DTC of Open-Circuit Fault Non-Sinusoidal Back-EMF PM BLAC Motor

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Abstract: - DTC of three-phase non-sinusoidal back-EMF permanent magnet brushless AC motor (PM BLAC) having one phase open-circuit fault is presented in this paper. The scheme employs an extra inverter leg in providing a current path for the two phase windings. Stator flux linkages and developed torque of the faulty motor are estimated using approximate model estimators. The models in \( \alpha \beta \) coordinates are obtained from the fundamental component of the rotor flux and measured stator currents. Voltage vectors and switching table of the DTC are described. Performances of the proposed strategy are evaluated by simulation and validated by comparing with the measured data.

Key-Words: - Direct torque control, Non-sinusoidal back-EMF, Permanent magnet brushless motor, Open-circuit fault

1 Introduction

Permanent magnet brushless motors can be manufactured into two types by back-EMF waveforms, sinusoidal back-EMF and non-sinusoidal back-EMF [1]. The non-sinusoidal back-EMF motor is usually referred to the motor with trapezoidal-like back-EMF waveform. The ratio of rms voltage to the peak voltage of the trapezoidal waveform is higher than that of sine wave. The trapezoidal-like back-EMF motors then offer higher torque density per ampere and also ease in manufacturing compared to the sinusoidal back-EMF motor [1-2]. As a result this type of motor is commonly used in industrial appliances such as in machine tools or in robotic systems etc. The non-sinusoidal back-EMF PM motors however pose a subtle disadvantage in torque ripple. The motors also require an appropriate current waveform, despite of sinusoidal function, to achieve constant torque operation.

DTC is proposed as a candidate algorithm in electric drive system [3]. Conventional voltage and current model flux estimator are derived based on symmetrical motor model. DTC is therefore relies on healthy system. Although stator fluxes linkage and torque of the non-sinusoidal back-EMF motor can be estimated the algorithms are normally complicated compared to those of the sinusoidal back-EMF motor [4-6]. DTC of three-phase PM BLAC motor having one phase open-circuit fault is presented in [7]. The method is however derived based on sinusoidal back-EMF motor.

Model of the three-phase non-sinusoidal back-EMF PMBL motor in \( \alpha \beta 0 \) coordinates is derived. The approximate models of torque and stator flux linkage of the faulty motor are established. Then DTC is presented subjected to the proposed configuration. Performances of the proposed method are shown.

2 System Modeling

Stator voltage of non-sinusoidal back-EMF PM BLAC motor in \( \alpha \beta 0 \) coordinates can be derived in (1)

\[
V_{\alpha 0} = R_{\alpha 0} i_{\alpha 0} + \frac{d(\psi_{\alpha 0})}{dt} \tag{1}
\]

where \( V_{\alpha 0} = \begin{bmatrix} V_\alpha & V_\beta & V_0 \end{bmatrix}^T \) and \( I_{\alpha 0} = \begin{bmatrix} I_\alpha & I_\beta & I_0 \end{bmatrix}^T \) denote stator voltage vector and stator current vector.

\( \psi_{\alpha 0} = \begin{bmatrix} \psi_{\alpha} & \psi_{\beta} & \psi_{0} \end{bmatrix}^T \) and \( R_{\alpha 0} = \begin{bmatrix} R_\alpha & 0 & 0 \\ 0 & R_\beta & 0 \\ 0 & 0 & R_0 \end{bmatrix} \)

are stator flux linkage vector and resistance matrix. Stator flux linkage vector \( \psi_{\alpha 0} \) can be indicated in (2).

\[
\psi_{\alpha 0} = L_{\alpha 0} i_{\alpha 0} + \psi_{\alpha 0} \tag{2}
\]

where \( L_{\alpha 0} = \begin{bmatrix} L_\alpha & 0 & 0 \\ 0 & L_\beta & 0 \\ 0 & 0 & L_0 \end{bmatrix} \) is the inductance matrix and \( \psi_{\alpha 0} \) is the stator flux linkages due to the rotor magnetic flux.

\[
\psi_{\alpha 0} = \sum_{n=1}^{2n} K_{\alpha 0} \psi_{\alpha 0} n \begin{bmatrix} \cos((2n-1)\theta) \\ \sin((2n-1)\theta) \\ \cos(2n-1)\theta) \end{bmatrix} \tag{3}
\]
\( \theta \) is the rotor angular position respected to phase \( a \) magnetic axis. \( \psi_{ra \beta 0}^{2n-1} \) and \( K_{ra \beta 0}^{2n-1} \) denote the stator flux linkage due to rotor magnet at the harmonic \( 2n-1 \) and their coefficients. The coefficients of the harmonic fluxes are shown in Table 1.

Table 1 Coefficient of rotor harmonic fluxes

<table>
<thead>
<tr>
<th>( n )</th>
<th>Harmonic order</th>
<th>( k_a )</th>
<th>( k_b )</th>
<th>( k_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
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<td>2</td>
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<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

2.1 Flux linkage estimator

Flux linkages of three-phase PM motor having phase \( a \) winding open-circuit fault can be described in (4) where the corresponding flux vectors are shown in Fig.1.

\[
\begin{align*}
\psi_{sa} &= M_i a + M_i c + \psi_{ra} \\
\psi_{sb} &= L_i b + M_i c + \psi_{rb} \\
\psi_{sc} &= L_i c + M_i a + \psi_{rc}
\end{align*}
\]

\( \psi_{sa}, \psi_{sb} \) and \( \psi_{sc} \) and \( \psi_{ra}, \psi_{rb} \) and \( \psi_{rc} \) are respectively the stator flux linkages produced from stator currents and permanent magnets in \( abc \) coordinates. \( L \) and \( M \) are respectively self inductance and mutual inductance.

![Space vector in abc coordinates of stator flux linkages produced from (a) stator currents (b) permanent magnet in rotor.](image)

The stator flux linkages can be spited into two parts, one produced from stator currents and the other obtained from permanent magnets in the rotor. Hence the separation calculation is applicable. If amplitude of the fundamental component of stator flux linkage and stator currents are significant their models can be simplified by omitting the higher order harmonic components. An approximate stator flux linkages in \( a\beta \) coordinates presented in (5) is obtained by considering only fundamental component of rotor flux and the stator current.

\[
\begin{align*}
\widetilde{\psi}_{sa} &= L_a \tilde{I}_a + \psi_{ra} \\
\widetilde{\psi}_{sb} &= L_b \tilde{I}_b + \psi_{rb}
\end{align*}
\]

By this way stator flux linkages of the non-sinusoidal back-EMF motor in \( a\beta \) coordinates take a form of the conventional current model flux estimator. \( \psi_{ra} \) and \( \psi_{rb} \) are calculated from (6) and \( \tilde{I}_a \) and \( \tilde{I}_b \) are obtained from Clark’s transformation of the stator currents.

\[
\begin{align*}
\tilde{\psi}_{ra} &= |\psi_{r1}| \cos(\theta_r) \\
\tilde{\psi}_{rb} &= |\psi_{r1}| \sin(\theta_r)
\end{align*}
\]

\[ |\psi_{r1}| \] is the magnitude of the fundamental component of rotor flux.

2.2 Torque estimator

Representing as a product of the stator current vector and an infinite series function of stator flux linkages, the developed torque of the non-sinusoidal back-EMF motor is not in the applicable form. If influences of the higher order harmonic currents and flux linkages can be ignored a simplified torque model may achieve. The approximate torque model given in (7) is obtained by employing the stator flux linkage (5).

\[
\tilde{T}_e = \frac{3}{2} P \left( \tilde{\psi}_{sa} \tilde{I}_b - \tilde{\psi}_{sb} \tilde{I}_a + \tilde{\psi}_{r0} \tilde{I}_0 \right)
\]

Torque equation given in (6) can be separated into two parts. The first term on the right hand side takes the form of the conventional torque estimator and the second term is an additional term which emerges due to the third harmonic rotor flux. The zero sequence rotor flux linkage is separately calculated as given in (8).

\[
\tilde{\psi}_{r0} = 2\psi_{r30} \sin(2\theta_r)
\]

\( \tilde{I}_0 \) is the zero sequence current which can be obtained from Clark’s transformation of stator currents.

3. DTC of Single phase Open-circuit Fault PM BLAC Motor

Schematic diagram of torque control loop of the proposed method is shown in Figure 2. It is assumed that phase \( a \) winding of the motor is open-circuit. The
power unit is the three-leg inverter where one leg is devoted as a current path for the motor neutral point. Estimated stator flux linkages and developed torque of the motor are respectively calculated from (5), (7). The zero-sequence rotor flux is separately calculated from (8). Two level hysteresis controllers are employed in the torque and flux control loops. Phase back-EMF of the motor is shown in Fig. 3.

According to the topology there are six active voltage vectors available. The voltage vectors obtained from (9) are indicated in Fig. 4. For the design purpose the voltage vectors are defined by V1-V6 where V1 is the voltage vector in direction of $\alpha$-axis and the rest are consecutive vectors located at 60 degree apart in anticlockwise direction. Sectors are defined in the similar fashion as used in conventional DTC of an AC motor where every sector has identically expanded angle with a voltage vector placing in the middle.

$$V_{\alpha} = \frac{2}{3} \left( -0.5V_{bn} - 0.5V_{cn} \right) \quad (9)$$

$$V_{\beta} = \frac{2}{3} \left( \sqrt{3} V_{bn} - \sqrt{3} V_{cn} \right)$$

where $V_{bn}$ and $V_{cn}$ are voltages across phase $b$ and $c$ respectively.

The control signal is obtained from the collaboration of switching table and hysteresis controller. The switching table of the proposed method is indicated in Table 2.

To complete the simulation speed control loop is also developed. The reference speed is set to 200 rpm. A PI controller is employed in the speed loop. The controller parameters are set up trial and error where the proportional gain $K_p$ and integral time constant $K_i$ are set to 0.005 and 0.025 respectively. The reference flux and load torque are set to 92.8 mWeb and 1 Nm respectively. Two-level comparators and switching table, Table 2, are utilized in the simulation. The simulated results of the proposed method are shown Figure 5. The results show that the system can be stabilized by the control algorithm. The two phase-

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**Table 2 The switching table**

<table>
<thead>
<tr>
<th>$d\varphi_s$</th>
<th>$dT_e$</th>
<th>Sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>V1 V5 V4 V6 V2 V3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>V2 V3 V1 V5 V4 V6</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>V5 V4 V6 V2 V3 V1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>V6 V2 V3 V1 V5 V4</td>
</tr>
</tbody>
</table>

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**Table 3 Motor parameters**

| Phase resistance | 0.35 Ohm. |
| D-q axis inductance | 4.64 mH. |
| Rotor magnetic flux | 79.4 mWeb. |
| Number of pole pairs | 5 |
| DC bus voltage | 36 Volt. |
| Rated speed | 400 rpm. |
| Mutual inductance | 0.0023 mH. |
| Number of slot | 12 |

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**Fig. 2** The proposed DTC fault tolerant control schematic diagram.

**Fig. 4** Voltage vectors and sectors.

**Fig. 3** Back-EMF waveform of the motor at the rotor speed 400 rpm.
winding motor can develop a torque balanced to the torque command. Sinusoidal waveforms of the stator flux linkage in $\alpha\beta$ coordinates are obtained from the approximate stator flux linkage estimator. Symmetrical sinusoidal stator flux linkages in $\alpha\beta$-coordinates indicate that the constant magnitude and angular speed rotating flux vector is established. Although phase currents are not control variables in DTC algorithm they are indirectly controlled to achieve the torque and flux demands. Constant rotor speed responses are also presented in the Figures.

To validate the performance of the proposed method as shown in the simulation results a laboratory test is implemented. The experimental system is implemented based on DSP TMS 320C31 environment. Rotor position is measured using incremental encoder accuracy 4000 pulses per revolution. 12 bit ADC interface cards are employed in signal conversion. The algorithm is coded in assembly language. The experimental results are shown in Fig.6.

Fig. 5 Simulated results of the proposed method

Fig. 6 Measured results of the proposed method.
5. Conclusions

DTC of three-phase non-sinusoidal back-EMF PM BLAC motor having one phase open-circuit fault is presented in this paper. Since flux linkages and developed torque of the motor are expressed in infinite harmonics functions they are inappropriate in application. Approximate estimators are proposed by avoiding the higher order harmonic components in flux and torque estimation. By this way the stator flux linkages and torque of the motor can be estimated by the conventional-like estimator. The simplicity in the algorithm can be preserved. The switching table and voltage vectors subjected to the faulty system are also presented. The simulations results show that torque and flux linkage are properly control by the proposed algorithms. The scheme is robust to the error due to the higher harmonic neglecting in the variable estimation procedure. Measurements are used in validation the applicability of the proposed method.

References: