Estimating the Asymmetric Pattern of Volatility in China Stock Markets. A New Proposal

M. C. García, G. Fernández-Avilés and J. M. Montero

Abstract—Over the past 25 years, volatility models and their forecasts have been the focus of both academic researchers and practitioners. However, the models proposed in the literature use to fail to detect the asymmetric pattern of volatility in a large number of cases. This is the reason why in this paper we propose a new asymmetric ARSV model: the Threshold Asymmetric Autoregressive Stochastic Volatility (TA-ARSV) model.

We focus our empirical study on China’s daily Stock Exchange returns. The are four main reasons for this: China can be considered a relatively “new” market; the growth potential of Chinese stock markets in emerging countries has attracted qualified foreign institutional investors and global investors in recent years; Chinese stock markets have experienced rapid growth associated with high risk; and there has been relatively little work done on modelling and forecasting stock market volatility in China with inconclusive results.

Keywords—Asymmetric pattern of volatility, financial returns, China, TARSV model.

I. INTRODUCTION

Volatility models and their forecasts are of interest to many types of economic agents, especially for financial risk management. Volatility is one of the most important aspects of financial market developments, providing an important input for portfolio management, option pricing and market regulation. In recent years, the tremendous growth of trading activity and the trading losses of well known financial institutions have led financial regulators and supervisory committees to quantify the risk.

Research on volatility of speculative prices is not new. On the contrary, it has a long history. References [1] and [2] first discovered that the changes of speculative prices and the rates of return are characterized by tranquil and volatile periods – large price changes tend to be followed by large changes, of either sign, and small price changes tend to be followed by small price changes —. That is, price change volatility showed itself as being clustered and variance changes over time. After that, many economists carried out extensive research on the characteristics of the volatility of speculative prices.

Over the past 25 years, volatility models and their forecasts have been the focus of both academic researchers and practitioners. Since [3] proposition of the Autoregressive Conditional Heteroskedastic (ARCH) model, there has been a large body of literature on volatility forecasting. Traditional models used in the econometric literature can be classified in two types: (i) the Generalized Autoregressive Conditional Heteroskedastic (GARCH) models proposed by [4] and (ii) the Autoregressive Stochastic Volatility (ARSV) models developed by [5] and [6].

However, empirical evidence has shown that the pattern of volatility is not the same when returns are positive or negative. This stylized fact is known as “leverage effect” in econometric literature. To explain this asymmetric pattern of volatility, two traditional asymmetric GARCH models in the framework of discrete time have been used, namely the Asymmetric Generalized Autoregressive Conditional Heteroskedasticity (AGARCH) model [7] and the Threshold Generalized Autoregressive Conditional Heteroskedasticity (TGARCH) model [8]. However, in our opinion these two strategies fail to detect the asymmetric pattern of volatility in a large number of cases.

This is the reason why in this paper we propose a new asymmetric ARSV model: the Threshold Asymmetric Autoregressive Stochastic Volatility (TA-ARSV) model. This new strategy captures the asymmetric pattern of volatility, establishing a threshold which indicates that the behaviour of volatility varies with shocks of different sign. The TA-ARSV model does not need to establish a correlation between the innovations of returns and the volatility equation to capture the leverage effect. There are different orders of the TA-ARSV model, but usually a simple TA-ARSV(1) is adequate to explain the behaviour of the daily returns series used in this research.

In this paper the novel strategy we propose competes with the leading models in the literature. For this purpose we use Chinese daily Stock Exchange returns. The reason for choosing China stock markets are: (i) that China can be considered a
“new” market—though China’s stock market has been developing at an amazing speed in recent years, it still has many deficiencies compared with other security markets in some developed countries; (ii) the growth potential of Chinese stock markets in emerging countries has attracted qualified foreign institutional investor and global investors in recent years. However, this rapid growth is associated with high risk. And (iii) there has been relatively little work done on modelling and forecasting stock market volatility in China [9] – [14] and the references therein are recommended references) and results are far from conclusive.

II. THE STYLIZED FACTS OF CHINESE STOCK MARKET

In order to analyze the existence of a different pattern of volatility when returns are positive and negative (but the same magnitude) in China we have considered the following six indexes: Hang Seng (HANG SENG), China Securities 300 (CHSS300), Dow Jones China Broad MKT Index (DJBDMKT), FTSE Xinhua China 25 Index (XINFTSE), Shanghai SE Composite (CHSCOMP) and Shenzhen SE Composite (CHZCOMP). Most of the literature focuses on CHSCOMP and CHZCOMP, but we prefer to have a global perspective of the situation.

Data have been obtained from the DataStream Data Base. The period analysed, with the exception of CHSS300 and XINFTSE, dates from 01/01/1990 to 30/06/2010.

As usual, returns are calculated as follows:

\[ y_t = 100(\log X_t - \log X_{t-1}) \]

where \( X_t \) is the index value at instant \( t \).

The daily returns series under consideration shows the following main stylised facts:

1) Returns fluctuate around a constant low level close to zero (see Table I).
2) Conditional variance is not constant, since some periods with high variability alternate with periods with low variability. This stylized fact is known as volatility clusters.
3) Returns are uncorrelated, because the autocorrelation (ACF) functions have no significant coefficients.
4) Returns are not independent, because the autocorrelation functions report significant squared return correlations.
5) Returns are not Gaussian, but both skewed and leptokurtic. CHSS300, XINFTSE returns are negatively skewed and HANG SENG, DJBDMKT, CHSCOMP, CHZCOMP positively skewed.

These stylized facts show that the behaviour of volatility presents certain regularities. But our interest is in whether the pattern of volatility is different for positive and negative shocks of the same magnitude. This is the reason why in the next section we propose a new strategy to capture a potential asymmetric pattern in volatility.

III. MODELLING VOLATILITY

We propose four models to explain the dynamics of the volatility of financial daily returns: the well-known AGARCH, TGARCH, ARSV models and the TA-ARSV strategy we propose. These four models have the same mean equation

\[ y_t = \sigma_t e_t \quad e_t \sim iid \ N(\mu, 1) \]  

(2)

where \( y_t \) are the daily returns of the corresponding Stock Exchange (calculated using daily average prices), \( \sigma_t \) represents conditional variance and \( e_t \), innovations, are independent and follow a Gaussian distribution with zero mean and unit variance.

Although the above-mentioned models share a common mean equation, they differ in the specification of conditional variance. In light of both the autocorrelation and partial autocorrelation functions of returns, we aim to explain the dynamics of energy return volatility using AGARCH(1,1), TGARCH(1,1), EGARCH(1,1), ARSV(1) and TA-ARSV(1). The specification of conditional variance in the above-mentioned models is reported in Table II.

### Table I

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean Equation</th>
<th>Variance Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGARCH(1,1)</td>
<td>( \mu_t = \alpha_0 + \alpha_1 y_{t-1} )</td>
<td>( \sigma_t = \alpha_0 + \alpha_1 y_{t-1} + \gamma \sigma_{t-1} + \beta \sigma_{t-1} )</td>
</tr>
<tr>
<td>TGARCH(1,1)</td>
<td>( \mu_t = \alpha_0 + \alpha_1 y_{t-1} )</td>
<td>( \sigma_t = \alpha_0 + \alpha_1 y_{t-1} + \gamma \sigma_{t-1} + \beta \sigma_{t-1} )</td>
</tr>
<tr>
<td>ARSV(1)</td>
<td>( \mu_t = \alpha_0 + \alpha_1 y_{t-1} )</td>
<td>( \sigma_t = \alpha_0 + \alpha_1 y_{t-1} + \gamma \sigma_{t-1} + \beta \sigma_{t-1} )</td>
</tr>
<tr>
<td>TA-ARSV(1)</td>
<td>( \mu_t = \alpha_0 + \alpha_1 y_{t-1} )</td>
<td>( \sigma_t = \alpha_0 + \alpha_1 y_{t-1} + \gamma \sigma_{t-1} + \beta \sigma_{t-1} )</td>
</tr>
</tbody>
</table>

In the AGARCH(1,1) model, \( \gamma \) is the parameter that accounts for the asymmetric behaviour of volatility and the asymmetric effects (if \( \gamma \neq 0 \) and significant) are captured by \( \alpha_1 (y_{t-1} - \gamma)^2 \). The TGARCH strategy incorporates an indicator...
variable, $d_{t+1} = \begin{cases} 1 & \text{if } e_{t+1} < 0 \\ 0 & \text{if } e_{t+1} \geq 0 \end{cases}$, representative of the good news ($e_{t+1} > 0$) and bad news ($e_{t+1} < 0$) in the market. Therefore, in the case of good news, the effect on volatility is $\alpha_i$ and in the case of bad news the impact is $\alpha_i + \gamma$. In these two specifications $\alpha_i$ is positive and both $\alpha_i$ and $\beta$ are nonnegative (to guarantee nonnegative variance). Additionally, the condition $\alpha_i + \beta < 1$ is satisfied to ensure the stationarity of the process $y_t$. In the ARSV(1) model, $\sigma = \sigma \exp(0.5h_t)$, where $h_t$ is log-volatility, $h_t = \log \sigma_t^2$, and $\sigma$, is a positive scale factor in the mean equation to avoid including a constant in the log-volatility equation. $\eta_t$ is a white noise process and it follows a Gaussian distribution with zero mean and variance $\sigma_{\eta}^2$, and both $\varepsilon_t$ and $\eta_t$ are independent, $E(\varepsilon_t, \eta_t) = 0$, $\forall t, s$.

This way of specifying the conditional variance guarantees the variance will be positive.

The TA-ARSV(1) model in the paper is the other strategy proposed to describe the dynamics of volatility in the series of energy product daily returns. The mean equation is defined as in the ARSV(1) model, but conditional variance is specified differently.

The asymmetric pattern of volatility rests on establishing a known threshold that changes the value of the parameters in the model. Therefore, obtaining the TA-ARSV model from an ARSV model implies:

a) Adding two new parameters, $\phi_1$ and $\phi_2$, which measure the effect of the positive and negative returns on volatility, respectively.

b) Adding two indicator variables, $I_{1t}$ and $I_{2t}$, defined as:

$$I_{1t} = \begin{cases} 1 & \text{when the price variation is zero or positive} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{2t} = \begin{cases} 1 & \text{when the price variation is negative} \\ 0 & \text{otherwise} \end{cases}$$

Therefore, the TA-ARSV(1) strategy we propose can be seen as a generalization of the ARSV(1) model that includes two additional parameters, which allow for an asymmetric pattern of volatility. Volatility is defined as an exponential function. Thus, the model is not linear. However, for estimation purposes it can be expressed in linear form by squaring the mean equation and taking logarithms. Following [15], we express the specification in a space state form to obtain the following linear model:

$$\begin{pmatrix} h_{t+1} \\ Y_{t+1} \end{pmatrix} = \delta_t + \Phi h_t + u_t$$

where $Y_t = \log y_t^2$ and $u_t \sim i.d.d. N(0, \Omega_t)$. $\delta_t = \begin{pmatrix} 0 \\ \ln \sigma_{\eta}^2 \end{pmatrix}$.

$$\Phi = \begin{pmatrix} \phi_1 I_{1t} + \phi_2 I_{2t} \\ \sigma_{\eta}^2 \end{pmatrix}, \Omega_t = \begin{pmatrix} \sigma_{\eta}^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}.$$
the TA-ARSV(1) model have been developed by the authors using the Ox 4.1 programming language and SsfPack 2.3.

Focusing on the leverage effect (Table 3), the three competing asymmetric models only agree in the symmetric pattern of volatility of CHZCOMP and the asymmetric pattern of HAN SENG. In fact, HAN SENG is the only index where asymmetric effects are detected with an AGARCH (1, 1) specification. TGARCH models also include CHZCOMP as having an asymmetric pattern of volatility. The T-ARSV strategy we propose also includes XINFSE, DJBDMKT, CHZCOMP and CHSS300 in the group of indexes with an asymmetric pattern of volatility.

In both AGARCH (1, 1) and TGARCH specifications, the sign of \( \phi_1 \) as expected in the literature, \((\phi_1 > \phi_1)\).  

\[ \hat{\phi}_1 > \hat{\phi}_1 \]

**TABLE III**

**ESTIMATION OF VOLATILITY WITH ASYMMETRIC AGARCH(1,1) MODEL**

<table>
<thead>
<tr>
<th>MODEL</th>
<th>( \alpha_0 ) (robust SE)</th>
<th>( \alpha_1 ) (robust SE)</th>
<th>( \beta ) (robust SE)</th>
<th>( \gamma ) (robust SE)</th>
<th>( \alpha_1 + \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HANG</td>
<td>0.0224</td>
<td>0.082</td>
<td>0.8979</td>
<td>0.6223</td>
<td>0.9800</td>
</tr>
<tr>
<td>SENG</td>
<td>(0.009)</td>
<td>(0.007)</td>
<td>(0.014)</td>
<td>(0.2175)</td>
<td></td>
</tr>
<tr>
<td>CHSS300</td>
<td>0.0294</td>
<td>0.0587</td>
<td>0.9365</td>
<td>0.9446</td>
<td>0.9953</td>
</tr>
<tr>
<td>DJBDMKT</td>
<td>0.0176</td>
<td>0.0724</td>
<td>0.9275</td>
<td>0.3261</td>
<td>1</td>
</tr>
<tr>
<td>XINFTSE</td>
<td>0.0283</td>
<td>0.0789</td>
<td>0.9122</td>
<td>0.3470</td>
<td>0.9909</td>
</tr>
<tr>
<td>CHSCOMP</td>
<td>0.1311</td>
<td>0.1910</td>
<td>0.8089</td>
<td>-0.0628</td>
<td>1</td>
</tr>
<tr>
<td>CHZCOMP</td>
<td>0.0429</td>
<td>0.0831</td>
<td>0.914</td>
<td>0.3174</td>
<td>0.9979</td>
</tr>
</tbody>
</table>

**TABLE IV**

**ESTIMATION OF VOLATILITY WITH ASYMMETRIC TGARCH(1,1) MODEL**

<table>
<thead>
<tr>
<th>MODEL</th>
<th>( \alpha_0 ) (robust SE)</th>
<th>( \alpha_1 ) (robust SE)</th>
<th>( \beta ) (robust SE)</th>
<th>( \gamma ) (robust SE)</th>
<th>( \alpha_1 + \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HANG</td>
<td>0.0437</td>
<td>0.0345</td>
<td>0.9052</td>
<td>0.0880</td>
<td>0.939</td>
</tr>
<tr>
<td>SENG</td>
<td>(0.013)</td>
<td>(0.007)</td>
<td>(0.012)</td>
<td>(0.0205)</td>
<td>1</td>
</tr>
<tr>
<td>CHSS300</td>
<td>0.0334</td>
<td>0.0522</td>
<td>0.9345</td>
<td>0.0143</td>
<td>0.866</td>
</tr>
<tr>
<td>DJBDMKT</td>
<td>0.0000</td>
<td>0.3832</td>
<td>0.6167</td>
<td>1.6875</td>
<td>1</td>
</tr>
<tr>
<td>XINFTSE</td>
<td>0.0343</td>
<td>0.0491</td>
<td>0.9161</td>
<td>0.051</td>
<td>0.965</td>
</tr>
<tr>
<td>CHSCOMP</td>
<td>0.1151</td>
<td>0.1843</td>
<td>0.8156</td>
<td>0.023</td>
<td>1</td>
</tr>
<tr>
<td>CHZCOMP</td>
<td>0.0504</td>
<td>0.068</td>
<td>0.9146</td>
<td>0.029</td>
<td>0.983</td>
</tr>
</tbody>
</table>

**TABLE V**

**ESTIMATION OF VOLATILITY WITH TA-ARSV AND ARSV STRATEGIES**

<table>
<thead>
<tr>
<th>MODEL</th>
<th>Estimated parameters</th>
<th>ARSV Estimated Parameters</th>
<th>LR1</th>
</tr>
</thead>
<tbody>
<tr>
<td>HANG</td>
<td>( \sigma ) ( \phi_1 ) ( \phi_1 ) ( \sigma^* ) ( \phi^* ) ( \phi^* ) ( \lambda )</td>
<td>( \sigma ) ( \phi_1 ) ( \phi_1 ) ( \sigma^* ) ( \phi^* ) ( \phi^* ) ( \lambda )</td>
<td></td>
</tr>
<tr>
<td>SENG</td>
<td>(0.08) (0.53) (0.24) 0.028 (0.07) (0.21) 1.6 4.2</td>
<td>(0.08) (0.53) (0.24) 0.028 (0.07) (0.21) 1.6 4.2</td>
<td></td>
</tr>
<tr>
<td>CHSS300</td>
<td>(0.16) (0.59) (0.61) 0.029 (0.26) (0.47) 3.1 5.4</td>
<td>(0.16) (0.59) (0.61) 0.029 (0.26) (0.47) 3.1 5.4</td>
<td></td>
</tr>
<tr>
<td>DJBDMKT</td>
<td>(0.07) (0.25) (0.29) 0.250 (0.23) (0.21) 1.9 3.3</td>
<td>(0.07) (0.25) (0.29) 0.250 (0.23) (0.21) 1.9 3.3</td>
<td></td>
</tr>
<tr>
<td>XINFTSE</td>
<td>(0.11) (0.27) (0.38) 0.200 (0.10) (0.29) 2.4 6.2</td>
<td>(0.11) (0.27) (0.38) 0.200 (0.10) (0.29) 2.4 6.2</td>
<td></td>
</tr>
<tr>
<td>CHSCOMP</td>
<td>(0.08) (0.41) (0.33) 0.225 (0.08) (0.16) 1.9 2.3</td>
<td>(0.08) (0.41) (0.33) 0.225 (0.08) (0.16) 1.9 2.3</td>
<td></td>
</tr>
<tr>
<td>CHZCOMP</td>
<td>(0.06) (0.13) (0.19) 0.259 (0.05) (0.11) 2.3 6.4</td>
<td>(0.06) (0.13) (0.19) 0.259 (0.05) (0.11) 2.3 6.4</td>
<td></td>
</tr>
</tbody>
</table>

1Likelihood Ratio Contrast (LR). \( \lambda = -2(\ln L_1 - \ln L_2) \). Critical value: 3.84 (5%).

The value between parenthesis for the ARSV model is the standard error.
As for persistence, it is high irrespective of the estimated model. However, the use of AGARCH(1,1) or TGARCH(1,1) specifications for estimating persistence in the volatility of DJBDMKT and CHSCOMP returns implies non-stationarity problems. This is another reason for using the proposed T-ARSV model for estimating volatility, since stationarity is guaranteed.

V. CONCLUSION

Reference [14], in view of the different findings of past research on asymmetry in the Chinese stock market and with respect to new market movements, suggested that more studies were needed to shed light on this issue, especially with more recent data. This study includes data until the first half of 2010 and we are ready to answer the two most important questions he considers. Do Chinese stock market volatilities react to shocks asymmetrically as in most mature stock markets in the world? The answer is yes, with the exception of CHSCOMP. And do volatilities react similarly to shocks in the Shanghai and Shenzhen stock markets? The answer is no. According to the T-ARSV model we propose, the pattern of volatility is symmetric in CHSCOMP and asymmetric in CHZCOMP. Note that, according to the AGARCH strategy, in five of the six indexes considered the pattern of volatility is symmetric. The number of indexes with symmetric patterns of volatility drops to four when we use a TGARCH model and to only one (CHSCOMP) if the selected strategy is T-ARSV. Thus, the use of a reliable model for estimating volatility is a core aspect for formulating the optimal strategies for portfolio selection and risk management.

On the other hand, in light of the capability of the T-ARSV strategy to detect an asymmetric pattern of volatility, more research should be done to determine the optimal threshold and to include more than one threshold in the model.

REFERENCES