Simulate the effect of a conductive wall on convection in a porous enclosure

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Abstract: Effects of a conductive wall on natural convection in a square porous enclosure having internal heating at a rate proportional to a power of temperature difference are studied numerically. Horizontal heating is considered, where the vertical walls are heated isothermally at different temperatures, while the horizontal walls are kept adiabatic. The Darcy model is used in the mathematical formulation for the porous layer. It is found that increasing the value of the thermal conductivity ratio and/or decreasing the thickness of solid wall can increase the maximum fluid temperature.

Key–Words: Conjugate heat transfer, Natural convection, Porous media, Darcy’s law

1 Introduction

Analysis of flow and convective heat transfer within the porous materials when the porous material provides its own source of heat has become a separate topic for research in the last thirty years. This interest is due to its importance in various engineering applications, such as in the self-induced heating of coal stockpiles [4] and [5] and bagasse-piles ([20], [19] and [8]). Convectional flows are governed by non-linear partial differential equations, which represent conservation laws for the mass, momentum and energy. Numerical treatment usually used to solve the equations or if possible by an analytic method. Numerical investigation of natural convection in porous enclosure having uniform internal heating firstly conducted by Haajizadeh et al. [10]. Then the study was continued by Rao and Wang [14] and analytically by Joshi et al. [11]. Recently, Mealey and Merkin [13] moved away from the study of uniform internal heating to that of non-uniform internal heating and solved the problem numerically and analytically. Mealey and Merkin [13] found that a strong non-uniform internal heating can generate significant maximum fluid temperatures above the heated wall.

In some situations of considerable practical importance, the heated or cooled wall thickness is finite having a thermal conductivity significantly different with the fluids in the porous material. This gives an alternative way in which a convective flow can be set up within the porous material. Such a situation can arise in the high performance insulation for buildings or the heated ground water. For example in the heated ground water due to hot intrusion may rise in a narrow fractured zone, as the heated water rises, it eventually encounters a cooler rock formation that sandwiches the permeable vertically slender space. This causes heat transfer between the hot water and the colder surrounding rocks. This coupled conduction-convection problem is known as conjugate convection. Conjugate natural convection in a rectangular porous enclosure surrounded by walls was firstly examined by Chang and Lin [6]. Their results show that wall conduction effects decrease the overall heat transfer rate from the hot to cold sides of the system. Chang and Lin [7] also studied the effect of wall heat conduction on natural convection in an enclosure filled with a non-Darcian porous medium. Baytas et al. [2] gave a numerical analysis in a square porous enclosure bounded by two horizontal conductive walls. Later, Saeid [15] studied conjugate natural convection in a square porous enclosure with two equal–thickness walls. Saeid [16] investigated the case when only one vertical wall is of finite thickness. Thermal nonequilibrium model to investigate the conjugate natural convection in porous media was reported by Saeid [17]. Varol et al. [21] studied a porous enclosure bounded by two solid massive walls from vertical sides at different thicknesses. Al-Amiri et al. [1] considered the two insulated horizontal walls of finite thickness and used the Forchheimer–Brinkman–extended Darcy model in the mathematical formulation. Very recently Varol et al. [22] analyzed heat flow of a cold water near 4°C in a thick bottom
2 Mathematical model

A schematic diagram of horizontal heating of a porous enclosure with finite wall thickness is shown in Figure 1. The choice of the spatial coordinates is also described in this figure, where the gravitational field is parallel to the x-axis. The left surface \((y = 0)\) of the impermeable wall is heated to a constant temperature \(T_h\) and the right surface \((y = 0)\) of the porous enclosure is cooled to a constant temperature \(T_c\), while the horizontal walls are kept adiabatic. Heat is assumed also to be generated internally within the porous medium at a rate proportional to \((T_p - T_c)^\lambda\) \((\lambda \geq 1)\). This relation, as explained by Mealey and Merkin [13], is an approximation of the state of some exothermic process. In the porous medium, Darcy’s law is assumed to hold, the Oberbeck-Boussinesq approximation is used and the fluid and the porous matrix are in local thermal equilibrium.

With these assumptions, the continuity, Darcy and energy equations for steady, two-dimensional flow in an isotropic and homogeneous porous medium are:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} &= gK\beta \frac{\partial T_p}{\partial y} \\
\end{align*}
\]

and the energy equation for the impermeable wall is:

\[
\frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2} = 0
\]

where the subscript \(p\) stands for porous layer and \(w\) stands for the wall. The above equations can be written in terms of the stream function \(\psi\) defined as \(u = \partial \psi / \partial y\) and \(v = -\partial \psi / \partial x\) together with the following non-dimensional variables:

\[
\begin{align*}
\Theta_p &= \frac{\psi}{\xi}, \\
\Theta_w &= \frac{\psi - \psi_D}{\xi}, \\
\end{align*}
\]

where \(\Delta T = T_h - T_c > 0\). The resulting non-dimensional forms of the governing Eqs. (1)-(4) are:

\[
\begin{align*}
\frac{\partial^2 \Theta_p}{\partial X^2} + \frac{\partial^2 \Theta_p}{\partial Y^2} &= \frac{\partial \Theta_p}{\partial Y} \\
Ra \left[ \frac{\partial \psi \partial \Theta_p}{\partial Y \partial X} - \frac{\partial \psi \partial \Theta_p}{\partial X \partial Y} \right] &= \frac{\partial^2 \Theta_p}{\partial X^2} + \frac{\partial^2 \Theta_p}{\partial Y^2} + \Theta_p^\lambda \\
\frac{\partial^2 \Theta_w}{\partial X^2} + \frac{\partial^2 \Theta_w}{\partial Y^2} &= 0
\end{align*}
\]

where \(Ra\) is the Rayleigh number defined as: \(Ra = g\beta K \Delta T \ell / (\nu \alpha)\) and \(\gamma\) is the internal heating parameter defined as: \(\gamma = Gf(\Delta T)^{-1}/(\alpha \rho c_p)\). The values of the non-dimensional stream function are zero in the wall region and on the solid-fluid interfaces. The boundary conditions for the non-dimensional temperatures are:

\[
\begin{align*}
\Theta_p(X, 0) &= 1, \quad \Theta_p(X, 1) = 0 \\
\partial \Theta_p(0, Y) / \partial X &= 0, \quad \partial \Theta_p(0, Y) / \partial X = 0 \\
\partial \Theta_p(1, Y) / \partial X &= 0, \quad \partial \Theta_w(1, Y) / \partial X = 0 \\
\Theta_p(X, D) &= \Theta_w(X, D); \\
\partial \Theta_p(X, D) / \partial Y &= Kr \partial \Theta_w(X, D) / \partial Y
\end{align*}
\]

where \(Kr = k_w/k_p\) is the thermal conductivity ratio. The physical quantities of interest in this problem are the average Nusselt number defined by:

\[
\begin{align*}
\overline{Nu}_w &= \int_0^1 \frac{1}{\partial \Theta_w / \partial Y} \bigg|_{Y=0,D} \, dY \\
\overline{Nu}_p(X, 0) &= \int_0^1 \frac{1}{\partial \Theta_p / \partial Y} \bigg|_{Y=D} \, dY \\
\overline{Nu}_p(X, 1) &= \int_0^1 \frac{1}{\partial \Theta_p / \partial Y} \bigg|_{Y=1} \, dY
\end{align*}
\]
where $\overline{Nu_w}$ is represents the dimensionless heat transfer through the solid wall, $\left( \overline{Nu}_p \right)_h$ and $\left( \overline{Nu}_p \right)_c$ represent the dimensionless heat transfer through the hot and cold porous wall respectively.

### 3 Computational methodology

The governing equations along with the boundary condition are solved numerically by the CFD software package COMSOL Multiphysics. COMSOL Multiphysics (formerly FEMLAB) is a finite element analysis, solver and simulation software package for various physics and engineering applications. We consider the following application modes in COMSOL Multiphysics. The Poisson’s Equations mode (poeq) for Eqs. (5), the Convection–Conduction Equations mode (cc) for Eq. (6) and the Conduction Equations mode (ht) for Eq. (7).

Several grid sensitivity tests were conducted to determine the sufficiency of the mesh scheme and to ensure that the results are grid independent. We use the COMSOL Multiphysics default settings for predefined mesh sizes, i.e. extremely coarse, extra coarse, coarser, coarse, normal, fine, finer, extra fine and extremely fine. In the tests, we consider the parameters $D = 0.1$, $Kr = 2.4$, $\gamma = 0$ and $Ra = 1000$ as tabulated in Table 1. Considering both accuracy and time, an extra fine mesh size was selected for all the computations done in this paper. Table 1 also shows that the relation [16], $\left( \overline{Nu}_p \right)_h = Kr \overline{Nu_w}$ is satisfied.

To validate the computation code, the previously published problems on natural convection in a square porous enclosure without conductive wall ($D = 0$) and no heat source ($\gamma = 0$) were solved. Table 2 shows that the average Nusselt number at $Ra = 100, 1000$ are in good agreement with the solutions reported by the literatures. These comprehensive verification efforts demonstrate the robustness and accuracy of the present computation.

### 4 Results and discussion

Figure 2 illustrates the effects of the wall thickness parameter $D$, for $Kr = 2.4$, $\gamma = 4$, $\lambda = 1$ and $Ra = 40$ on the flow fields and thermal fields in the porous enclosure and the solid wall. As can be seen in 2(a)–(c), the parameter $D$ affects the flow characteristics and the fluid temperatures as well as the solid wall temperatures. Strong internal heat source gives maximum fluid temperature in the porous enclosure more than the heated solid wall. As the wall thickness increases $\Theta_p$ decreases significantly and lower than the heated solid wall as shown in 2(b) and (c). The strength of the flow circulation of the fluid-saturated porous medium is much higher for a thin solid wall. It is important to note that the Rayleigh number in the present study is based on the total height of the wall and not on the thickness of the porous layer.

### 5 Conclusion

The present numerical simulation studies the effects of a conductive wall on natural convection in a square porous enclosure having internal heating at a rate proportional to the local temperature difference. Detailed computational results for flow and temperature fields have been presented in graphical form. The main conclusion of the present analysis is that the strength of the flow circulation of the fluid saturated porous medium is much higher with thin walls and/or a higher value of the solid to fluid thermal conductivity ratio.
Figure 2: Streamlines (left), isotherms (right) at $Ra = 40$, $Kr = 2.4$, $\gamma = 4$ and $\lambda = 1$.

References:


