Abstract: - Flow data are viewed as cross-classified data, and spatial interaction models are reformulated as log-linear models. According to this view, we introduce a spatial panel data model and we derive a Generalized Maximum Entropy – based estimation formulation. The estimator we propose has the advantage of being consistent with the underlying data generation process and eventually with the restrictions implied by some non sample information or by past empirical evidence by also controlling for collinearity and endogeneity problems.

Key-Words: - Maximum entropy estimation, spatial panel data models, spatial Interaction models.

1 Introduction

The analysis of macro data at the country or industry level in trade has a long tradition in the empirical literature. More specifically, spatial interaction models or gravity models (Linneman, 1966; Ord, 1975) have recently become an useful tool for applied trade analysis. These models relate bilateral trade to the aggregate supply of the exporting country, the aggregate demand of the importing country, transport and transaction costs, and other specific trade factors.

The recent empirical trade literature (Fischer et al., 2006; Cheng et al., 2005; Egger et al., 2003; LeSage et al., 2005) seems to suggest: (i) panel data models which have the advantage to control for heterogeneity among the countries and cross sectional correlation, and (ii) to take into account the potential endogeneity between trade and output. At the same time, stressing the importance of the cross-sectional spatial dependence term that can represent the spatial effect of regional governance on trade flows, an appropriate estimation methodology that allows for such an effect is needed.

In the estimation of gravity models, this paper addresses these issues and also the problem of zero trade flows. Two common approaches to handle the presence of zero trade involve either discarding the zeros from the sample or to add a constant factor to each observation on the dependent variable, introducing a log-transformation of the model with a multiplicative exponential error term and assuming that the pattern observed in the empirical distribution is better represented by lognormal distribution. If the zeros are not randomly distributed, the first strategy induces a selection bias. With reference to the second strategy, the heteroskedasticity inherent in the log-linear formulation of the model can produce both biased and inefficient OLS estimates.

In our approach, trade flows, from incomplete data, are represented by spatial interaction models which were originally developed in geography. Flow data were viewed as cross-classified data, also referred to as contingency tables, and in this perspective spatial interaction models were reformulated as log-linear models. In this view, we proceed by introducing a panel data model specification which recovers information on trade flows from incomplete data and by estimating the spatial econometric flow model by using a Generalized Maximum Entropy estimation approach.
The proposed estimator has the advantage of being consistent with the underlying data generation process and eventually with the restrictions implied by some non sample information or by past empirical evidence while controlling for collinearity and endogeneity problems.

The paper is structured as follows. The next session introduces spatial panel interaction models. The third section formalizes the proposed generalized maximum entropy estimation approach. The forth section shows an empirical application and the last section concludes.

2 Spatial panel interaction model specification

The most common formulation of the models for origin i to destination j flows start by vectorizing the n by n square matrix of interregional flows from each of the n origin regions to each of the n destination regions (with i=1,...,n and j=1,...,n). An \( n^2 \) -components vector of flows is then obtained by stacking the columns of the flow matrix into a variable vector that we designate as \( Y \). The objective of flow models is to explain variation in the magnitude of flows between each origin-destination pair. Following LeSage and Pace (2005), our focus is on a formal methodology for accounting for spatial dependence in the origin-destination flows. The basic idea is that: i) large commodity flows from region O (origin) to region D (destination) might be accompanied by similar large flows from neighbors to region O to region D; ii) large commodity flows from region O to region D might be accompanied by similar large flows from region O to neighbors of region D; and iii) large commodity flows from region O to region D might be accompanied by large flows from neighbors to region O to neighbors of region D. In accordance with this: i) is labeled as origin-based dependence, ii) as destination-based dependence and iii) as origin-destination dependence.

Conventional gravity models use explanatory variables containing characteristics of both the origin and destination regions in an attempt to explain variation in the vector \( Y \) containing interregional flows. In addition, an intercept term and a \( n^2 \) by 1 vector of distances between all origins and destinations are typically used as additional variables.

With reference to a panel data framework, by applying a log-transformation to the standard gravity model, the resulting structural model takes the following log-additive specification:

\[
Y = \alpha t + X_o \beta_d + X_o \beta_o + D \gamma + \varepsilon
\]  

(1)

where \( Y \) is a vector of dimension \( n^2T \) where observations 1 to n reflect flows from origin 1 to all n destinations for all times T.

In (1), the explanatory variable matrices \( X_d \), \( X_o \) represent \( n^2T \) by \( K \) matrices containing destination and origin characteristics respectively and the associated \( K \) parameter vectors are \( \beta_d \) and \( \beta_o \). The matrix \( X_d \) is constructed using characteristics of the destination node for each of the origin-destination (O-D) pair observations, and the matrix \( X_o \) is similarly constructed from the origin node in the O-D pairs representing the sample of observations. The vector \( D \) denotes the \( n^2T \) origin-destination distances and the scalar parameter \( \gamma \) reflects the effect of distance D, \( \alpha \) denotes the constant term parameter and \( t \) is a vector of ones of dimension \( n^2T \). Typically these regression models assume \( \varepsilon \sim N(0; \sigma^2 I n^2_T) \). When spatial dependence is introduced, the correlation across cross-sectional units is non-zero, and the pattern of non-zero correlations follows a certain spatial process. With a few exceptions, use of spatial lags typically found in spatial econometric methods have not been used in the spatial interaction models. models where each observation represents a region rather than an origin-destination pair.

The family of models introduced by LeSage and Pace (2005) rely on a spatial autoregression filtering shown in (2) that takes into account origin, destination, and origin-to-destination dependence:

\[
y = \alpha t + X_o \beta_d + X_o \beta_o + D \gamma + \rho_1 W_o y + \rho_2 W_d y + \rho_3 W_d y + \varepsilon
\]  

(2)

where, \( W_o = I_o \otimes W \), where \( I_n \) is an identity matrix of dimension \( n \) and \( \otimes \) denotes the Kronecker product. \( W \) represents an \( nT \) by \( nT \) spatial weight matrix whose diagonal elements are zero. The specification of the spatial weights is typically driven by geographic criteria, such as contiguity (sharing a common
3 Generalized Maximum Entropy estimation

In the context of this work, we adapt the approach presented in Bernardini Papalia (2009) to the spatial interaction model. More specifically, a spatial lag model specification, as specified in equation (2), which includes a spatially lagged dependent variable is here considered:

\[
y = \alpha t + X_d\beta_d + X_o\beta_o + D\gamma + \rho_1 W_o y + \rho_2 W_d y + \rho_3 W_w y + \epsilon
\]

(3)

This model can also be written as:

\[
Y = \rho_1 W_o y + \rho_2 W_d y + \rho_3 W_w y + X\beta + \epsilon;
\]

(4)

where \( X = [t; X_o; X_d; D] \) and \( \beta = [\alpha; \beta_o; \beta_d; \gamma] \) and the other notation is as before.

It is assumed that: spatial effects are not identical across spatial units; differentiated spatial effects within and between spatial units are taken into account.

In empirical applications, it is common practice to derive spatial weights for \( W_o, W_d \) and \( W_w \) from the location and spatial arrangements of observation by means of a geographic information system. In this case, units are defined 'neighbors' when they are within a given distance of each other, i.e. \( w_{ij} = 1 \) for \( d_{ij} \leq \delta \) and \( w_{ij} = 0 \) otherwise. The spatial weights matrix \( W \) reflects type 3 dependence that we referred to as origin-based dependence, or destination-based dependence.

For \( \delta \) being a critical cut-off value. More specifically, a spatial weights matrix \( W^* \) is defined as follow:

\[
w^*_y = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } d_{ij} \leq \delta, i \neq j \\ 0 & \text{if } d_{ij} > \delta, i \neq j \end{cases}
\]

(5)

and the elements of the row-standardized spatial weights matrix \( W \) (with elements of a row sum to one) result:

\[
w_y = \frac{w^*_y}{\sum_{j=1}^{N} w^*_y}, \quad i, j = 1, ..., N.
\]

(6)

Under the GME framework the objective is to recover simultaneously the unknown parameters, the unknown errors by defining an inverse problem, which is based only on indirect, partial or incomplete information. In this respect, the parameters of the spatial interaction model are estimated with minimal distributional assumptions.

Each parameter is treated as a discrete random variable with a compact support \( Z \) and \( M \) possible outcomes, \( 2 \leq M \leq \infty \). The uncertainty about the outcome of the error process is represented by treating each error as a finite and discrete random variable with \( J \) possible outcomes, \( 2 \leq J \leq \infty \). A set of discrete points, the support space \( V = [v_1, v_2, ..., v_J] \) of dimension \( J \geq 2 \), that are at uniform intervals and symmetric around zero, are chosen and each error term has corresponding unknown weights \( r_i = [r_{i1}, r_{i2}, ..., r_{ij}] \) with the properties of probabilities \( 0 \leq r_{ij} \leq 1 \) and \( \sum r_{ij} = 1 \). In practice, discrete support spaces for both the parameters and errors are defined according to economic theory or other prior information.

In matrix notation, the unknown parameters and errors are reparameterized as:

\[
\rho_1 = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix}, \quad \rho_2 = \begin{pmatrix} r_{31} & r_{32} \\ r_{41} & r_{42} \end{pmatrix}, \quad \rho_3 = \begin{pmatrix} r_{51} & r_{52} \\ r_{61} & r_{62} \end{pmatrix}, \quad \beta = [\beta_1; \beta_2; \beta_3]
\]

yielding the following GME specification:
\[ Y = (Z^\alpha p^\alpha) W_w Y + (Z^\beta p^\beta) W_d Y + X(Z^\beta p^\beta) + (V_r) \] 

(7)

where \( p^\alpha = \text{vec}(p^a, p^b, p^d, p^\gamma) \) consists of vectors of weights \( p^a, p^b, p^d, p^\gamma \), each having nonnegative elements summing to unity and \( r \) are vectors of proper probability distributions for parameters and errors, respectively. Given the data consistency (8), the GME objective function \( H(.) \), relative to our formulation problem, may be formulated as:

\[ H(p^\beta, r) = -\ln p^\beta - r'\ln r \] 

(8)

subject to:

- data consistency conditions (7):
  - (ii) adding-up constraints:
    \[ l' p_k^\beta = 1 \forall k; \]
    \[ l' r_i = 1 \forall i. \] 

(9)

The advantages of the GME estimation approach for spatial panel gravity models are:
- first, it is possible to obtain consistent estimates of the individual fixed effects when \( N \to \infty \) (the incidental parameter problem); second, this estimation procedure deals with the problem of endogeneity of the spatial lag term as specified in spatial lag interaction models and within a panel data framework.

4 An application to commodity flows between Italy and European countries of the Balkanic area

To illustrate the ideas discussed in Section 2 we produced GME estimates for the spatial lag model in (2) using commodity flows between Italy and European countries of the Balkanic area that covers a total of seven countries (Albania, Bosnia, Erzegovina, Bulgaria, Croazia Macedonin, Romania, Serbia Montenegro) during the years from 1998 to 2004. It is assumed that the volume of exports between Italy and each of the other countries is determined by the following set of explanatory variables \( (Xo \text{ and } Xd) \): gross domestic product (GDP), openness of treading countries, GDP per capita differential, distance as a proxy of transportation costs, and a set of dummies variables either facilitating or restricting trade between pairs of countries or specific subgroups of countries that identify some communication or transportation networks. The weight matrix is computed by means of the distance between the capital cities, with a critical cut-off value equal to the first quartile. We would expect that changes in per capita GDP would exhibit positive signs, leading to higher levels of commodity flows at both the origin and destination regions. The coefficient estimate on distance should be negative indicating a decay of flows with distance.

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Table 1 shows that Italy’s export is positively determined by the size of the economies, and openness of the countries involved. With regards to the country specific effects, we observe that these effects are strongly significant for all countries. It has been found that transportation costs are significant factors in influencing Italy’s exports negatively. This implies Italy would be influenced to a greater extent by the border between the Balkanic area and Italy.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficient</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln per cap GNP Diff</td>
<td>0.6097</td>
<td>0.0481</td>
</tr>
<tr>
<td>ln impj/GDPj</td>
<td>0.0018</td>
<td>0.0863</td>
</tr>
<tr>
<td>ln openness</td>
<td>0.0463</td>
<td>0.0987</td>
</tr>
<tr>
<td>ln distance</td>
<td>0.6304</td>
<td>0.0890</td>
</tr>
<tr>
<td>spatial dep. Var</td>
<td>0.9088</td>
<td>0.0046</td>
</tr>
<tr>
<td>2000 dummy</td>
<td>0.0336</td>
<td>0.0024</td>
</tr>
<tr>
<td>2001 dummy</td>
<td>0.0077</td>
<td>0.0024</td>
</tr>
<tr>
<td>2002 dummy</td>
<td>0.0048</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

Note: Dependent variable, Italian exports to country j (in logs); time effects to control for business cycles are included;
5 Conclusion

Despite numerous applications in empirical trade analysis, there are still open issues related to the estimation of gravity models. First, cross-sectional correlation is present already because of the construction of this kind of models, which involve bilateral trade flows and aggregate national variables. Second, even if few authors estimate dynamic panel data models in order to catch the relevance of persistence in bilateral trade patterns, the introduction of dynamics in a panel data model produces inconsistency of the estimators due to the endogeneity of the lagged dependent variable.

In analyzing trade dynamics throughout spatial interaction models, the contribution of this study comes from the combination of (i) the use of panel data with spatial unobserved heterogeneity which allows estimating elasticity of trade with respect to its determinants, and (ii) an adequate estimation technique which deals with problems of potential endogeneity of the gross domestic product (GDP) variables when bilateral specific effects are accounted for or when a dynamic panel data model specification is assumed.

From a more general point of view, the GME estimation method has advantages in cases involving small samples or ill-posed problems and is computationally efficient and robust for given support points chosen according to prior information and/or past empirical evidence. An illustrative application of the spatial GME estimator in the context of the analysis of commodity flows between Italy and European countries of the Balkanic area that covers a total of seven countries (Albania, Bosnia, Erzegovina, Bulgaria, Croazia Macedonia, Romania, Serbia Montenegro) and refers to the period of 1998 to 2004, has been provided. Our results have shown that Italian’s trade is determined by the size of the economies, per capita GNP differential of the countries involved, openness of the trading countries and distance between the two countries’ capitals (proxy of transportation costs). The role of both the spatial lag dependent variable and the lagged bilateral trade variable seems to be confirmed.

Further empirical investigations could be implemented with the aim of considering alternative formulations for spatial weights, based on different geographic criteria (great circle distance, k nearest neighbors) as well as derived from aggregate trade flows between countries.

References:


