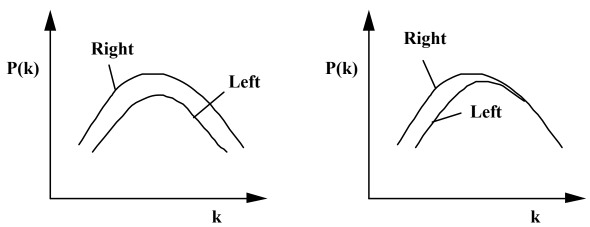
To reveal the truly universal (Fig.1) nature of quantum mechanics one notes that for each statistical field shown in Figs.1 and 3 one can write a *Bernoulli* equation and by (62)-(65) arrive at a *Schrödinger* equation. At the *Planck* scale $(\hbar G / c^3)^{1/2} \simeq 10^{-35}\,\mathrm{m}$ (see Fig.3) the hydrodynamic field represents the physical space [95] or *Casimir* vacuum [49] with its fluctuations and energy spectrum. According to definitions (27)-(28), the universal *Planck* and *Boltzmann* constants (h, k) respectively relate to spatial (λ) and temporal (ν) aspects of vacuum fluctuations. Also, at cosmic scales 10^{35} with $\beta = g$ (Figs.1, 3), the wave function Ψ_g will correspond to the wave function of the universe [96, 97]. The wave-particle duality of galaxies has been established by their observed quantized red shifts [98]. Indeed, it has been suggested by *Laughlin and Pines* [98] that *Schrödinger* equation may account for a part of the final theory, i.e. the theory of everything (TOE). Also, *Feynman et. al.* [100] suggested that *Schrödinger* equation may very possibly describe life itself.

In regards to the universality of quantum mechanics discussed above, it is interesting to examine the connections between classical and quantum mechanics. It seems that the new paradigm of quantum mechanics presented herein is in harmony with the perceptions of *Heisenberg* [101]

"We no longer say "Newtonian mechanics is false and must be replaced by quantum mechanics, which is correct." Instead we adopt the formula "Classical mechanics is a consistent self–enclosed scientific theory. It is strictly correct description of nature wherever its concepts can be applied"

This is because according to (62)-(66) quantum mechanics concerns behavior of virtual oscillators, wave packets in statistical fields such as in cosmology, hydrodynamics, molecular dynamics, electrodynamics or optics. Classical Newtonian mechanics is often concerned with motion of a few bodies such as the earth-moon-sun three-body problem.

According to Fig.3, a factor of 10^{-17} seems to separate the spatial scales of the stochastic fields of chromodynamics (10^{-35}), electrodynamics (10^{-17}), hydrodynamics (10^{0}), astrophysics (10^{17}), and cosmology (10^{35}). However, there are no mathematical or physical reasons to limit either large or small ends of the hierarchies shown in Fig.3. If one assumes with *Newton* that space is infinite and our universe is just one of many universes as suggested by Fig.3, then the recent observed asymmetry in the power spectrum from the right versus the left side of our universe [102] schematically shown in Fig.17.



(a) **(b)**

Fig.17 Asymmetry in measured cosmic power spectrum (a) calculated (b) measured [102].

appears to suggest that our universe is rotating in harmony with *Kerr's* perceptions [103] as well as all statistical fields of lower scales shown in Fig.1.

One may now introduce a new paradigm of the physical foundation of quantum mechanics according to which *Bohr's* stationary states will correspond to the statistically stationary sizes of clusters, *de Broglie* wave packets, which will be governed by *Maxwell-Boltzmann* distribution function (51) as shown in Fig.6 [33]. Next, different energy levels of quantum mechanics are identified as different size clusters (elements). For example, in EED field one views the transfer of a cluster from a small rapidly oscillating eddy (j) to a large slowly oscillating eddy (i) as transition from the high energy level (j) to the low energy level (i), see Fig.6, as schematically shown in Fig.18.

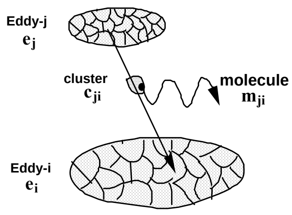


Fig.18 Transition of cluster c_{ij} from eddy-j to eddy-i leading to emission of molecule m_{ij} [34].

Such a transition will be accompanied with emission of a "sub-particle" that will be a "molecule" (see Fig.18) to carry away the excess energy [34]

$$\Delta\varepsilon_{ji\beta} = \varepsilon_{j\beta} - \varepsilon_{i\beta} = h(\nu_{j\beta} - \nu_{i\beta}) \qquad (67)$$

in harmony with *Bohr's* theory of *atomic* spectra [48]. Therefore, the reason for the *quantum nature* of "molecular" energy spectra in equilibrium isotropic turbulence is that transitions can only occur between eddies with energy levels that satisfy the criterion of *stationarity* imposed by *Maxwell-Boltzmann* speed distribution function [95].

7 Compressibility of Physical Space and its impact on Special Theory of Relativity

The invariant time dependent *Schrödinger* equation (65) was derived from *Brenoulli* equation for an incompressible fluid. According to the scale-invariant statistical theory of fields schematically shown Fig.1 *physical space* is identified as a tachyonic fluid [95] that is *Dirac's* stochastic ether [104] or *de Broglie's* "hidden thermostat" [3]. Photons are considered to be composed of a large number of much smaller particles [95] called *tachyons* [105]. The importance of *Aristotle's* ether to the theory of electrons was emphasized by *Lorentz* [106, 107]

"I cannot but regard the ether, which is the seat of an electromagnetic field with its energy and its vibrations, as endowed with certain degree of substantiality, however different it may be from all ordinary matter"

Since the velocity of light is the mean thermal speed of tachyons, $u_k = c = v_t$, at least some of the tachyons must be *superluminal*.

The tachyonic fluid that constitutes the physical space is considered to be *compressible* in accordance with *Planck's* compressible ether [105]. If the compressible tachyonic fluid is viewed as an ideal gas, its change of density when brought isentropically to rest will be given by the expression involving *Michelson* number $Mi = v / c$ [108]

$$\rho = \rho_o \left[1 + \frac{\gamma - 1}{2}\frac{v^2}{c^2}\right]^{\frac{1}{\gamma-1}} = \rho_o \left[1 + \frac{\gamma - 1}{2} Mi^2\right]^{\frac{1}{\gamma-1}} \qquad (68)$$

With $\gamma = 4/3$ for photon gas, (68) leads to *Lorentz-FitzGerlad* contraction [108]

$$\lambda = \lambda_o \sqrt{1 - (v / c)^2} \qquad (69)$$

that accounts for the null result of *Michelson–Morley* experiment [109].

Therefore, supersonic $Ma > 1$ (superchromatic $Mi > 1$) flow of air (tachyonic fluid) leads to the formation of *Mach* (*Poincaré-Minkowski*) cone that separates the zone of sound (light) from the zone of silence (darkness). Compressibility of physical space can therefore result in *Lorentz-FitzGerald* contraction [106], thus accounting for relativistic effects [84-86, 106, 109-112] and providing a *causal explanation* [110] of such effects in accordance with the perceptions of *Poincaré* and *Lorentz* [84-86, 106, 112].

In view of the above considerations and in harmony with ideas of *Darrigol* [113] and *Galison* [114], one can identify two distinct paradigms of the Special Theory of Relativity [108]:

(A) Poincaré-Lorentz
Dynamic Theory of Relativity
Space and time (x, t) are altered due to causal effects of motion on the ether.

(B) Einstein
Kinematic Theory of Relativity
Space and time (x, t) are altered due to the two postulates of relativity:

1- The laws of physics do not change form for all inertial frames of reference.
2- Velocity of light is a universal constant independent of the motion of its source.

The result (69) and the constancy of the speed of light

$$\lambda\nu = \lambda_o \nu_o = c \qquad (70)$$

lead to the frequency transformation

$$\nu = \nu_o / \sqrt{1 - (v / c)^2} \qquad (71)$$

The relativistic transformation of frequency in (71) may also be expressed as contraction of time duration or transformation of period $\tau = 1/\nu$ as

$$\tau = \tau_o \sqrt{1 - (v / c)^2} \qquad (72)$$

Hence, time durations and space extensions contract by (72) and (69) such that the speed of light remains invariant (70). It is emphasized however that the relation between time and space according to the dynamic theory of relativity of *Poincaré-Lorentz* is causal as emphasized by *Pauli* [110] and is induced by compressibility of physical space itself rather than being a purely kinematic effect as suggested by *Einstein* [111] according to paradigm **(B)** above.

Parallel to ideas of *Lorentz* [106], the concept of ether always played a crucial role in *Poincaré*'s perceptions of relativity [113, 114] as he explicitly stated in his Principle of Relativity [115]

"We might imagine for example, that it is the ether which is modified when it is in relative motion in reference to the material medium which it penetrates, that when it is thus modified, it no longer transmits perturbations with the same velocity in every direction."

As opposed to *Einstein* who at the time found the ether to be superfluous [111], *Poincaré* anticipated the granular structure of the ether and its possible role in electrodynamics [115]

"We know nothing of the ether, how its molecules are disposed, whether they attract or repel each other; but we know this medium transmits at the same time the optical perturbations and the electrical perturbations;"

"The electrons, therefore, act upon one another, but this action is not direct, it is accomplished through the ether as intermediary."

Also, the true physical significance of *Lorentz's* local time [106] was first recognized by *Poincaré* [116]. In his lecture delivered in London in 1912 shortly before he died *Poincaré* stated [114, 117]

"Today some physicists want to adopt a new convention. It is not that they are constrained to do so; they consider this new convention more convenient; that is all. And those who are not of this opinion can legitimately retain the old one in order not to disturb their old habits. I believe, just between us, that this is what they shall do for a long time to come."

The definitions of lengths $(L_\beta, \lambda_\beta, \ell_\beta)$ in (21) and velocities $(\mathbf{w}_\beta, \mathbf{v}_\beta, \mathbf{u}_\beta)$ in (1)-(2) result in the following definitions of system, element, and atomic "times" $(\Theta_\beta, \tau_\beta, t_\beta)$ for the statistical field at scale β [30]

$$\Theta_\beta = L_\beta / w_\beta = \tau_{\beta+1} \qquad (73a)$$

$$\tau_\beta = \lambda_\beta / v_\beta = t_{\beta+1} \qquad (73b)$$

$$t_\beta = l_\beta / u_\beta = \tau_{\beta-1} \qquad (73c)$$

where l_β, and λ_β are the free paths of atoms, and elements, and $L_\beta = \lambda_{\beta+1}$ is the system size. Atomic time (73c) could also be based on the rotation velocity of particles since the equipartition principle of *Boltzmann*

$$m_\beta u_{t\beta}^2 / 2 = I_\beta \omega_\beta^2 / 2 = m_\beta r_\beta^2 \omega_\beta^2 / 2$$

results in $u_{t\beta} = 2\pi r_\beta \nu_\beta = u_{r\beta}$ that leads to

$$t_\beta = 1/\nu_\beta = 2\pi r_\beta / u_{r\beta} = \ell_\beta / u_{r\beta} \qquad (73d)$$

Therefore, there exists an *internal clock* associated with the random thermal motions of atoms $t_\beta = \tau_{\beta-1}$ for each statistical field from cosmic to tachyonic scales [95]

$$\ldots \ \tau_e > \tau_c > \tau_m > \tau_p > \tau_s > \ldots \qquad (74)$$

The physical model schematically shown in Fig.1 suggests a hierarchy of embedded clocks each associated with its own *periodic motions* as schematically shown in Fig.19.

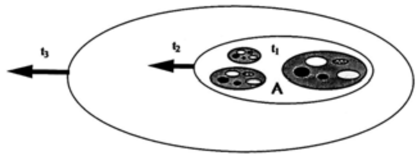


Fig.19 Hierarchies of embedded clocks.

Clearly, the problem of time reversal at any scale β is now much more complex and requires reversal of the entire hierarchies of times of lower scales (74).

The most fundamental and universal physical time is the time associated with the tachyon fluctuations $\tau_t = t_k$ [95] of *Casimir* vacuum [49] at *Planck* scale. One may associate the absolute mathematical time of *Newton* to the equilibrium state of tachyon-dynamics (t_t) that in the absence of any non-homogeneity (light) will be a timeless (eternal) world of darkness irrespective of its stochastic dynamics because, in accordance with the perceptions of *Aristotle* [118], the concept of time without any change is meaningless.

The classical problem of time emphasized by *Aristotle* [118] concerns the nature of time as past, present, and future that was most eloquently described by *St. Augustine* [118]

"But the two times, past and future how can they *be*?, since the past is no more and the future is not yet?"

"Thus we can affirm that time *is* only in that it tends towards not-being"

"Yet Lord, we are aware of periods of time; we compare one period with another and say that some are longer, some shorter"

"Does not my soul speak truly to You when I say that I can measure time? For so it is, O Lord my God. I measure it and I do not know what it is that I am measuring"

Now, the classical problem of how could a finite time be constructed from multitudes of instants, "nows", that do not exist can be addressed by the invariant definition of atomic time in (73c)

$$\tau_\beta = \Sigma t_\beta = \Sigma \tau_{\beta-1} \quad (75)$$

This is because the "atomic" instant $t_\beta = 0$ of scale β has a finite duration $\tau_{\beta-1}$ at the lower scale $\beta-1$ (73c). Thus, one uses clocks of $\beta-1$ scale to measure time of β scale, clocks of $\beta-2$ scale to measure time of $\beta-1$ scale, and so on at infinitum. Therefore, similar considerations employed for description of analysis in space continuum [119] will be required to describe the temporal continuum discussed by *Weyl* [120]

"Exact time- or space-points are not the ultimate, underlying, atomic elements of the duration or extension given to us in experience. On the contrary, only reason, which thoroughly penetrates what is experientially given, is able to grasp exact ideas"

Accordingly, if one does not wish to allow for infinite divisibility of time and space, then following *Leibniz* [119] one must introduce the *temporal monad* just like the *spatial monad* to represent the absolute smallest "atom" of time or *chronon*. Also, parallel to *Heisenberg's spatial uncertainty principle* [121]

$$\Delta\lambda_\beta \; \Delta p_\beta \geq h \quad (76)$$

that limits the resolution of spatial measurements, the *temporal uncertainty principle* [95]

$$\Delta\nu_\beta \; \Delta p_\beta \geq k \quad (77)$$

limits the resolution of time measurements.

Since time is identified as a physical attribute of the dynamics of tachyonic atoms (73c) and space is identified as this compressible tachyonic fluid itself, it is clear that the causal connections between space and time in relativistic physics become apparent. For example, in the classical problem of *twin paradox* of the special theory of relativity, the different times experienced by the twins could be attributed to the different rates of biological reactions in their body induced by the compressibility of physical space. According to the causal dynamic theory of relativity of *Poincaré-Lorentz*, the reason for the coincidence of directions of biological and cosmological times [122] becomes apparent.

The physical space or *Casimir* vacuum [49] when identified as a compressible tachyonic fluid provides a new paradigm for the physical foundation of quantum gravity [95]. The general implications of the model to time reversibility in quantum cosmology and to *Everett's* many-universe theory [96, 123-124] require further future investigations. Because of the definition of atomic time in (73c), quantum theories of gravity [123-130] may have wave functions Ψ_g that instead of *Wheeler-DeWitt* equation [96, 123, 124, 126, 130]

$$\mathrm{H}\Psi_g = 0 \quad (78)$$

satisfy the *modified Wheeler-DeWitt* equation

$$i\hbar \frac{\partial \Psi_g}{\partial t_{\beta-1}} = \mathrm{H}\Psi_g \quad (79)$$

that is *Schrödinger* equation (65). The resurrection of time in (79) is made possible because the new "*atomic*" *time* arises from internal degrees of freedom, permitting Ψ_g (x_1, x_2, x_3, t_β, $t_{\beta-1}$) and the associated $g_{ij}(x_1, x_2, x_3, t_\beta, t_{\beta-1})$, that by thermodynamic considerations is related to the temperature of the field [95].

The definition of time in (73) is in accordance with *ephemeris* time in astronomy [128, 131]

$$\delta t = \sqrt{\frac{\sum_i m_i (\delta d_i)^2}{2(E - V)}} \quad (80)$$

Substituting in (80) the kinetic energy $K = E - V = m\mathrm{v}^2/2$ and mass fraction $Y_i = m_i/m$ results in

$$\delta t = \sqrt{\frac{\sum_i m_i (\delta d_i)^2}{2(E - V)}} = \sqrt{\frac{m\sum_i Y_i (\delta d_i)^2}{m\mathrm{v}^2}} = \sqrt{\frac{m(\delta d)^2}{m\mathrm{v}^2}} = \frac{\delta d}{\mathrm{v}} \quad (81)$$

that is in accordance with (73). The mean extension δd and duration δt are mass-average of the component extension δd_i and duration δt_i and the corresponding velocity defined as

$$\delta t_i = \sqrt{\frac{m_i (\delta d_i)^2}{2(E_i - V_i)}} \quad , \quad \mathrm{v}_i = \frac{\delta d_i}{\delta t_i} \quad (82)$$

where $E_i = K_i + V_i$, $K_i = m_i \mathrm{v}_i^2/2$, and $V_i = -Gm_i m_j / r_{ij}$. The invariant definition of time in (73) suggests that the description of temporal continuum [120] requires introduction of *"temporal measure"* that relates to the important problems of duration and simultaneity identified by *Poincaré* in 1898 [132] as emphasized by *Barbour* [128].

8 Implications to Dirac Relativistic Wave Equation

It is interesting to explore the implications of the quantum mechanical wave function (64) to *Dirac's* relativistic wave equation [133]. To do this one notes that since the peculiar velocity of particle at scale β

$$\mathbf{V}'_\beta = \mathbf{u}_\beta - \mathbf{v}_\beta \quad , \tag{83}$$

is the diffusion velocity of scale (β−1) one can express (83) in terms of the velocities of the lower scale β−1 as in (3)

$$\mathbf{V}_{\beta-1} = \mathbf{v}_{\beta-1} - \mathbf{w}_{\beta-1} \tag{84}$$

Thus, the results (83) and (64) relate the quantum mechanic wave function to the equation of motion at scale (β−1) through diffusion velocity (84).

Hence, one starts with *Cauchy* equation of motion (7) at scale (β−1)

$$\frac{\partial \mathbf{p}_{i\beta-1}}{\partial t} + \frac{\partial}{\partial x_j}(\mathbf{p}_{i\beta-1}\mathbf{v}_{j\beta-1}) = -\nabla.\mathbf{P}_{ij\beta-1} \tag{85}$$

Next, the stress field within the particle at the lower scale (β−1) is considered to be uniform such that $\nabla.\mathbf{P}_{ij\beta-1} = 0$ and (85) reduces to

$$\frac{\partial \mathbf{p}_{i\beta-1}}{\partial t} + \frac{\partial}{\partial x_j}(\mathbf{p}_{i\beta-1}\frac{\partial x_j}{\partial t}) = 0 \tag{86}$$

Parallel to derivation of *Schrödinger* equation [33], one next considers the moving coordinates

$$\mathbf{x}_{j\beta} = \mathbf{z}_{j\beta} + \boldsymbol{\alpha}_j W_\beta t \tag{87}$$

that for uniform velocity W results in

$$\mathbf{v}_{j\beta-1} = \mathbf{V}_{j\beta-1} + \boldsymbol{\alpha}_j W_{\beta-1} = \mathbf{V}_{j\beta-1} + \mathbf{W}_{j\beta-1} \tag{88}$$

in accordance with (84). The components of convective velocity (87) are expressed as

$$\mathbf{W}_{j\beta-1} = \boldsymbol{\alpha}_j W_{\beta-1} \tag{89}$$

such that

$$W^2_{\beta-1} = \sum \alpha_j^2 W^2_{\beta-1} = \sum W^2_{j\beta-1} \tag{90}$$

Expressing (86) in terms of the uniformly moving coordinate (87) and substituting from (13c) gives

$$\frac{\partial \mathbf{p}_{i\beta-1}}{\partial t} + \alpha_j W_{\beta-1}\frac{\partial \mathbf{p}_{i\beta-1}}{\partial z_j} = \nu_\beta \frac{\partial^2 \mathbf{p}_{i\beta-1}}{\partial z_j^2} \tag{91}$$

For a coordinate system moving with uniform convective velocity W (88) becomes

$$\mathbf{v}_{i\beta-1} = \mathbf{W}_{i\beta-1} + \mathbf{V}_{ij\beta-1} = A + \mathbf{V}'_{ij\beta} \tag{92}$$

where A is a constant. Also, for an incompressible fluid with $\rho_{\beta-1}A = B$ one obtains from (92)

$$\rho_{\beta-1}\mathbf{v}_{i\beta-1} = B + \rho_{\beta-1}\mathbf{V}'_{ij\beta} = B + Y_{\beta-1}\nabla_i \rho_\beta \Phi'_\beta \tag{93}$$

that by (64) gives

$$\mathbf{p}_{i\beta-1} = B + Y_{\beta-1}\nabla_i \Psi_\beta \tag{94}$$

Substituting from (94) into (91) leads to

$$\frac{\partial \Psi_\beta}{\partial t} + \alpha_j W_{\beta-1}\frac{\partial \Psi_\beta}{\partial z_j} = \alpha_m \frac{\partial^2 \Psi_\beta}{\partial z_j^2} \tag{95}$$

where the viscosity tensor α_m by (13c) is defined as

$$\nu_\beta = \nu_{ij\beta} = \alpha_m \quad , \quad \mathbf{V}_{ij\beta} = -\nu_{ij\beta}\frac{\partial \ln \mathbf{p}_{j\beta}}{\partial x_i} \tag{96}$$

Following *Dirac* [133], by (96), and (84)-(89) the parameters (α_j, α_m) are considered to be tensors representing 4×4 matrices. However, since according to *Dirac* one of his four equations is redundant [133], the final wave equation will only involve three components such that by (90)

$$1 = \alpha_1^2 + \alpha_2^2 + \alpha_3^2 \tag{97}$$

leading to $\alpha_1^2 = \alpha_2^2 = \alpha_3^2 = 1/3$ for an isotropic field as compared to $\alpha_1^2 = \alpha_2^2 = \alpha_3^2 = 1$ [133]. The definitions (89) and (96) are in harmony with the perceptions of *Dirac* [133] who anticipated that (α_j, α_m) may be related to some coordinates associated with internal degrees of freedom.

Finally, one introduces the wave function

$$\Psi_\beta = \exp[(\sqrt{i m_{\beta-1} W^2_{\beta-1}/\hbar})z_j] \tag{98}$$

into (95) to obtain the scale invariant relativistic wave equation

$$\left\{ i\hbar[\frac{1}{W_{\beta-1}}\frac{\partial}{\partial t} + \alpha_j \frac{\partial}{\partial z_j}] + \alpha_m m_{\beta-1} W_{\beta-1} \right\} \Psi_\beta = 0 \tag{99}$$

At the important scale of electrodynamics β = s (ESD in Fig.1) when $W_{\beta-1} = W_k = c$ is the speed of light c, (99) becomes *Dirac* relativistic wave equation for massive particles [133]

$$\left\{ i\hbar[\frac{1}{c}\frac{\partial}{\partial \mathrm{t}} + \alpha_{\mathrm{j}}\frac{\partial}{\partial \mathrm{z}_{\mathrm{j}}}] + \alpha_{\mathrm{m}}\mathrm{mc} \right\} \Psi_{\beta} = 0 \qquad (100)$$

Because it was by shear genius and mathematical intuition that *Dirac* arrived at his relativistic wave equation (100), the physical basis of this equation remains quite abstract and mysterious. Therefore, the simple derivation of *Dirac* equation presented above may help the understanding of this important equation. It is also noted that the wave function Ψ_β in (64) remains well defined even in the presence of spin as long as $\nabla \times \mathbf{u}_\beta = \nabla \times \mathbf{v}_\beta$ such that $\nabla \times \mathbf{V}'_\beta = 0$ by (83). The true significance of the tensors (α_j , α_m) in (89) and (96) as well as the wave function (98) require further future investigations.

9 Concluding Remarks

A modified statistical theory of turbulence was presented and the connections between the problems of turbulence and quantum mechanics were further explored. New paradigms for physical foundations of invariant *Planck* law, *Schrödinger* equation, and *Dirac* relativistic wave equation were presented. The predicted velocity profiles for flow over a flat plate were compared with measurements for LED, LCD, LMD, and LAD scales. The universal nature of turbulence across broad range of spatio-temporal scales is in harmony with occurrence of fractals in physical science emphasized by *Takayasu* [134].

Acknowledgments: This research was in part supported by NASA grant No. NAG3-1863.

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