Traffic Flow Speed Controller Design of Automated Freeway System

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Abstract: In an automated freeway system environment, computers replace artificial driving and control vehicles accompany movement, which can greatly reduce randomness. We propose a traffic flow speed controller which can track desired control rate of speed. The controller can send speed commands to regulate speeds of each section on a freeway. At the same time, the smooth traffic densities of each section will be achieved. Simulations show that this controller can effectively reduce congestion and helps to achieve a smooth traffic flow.

Key-Words: speed controller, automated freeway system, traffic control, nonlinear integrator backstepping, exponential convergence, traffic flow model

1 Introduction
The freeway is not only an important routeway between cities, but also a key carrier of the urban traffic. It is a complex system which contains interaction between humans, vehicles and roads. By regarding this kind of system as a nonlinear system with some certain or random law, and applying the theory and technology in the fields of automatic control, computer and communication, we can transform the system into a controllable running system and maximize the system’s performance.

On-ramp metering[1,2] at freeway entrance ramps is a primary method to regulate the flow of incoming traffic to a freeway. But this approach provides very little feedback information and the traffic system is most of the time open-loop. Therefore, the freeway system is apt to present various types of instabilities.

Traffic flow instabilities such as traffic congestion are often caused by inappropriate speeds and headways that people choose when they operate their vehicles. In an automated highway system (AHS) environment, the driver’s actions are replaced by those of a computer control system that is designed to optimize traffic flow, which will reduce people’s subjective factors as well as greatly decrease the randomicity.

Current research[3,4,5,6,7] on automated freeway system is concerned with providing appropriate feedback control commands to the traffic system on the microscopic level. The goal of the macroscopic control approach is to prevent congestion, or at least avoid its amplification caused by traffic inhomogeneities.

Reference [3] proposes a traffic density controller for automated highway systems. The controller selects the traffic density as controlled variable and achieves tracking the desired density at each section. It drastically reduces congestion and helps to achieve a smooth traffic flow on a congested freeway. But, this controller need more computational efforts to solve the inverse matrix and probably gets a non-unique solution.

This paper proposes an improved controller design method for the controller proposed in reference [3]. This design selects the traffic flow speed as controlled variable, uses the theory of nonlinear integrator backstepping and can guarantee exponential convergence of the traffic flow speed at each section to the desired speed. It indirectly achieves correspondingly homogeneous traffic density, drastically reduces congestion and helps to achieve a smooth traffic flow on a congested freeway.

This paper is organized as follows. In sections 2 and 3, we give the problem statement and a discrete traffic flow model. In section 4, we present the detail of the controller design and analysis. In section 5, we provide the simulation results which show the benefits of our design. Conclusion is given in section 6.

2 Problem Description
Consider a freeway’s a long segment which is divided into \( N \) sections. The length of each section is \( L_i \); \( i = 1,2,\ldots,N \). The initial traffic volume entering section 1 is assumed to be \( q_0(n) \) vehicles per hour at sampling time \( nT \) (\( n = 1,2,\ldots \)) (\( T \) is the time-discretization step size.). The desired space mean speed of traffic flow (traffic flow speed) in section \( i \) at sampling time \( nT \) is assumed to be \( v_{di}(n) \).

The objective of the macroscopic control approach is to select an appropriate control law \( u_i(n) \) which can guarantee exponential convergence of the traffic flow speed at each section to the desired speed, namely, \( v_i(n) \rightarrow v_{di}(n) \) as \( n \rightarrow \infty \).

3 Traffic Flow Model

Papageorgiou et al. \[8,9,10,11\] (1983,1989,1990a) proposed and improved a traffic flow model which has been tested and validated by real traffic data from the Boulevard Peripherique in Paris. However, Karaaslan et al. \[3\] (1990) demonstrated several shortcomings of Papageorgiou’s model and proposed a more realistic one. This model’s description is in the following form:

\[
q_i(n) = \alpha(n) + (1 - \alpha)k_{i+1}(n)v_{i+1}(n) \quad (1)
\]

\[
k_{i}(n + 1) = k_i(n) + \frac{T}{L_i}[q_{i-1}(n) - q_i(n) + \rho_j(n) - s_j(n)] \quad (2)
\]

\[
v_i(n + 1) = v_i(n) + \frac{T}{\tau}[v_{i+1}(n) - v_i(n)] + \frac{T}{L_i}k_{i+1}(n) + k_{i+1}(n + 1)
\]

\[
x_i(n) = \left[v_{i+1}(n) - v_i(n)\right] - \frac{\mu(n)T}{\sigma_i}w_i(n) \quad (3)
\]

Where

\[
V_i(k_i) = v_f(1 - \left(\frac{k_i}{k_{jam}}\right)^p) \quad (4)
\]

\[
\mu(n) = \begin{cases} \frac{\rho}{k_{jam} - k_{i+1}(n) + \alpha} & \text{if } k_{i+1}(n) > k_i(n) \, , \\ \frac{\mu_2}{w_i(n)} = \frac{k_{i+1}(n) - k_i(n)}{k_i(n) + \kappa} & \text{otherwise} \end{cases} \quad (5)
\]

The parameters’ meanings in above formulas are as same as reference \[3\]'s.

In an automated freeway systems, the designed control law \( u_i(n) \) replaces the last term \( \frac{\mu(n)T}{\sigma_i}\) in formula (3), which indicates the controller sends the control command \( u_i(n) \) to the vehicles in section \( i \) at sampling time \( nT \) and \( u_i(n) \) will adjust the \( v_{i+1}(n + 1) \).

Let

\[
f_j(n) = v_j(n) + \frac{T}{\tau}[v_{j+1}(n) - v_j(n)] + \frac{T}{L_j}k_{j+1}(n) + k_{j+1}(n - j)
\]

\[
x_i(n) = \left[v_{i+1}(n) - v_i(n)\right] - \frac{\mu(n)T}{\sigma_i}w_i(n) \quad (7)
\]

On the basis of the above analysis, in this paper, the complete behavior of traffic flow in AHS is governed by the following equation:

\[
q_i(n) = \alpha(n) + (1 - \alpha)k_{i+1}(n)v_{i+1}(n) \quad (8)
\]

\[
k_{i}(n + 1) = k_i(n) + \frac{T}{L_i}[q_{i-1}(n) - q_i(n) + \rho_j(n) - s_j(n)] \quad (9)
\]

\[
v_i(n + 1) = v_i(n) + \frac{T}{\tau}[v_{i+1}(n) - v_i(n)] + \frac{T}{L_i}k_{i+1}(n) + k_{i+1}(n + 1)
\]

\[
V_i(k_i) = v_f(1 - \left(\frac{k_i}{k_{jam}}\right)^p) \quad (11)
\]

The boundary conditions are as same as reference \[3\]'s.

\[
k_{0}(n) = \frac{q_0(n)/v_1(n) - (1 - \alpha)k_1(n)}{\alpha} \quad (12)
\]

\[
v_0(n) = v_1(n) \quad (13)
\]

\[
k_{N+1}(n) = k_N(n) \quad (14)
\]

\[
v_{N+1}(n) = v_N(n) \quad \forall n. \quad (15)
\]

4 Traffic Flow Speed Controller

We propose a macroscopic roadway traffic flow speed controller for AHS. The controller uses nonlinear integrator backstepping \[12,13,14\] to achieve the control law needed to track a desired speed profile. The following general lemma is used in the design and analysis of the proposed roadway controller.

4.1 Lemma

Consider the following discrete-time system:

\[
z(n + 1) = cz(n) + u(n) \quad , \quad z(0) = z_0 \quad (16)
\]

Where \( c \) is a constant and \( |c| < 1 \). Then \( u(n) \to 0 \) exponentially implies \( z(n) \to 0 \) exponentially.

4.1 Speed Controller

The main idea of the controller design is to apply backstepping and use Lemma twice. Selecting the traffic flow speed as controlled variable, designing the control input \( u_i(n) \), we can achieve the desired \( v_{di}(n) \) utilizing formula (10).

The controller design consists of two steps.

Step 1.

Define the track error for section \( i \) as

\[
\xi_i(n) = v_i(n) - v_{di}(n) \quad \forall n = 1,2,\ldots \quad (17)
\]

Then, with (12), it follows that
\[ \xi_i(n+1) = v_i(n+1) - v_{di}(n+1) = f_i(n) - u_i(n) - v_{di}(n+1) = c_\varepsilon \xi_i(n) + \eta_i(n) \]  
\tag{18} \]

Where 
\[ \eta_i(n) = f_i(n) - u_i(n) - v_{di}(n+1) - c_\varepsilon \xi_i(n) . \]  
\tag{19} \]

From Lemma, we have \( \xi_i(n) \to 0 \) as \( n \to \infty \) if \( |c_\varepsilon| < 1 \) and \( \eta_i(n) \to 0 \) as \( n \to \infty \). The goal of the next step is to choose the control input \( u_i(n) \) that guarantees \( \eta_i(n) \to 0 \) as \( n \to \infty \).

Step 2.
From the Lemma and (19), we have 
\[ \eta_i(n) = c_\eta \eta_i(n-1) + \beta_i(n-1) , \]  
\tag{20} \]
\[ \beta_i(n-1) = f_i(n) - u_i(n) - v_{di}(n+1) - c_\varepsilon \xi_i(n) - c_\eta \eta_i(n-1) . \]  
\tag{21} \]

Let 
\[ \xi_i(0) = \xi_i(1) = v_i(1) - v_{di}(1) \]  
\tag{22} \]
then 
\[ \eta_i(0) = \xi_i(1) = c_\varepsilon \xi_i(0) \]  
\tag{23} \]

If 
\[ u_i(n) = f_i(n) - v_{di}(n+1) - c_\varepsilon \xi_i(n) - c_\eta \eta_i(n-1) \]  
\tag{24} \]
Then 
\[ \beta_i(n-1) = 0 \]  
\tag{25} \]
From Lemma, \( \eta_i(n) \to 0 \) as \( n \to \infty \).

Then, the control law is 
\[ u_i(n) = f_i(n) - v_{di}(n+1) - c_\varepsilon \xi_i(n) - c_\eta \eta_i(n-1) . \]  
\tag{26} \]

Compared with the method in reference [3], the proposed controller need not solve inverse matrix, and therefore, reduces algorithm’s complexity, achieves smaller computational efforts, can not get non-unique solution and automatically satisfies the controllability condition.

5 Numerical Simulation
Consider a freeway’s a long segment which is divided into 12 sections. The length of each section is 500m. The parameters of traffic flow model are as follows:

\[ v_f = 93.1 \text{km} \cdot \text{h}^{-1} , \]
\[ k_{\text{jam}} = 110 \text{vehicles} \cdot \text{km}^{-1} \cdot \text{lane}^{-1} , \]
\[ T = 15 / 3600 \text{h} , l = 1.86 , m = 4.05 , \]
\[ \alpha = 0.95 , \kappa = 50 \text{vehicles} \cdot \text{km}^{-1} \cdot \text{lane}^{-1} , \]
\[ \kappa^1 = 55 \text{vehicles} \cdot \text{km}^{-1} \cdot \text{lane}^{-1} , \]
\[ \mu_1 = 12 \text{km}^2 \cdot \text{h}^{-1} , \mu_2 = 6 \text{km}^2 \cdot \text{h}^{-1} , \]
\[ \tau = 20.4 / 3600 \text{h} , \]
\[ \rho = 120 \text{vehicles} \cdot \text{km}^{-1} \cdot \text{lane}^{-1} , \]
Controller’s parameters: \( c_\varepsilon = 0.9 , c_\eta = 0.9 \).

The initial traffic volume entering section 1 is assumed to be 1500 vehicles per hour. The initial density and mean speed of each section are as same as Table 2 of reference [3].

We use the proposed controller to achieve desired traffic flow speeds of 60km/hour respectively. The simulation results shown in Fig.1 and Fig.2 demonstrate that the initial congested conditions are quickly dampened out by the proposed controller and the traffic flow is regulated to achieve the desired traffic flow speeds. At the same time, traffic densities are indirectly influenced and get to a correspondingly homogeneous state.

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6 Conclusion
This paper proposes a design method of traffic flow speed controller which can achieve homogeneous traffic flow speed in an automated freeway system environment. Compared with the former method, the proposed controller need not solve inverse matrix, reduces algorithm’s complexity and automatically satisfies the controllability condition. Improving real-time performance, this controller will help a computer system to control more large-scale automated freeway system. Simulations show that the controller can guarantee exponential convergence of the traffic flow speed at each section to the desired speed and can effectively reduce traffic flow instabilities. It releases the pressure of congestion and helps to achieve a smooth traffic flow on a congested freeway.

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