A probabilistic approach of the wind energy system performance

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Abstract: - The main objective of the paper is to develop a probabilistic model for capacity factor estimation for a wind turbine located in a specific area, model based on the output power distribution of wind turbine. This model was applied for a wind turbine located to a region in the North-East of Romania, the model results being validate with results from Monte-Carlo simulation. Finally, the model was used to evaluate the effects of wind turbine generator parameters, for a given wind speed regime, on the capacity factor values.

Key-Words: - wind energy, wind speed, Weibull distribution, output power distribution, capacity factor

1 Introduction

In the last time, a growing interest in renewable energy resources has been observed. Unlike other renewable energy sources, wind energy has become competitive with conventional power generation sources and therefore application of wind turbine generators has the most growth among other sources. Wind is one of fastest growing energy source and is considered as an important alternative to conventional power generating sources.

The energy production from a wind turbine or a wind park, in a given area, depends by many factors. These factors include the wind speed conditions in the area, and most importantly, the characteristics of the wind turbine generator itself, particularly the cut-in, rated and cut-off wind speed parameters.

The power output of a wind turbine generator does not vary linearly with the wind speed. The output power increases with the wind speed between the cut-in speed and the rated wind speed, after that the power output remains constant at the rated power level [1,2].

A measure of the wind energy potential, for a specific area, is given by the capacity factor value. The capacity factor is defined as the ratio of the expected output power over a period of time to the rated power of wind turbine generator. The capacity factor is not an indicator of efficiency. Efficiency is the ratio of the useful output energy to the input wind energy. All power plants have capacity factors, and they vary depending on resource, technology and purpose. Typical wind power capacity factors are 20-40%, with values at the upper end of the range in particularly favorable areas [4].

In literature are presented various approaches for capacity factor estimation, mostly obtained from simulations techniques based on wind speed data [5,6] and sometimes from computational models [1,7], that needs numerical integration techniques or some approximations.

Having in view the continuously variable character of the primary energy, the probabilistic methods can be proper solutions for a capacity factor evaluation. In the paper, a probabilistic model is developed to evaluate the capacity factor values and to analyze the dependence of the wind turbine generator characteristics. The proposed model is based on probability density function of output power generated by the wind turbine. In order to validate the model, the model was compared with the results of the other model, namely with Monte Carlo technique. The model has the advantage that can be easily implemented in computer programs and require a computing time considerably less than in the case of simulation or numerical methods.

Different types of wind turbines are commercially available on the market. It is therefore desirable to select a wind turbine which is best suited for a particular area in order to obtain a maximum capacity factor. Selection of the optimal wind turbine was discussed in different manner in various papers, among which the maximization of capacity factor [5,6,7]. The choice of turbine involves choosing parameters that lead to maximizing capacity factor. The turbines must be chosen with the parameters that match those of wind profile area. Based on these issues, in the paper, a capacity factor analysis is realized to have a comparison between effects of different parameters of the wind turbine generator on capacity factor values, analyzing the importance and weight of each parameter on the capacity factor values.

2 Wind and output power wind turbine probabilistic characteristics

The output power from a wind turbine depends by the availability of the energy source, namely the wind speed and the power-wind characteristics of the wind turbine generator.

2.1 Probabilistic model of wind speed

The speed of the wind is continuously changing, making it desirable to be described by the probabilistic models. The probability density function (pdf) of wind speed is important in numerous wind energy applications.

A large number of studies have been published in scientific literature related to wind energy, which propose the use of a variety of pdf-s to describe wind speed frequency distributions [9,10]. The conclusion of these studies is that the Weibull distribution of two parameters may be successfully utilized to describe the principle wind speed variation. The Weibull probability density and cumulative distribution function are given by:

$$f_w(v)=\frac{\beta}{\alpha}\left(\frac{v}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{v}{\alpha}\right)^{\beta}\right];\ F_w(v)=1-\exp\left[-\left(\frac{v}{\alpha}\right)^{\beta}\right] \quad (1)$$

The scale parameter α (m/s) and a shape parameter β (dimensionless) of the Weibull distribution can be found using different estimation methods [2,4]. If the wind speed data is available, the Maximum Likelihood method is recommended for estimating the Weibull distribution parameters of wind speed probability distribution function. In many studies, the shape parameter is often chose to 2 and therefore a Rayleigh distribution can be used, with a same accuracy, but with a simpler model.

The wind blows faster at higher altitudes because it is reduced the influence of drag of the surface and lower air viscosity. The increase in velocity with altitude is most dramatic near the surface and is affected by topography, surface roughness and upwind obstacles such as trees or buildings. The most common expression for the variation of wind speed with hub height is the power law having the following logarithmic profile model [8,9].

$$v(z)=v(z_r)\cdot\left(\ln\left(\frac{z}{z_0}\right)\Big/\ln\left(\frac{z_r}{z_0}\right)\right) \quad (2)$$

where $v(z)$ and $v(z_r)$ are the wind speeds at a desired z and registered z_r height, and z_0 is the surface roughness length, a characterization of a ground terrain.

2.2 The output power characteristic of a wind turbine

The output electric power of a wind turbine is a function of the wind speed. The power curve gives a relation between the wind speed and the output electric power, a typical curve of the wind turbine generator is nonlinear related to the wind speed. However, the assumption of the linear characteristic of power with the wind speed, brings a significantly simplifies of calculations, without roughly errors. The power output characteristic can be assumed in such way:

- it start generating power when the speed wind exceeds the minimum wind speed, namely cut-in speed (v_{cut-in});
- the power output increase with the wind speed, when wind varies between cut-in and rated speed wind (v_{rated}), value for that the power achieves the rated power (P_{rated}).
- the rated power of a wind turbine, generally the maximum power output of a generator at highest efficiency, is produced when the speed lies between rated and cut-off wind speed ($v_{cut-off}$).
- cut-off wind speed is the maximum wind speed at which the turbine is allowed to produce power, usually limited by engineering design and safety constraints.

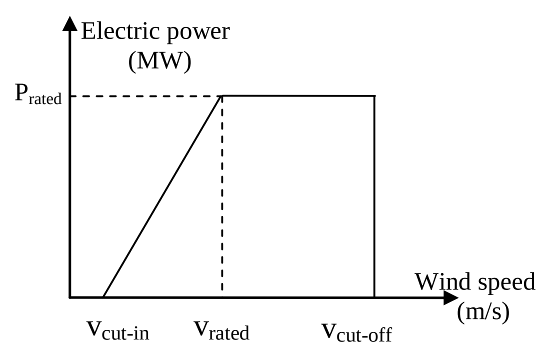


Fig. 1 – The power curve of a wind turbine

This curve comes available from the wind turbine manufacturer or plotted using recorded wind speed and corresponding output power data, a typical curve of the wind turbine generator is shown in the figure 1. Thus, the electric power P_{WT} may be calculated from the wind speed as follows:

$$P_{WT}(v)=\begin{cases}\dfrac{P_{rated}\cdot(v-v_{cut-in})}{(v_{rated}-v_{cut-in})} & for \quad v_{cut-in}<v<v_{rated}\\ P_{rated} & for \quad v_{rated}<v<v_{cut-off}\\ 0 & other\ else\end{cases} \quad (3)$$

2.3 Probabilistic model of wind turbine output power

The distribution function of the generate power of the wind turbine can be obtained using the analytical dependence between the generated power and Weibull distribution, combined in the random variable transformation theorem [14,15].

Lets assume that v is a continuous random variable with cumulative distribution function, $F_w(v)$, and that $P_{WT}=J(v)$ defines a one-to-one transformation from a region of the wind-space, to a region of the electric power-space, with inverse transformation $v=J^{-1}(P_{WT})$, as is presented in figure 2. The cumulative distribution function of output power of wind turbine is given by:

$$\begin{aligned} F_{WT}(P) &= \Pr(P<p)=\Pr(J(X)<p)= \\ &= \Pr(X<J^{-1}(p))=F_W(J^{-1}(p)) \end{aligned} \quad (4)$$

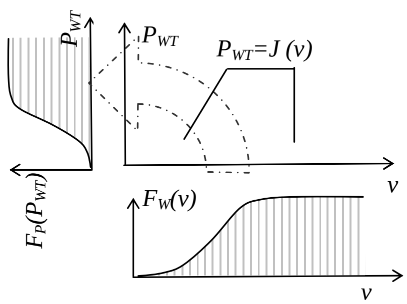


Fig.2 - Transformation of the wind speed variable

The output power probabilistic model for a wind turbine and its practical evaluations were developed and evaluated by authors in [11,12]. The possible values of $F_{WT}(P)$ may be roughly classified in 0, P_{rated} and in the interval that lies between mentioned values, respectively. Each possible value has been evaluated, having in view the probability to achieve that value. The cumulative distribution function of the output power of the wind turbine is:

$$F_{WT}(P)=\begin{cases} 1-\left[F_W(v_{cut\text{-}off})-F_W(v_{cut\text{-}in})\right] & for\ \ P=0 \\ F_{WT}(0)+F_W(W)-F_W(v_{cut\text{-}in}) & for\ \ 0<P<P_{rated} \\ 1 & for\ \ P=P_{rated} \end{cases} \quad (5)$$

The probability density function results from differential of cumulative distribution function:

$$f_{WT}(P)=\begin{cases} \Re 1 & for \quad P=0 \\ \left[\dfrac{(v_{rated}-v_{cut\text{-}in})}{P_{rated}}\right]\cdot f_W(W) & for\ \ 0<P<P_{rated} \\ \Re 2 & for \quad P=P_{rated} \end{cases} \quad (6)$$

where:

- $\Re 1=1-\left[F_W(v_{cut-off})-F_W(v_{cut-in})\right]=F_{WT}(0)$, represents the value of output power cumulative distribution function in the 0 point,
- $\Re 2=F_W(v_{cut-off})-F_W(v_{rated-0})=1-F_{WT}(P_{rated-0})$, represents the increase value of the cumulative distribution function in the P_{rated} value, and
- $W=\left((v_{rated}-v_{cut-in})\cdot\dfrac{P}{P_{rated}}+v_{cut-in}\right)$.

3 Capacity factor evaluations

To illustrate the effect of wind turbine parameters on capacity factor, a probabilistic model for capacity factor evaluation is developed. The capacity factor (*CF*) of wind power is the ratio of expected output power over a period of time to rated power. The expected output power of a wind turbine depends by the output power values and the probability to achieve that power, described by the probability distribution functions of output power. The expected output power (*E(P)*) from a wind turbine generator can be estimated from its power-wind and wind characteristics, being represented by the probability distribution of wind speeds. This is given by [3] as:

$$E(P)=\int_0^{P\,rated} P\cdot f_{WT}(P)dP \quad (7)$$

where P is the output power function of the wind speed and $f_{WT}(P)$ is a probability density function of output power from wind turbine.

Having in view the expression of output power distribution function, from eq.(6), the capacity factor can be obtained by:

$$\begin{aligned} CF &= \frac{1}{P_{rated}}\cdot\left[0\cdot\Re 1+\int_0^{\Pr ated} P\cdot f_{WT}(P)dP+P_{rated}\cdot\Re 2\right]= \\ &= \int_{v_{cut-in}}^{v_{rat}} \frac{v-v_{cut-in}}{v_{rated}-v_{cut-in}}\cdot f_W(v)\cdot dv+\Re 2 \end{aligned} \quad (8)$$

where $f_W(v)$ is a probability density function of wind speed.

Having in view that $dF_W(v)=f_W(v)\cdot dv$, the expression (8) may be rewrite:

$$\begin{aligned} CF &= F_W(v_{cut-off})-\int_{v_{cut-in}}^{v_{rated}} \frac{1-\exp(-(v/\alpha)^\beta)}{v_{rated}-v_{cut-in}}dv= \\ &= \int_{v_{cut-in}}^{v_{rat}} \frac{\exp(-(v/\alpha)^\beta)}{v_{rated}-v_{cut-in}}dv \quad -\exp(-(v_{cut-off}/\alpha)^\beta) \end{aligned} \quad (9)$$

The integration can be accomplished by making the change in variable $y=(v/\alpha)^\beta$, and therefore $dy=\beta(v/\alpha)^{\beta-1}d(v/\alpha)$.

After substitution of integration limits and their reduction to the minimum number of terms, the result is:

$$CF = \frac{\frac{\alpha}{\beta}\Gamma\left(\frac{1}{\beta}\right)}{v_{rated} - v_{cut-in}} \cdot \left[P\left(\left(\frac{v_{rated}}{\alpha} \right)^{\beta}, \frac{1}{\beta} \right) - P\left(\left(\frac{v_{cut-in}}{\alpha} \right)^{\beta}, \frac{1}{\beta} \right) \right] - \exp\left(-(v_{cut-off}/\alpha)^{\beta}\right) \quad (10)$$

where $\Gamma(\ )$ and $P(\ )$ are the gamma and the lower incomplete gamma functions, respectively [13].

The equation (10) express the result of authors research based on laborious calculations and detailed analysis of statistical distributions. This relationship represents an equation which shows the effects of cut-in, rated, and cut-off speeds parameters on the capacity factor value. For a given wind regime, with known α and β parameters, can be selected that values of v_{cut-in}, v_{rated} and $v_{cut-off}$ that maximize the expected output power, and thereby maximize the capacity factor.

4 Model validation and numerical example

For validation of probabilistic model, its results have been compared with results from other model. The Monte Carlo simulation has been used to provide information relates to the average values of the capacity factor. A Matlab program has been developed to validate the model previous developed.

The program has been structured by two main functions. First function is developed based on probabilistic model previous developed and modelled with eq.10. Second function has been developed based on Monte Carlo simulations technique. This technique generates different values of wind speed, in accordance with their Weibull distribution (with the shape and scale parameters estimated from the real data base) and these wind values are used to generate the output power, having in view the characteristics of the wind turbine generator. The expected output power from a wind turbine is the power produced at each wind speed sample, integrated over all possible wind speeds. The required capacity factor value may be observed from the average of all output power values, over a long number of samples. The simulation can be stopped when a specified degree of confidence has been achieved.

The methodology presented in this paper was applied to a real wind turbine and for a real wind speed database, to validate the probabilistic model and to evaluate the influence of main parameters of wind turbine generator to capacity factor. The wind speed database is collected from the north-east area of Romania, for a measurement interval to one hour for the year 2008. The figures 3.a,b present the wind speed collected to wind station height (10m) and adjusted to the hub wind turbine height (80m).

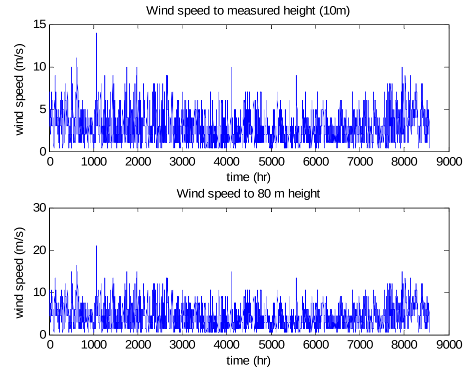


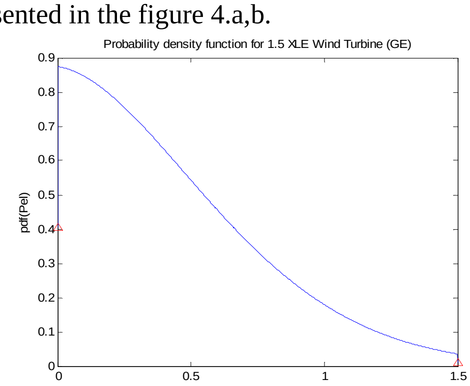
Fig. 3.a,b - Wind speed data base from the north-east area of Romania, for 10m, respectively 80 m height

The parameters of the Weibull distribution have been estimated using the hourly wind data base in 1m/s steps. The probability distribution function used to fitting is a Weibull distribution with scale parameter α=4.82253 m/s and a shape parameter β=1.8656. The wind turbine chose for analyze is a 1.5 XLE GE-Energy, manufactured by GE Energy [16], and their technical specifications are presented in table 1:

Table 1 Technical specifications of wind turbine GE Energy

Turbine Model	Rated power (MW)	Cut-in speed (m/s)	Rated speed (m/s)	Cut-off speed (m/s)	Hub Heights (m)
1.5xle - GE Energy	1.5	3.5	11.5	20	80

The probability density function (pdf) and cumulative distribution function (cdf) of the output power for a 1.5 XLE wind turbine, considering a Weibull distribution with mentioned parameters are presented in the figure 4.a,b.



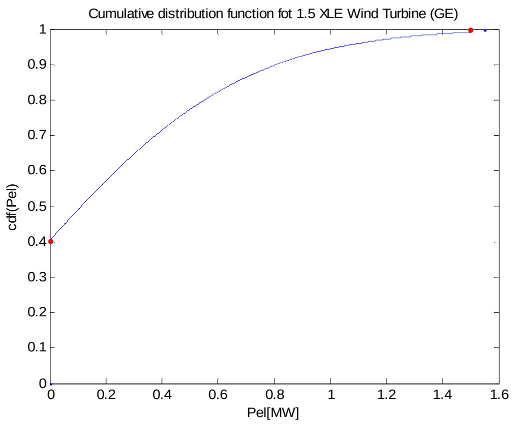


Fig.4a,b-The PDF and CDF of output power wind turbine

In the following is presented an example of capacity factor evaluation, for a 1.5 XLE GE-Energy wind turbine, using the Monte Carlo simulation technique. This technique creates a fluctuating convergence coefficient of variation range for various numbers of samples. Number of simulations results from condition that the deviation of the coefficient of variation of CF ranges to expected value to be under a settled value. Using simulations techniques, for a settled value (0,01%) is obtaining about 10.000 necessary samples, the convergence process of CF being presented in the figure 5.

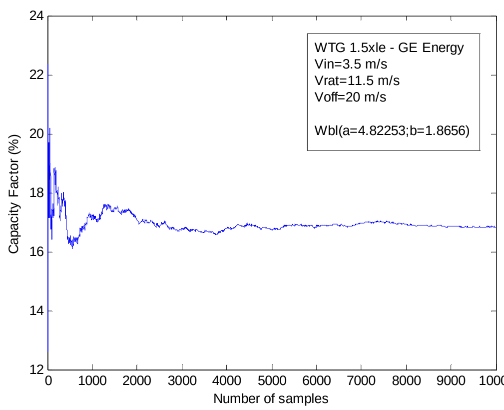


Fig. 5 – CF result from Monte Carlo simulation.

The capacity factor values provided by probabilistic model and sequential Monte Carlo simulation are shown in Table 2. For a better comparison between models, three ranges of speed parameters of wind turbine were considered in capacity factor evaluation. Commercial wind turbines typically have cut-in speeds between 2.5 and 4.5m/s, a rated wind speeds between 10 and 15 m/s and a cut-off speeds between 20 and 25 m/s.

Table 2. Capacity factor values from probabilistic model (PM) and Monte Carlo simulation (MCS)

Capacity Factor v_{rat}=11.5m/s $v_{cut-off}$=20m/s			Capacity Factor v_{cut-in}=3.5m/s $v_{cut-off}$=20m/s			Capacity Factor v_{cut-in}=3.5m/s v_{rat}=11.5m/s		
v_{cutin}	PM	MCS	v_{rat}	PM	MCS	v_{cutoff}	PM	MCS
2.5	22.3301	22.2012	10	20.4575	20.3350	20	16.8492	16.7547
3	19.5020	19.6152	11	17.9203	17.9318	21	16.8492	16.9726
3.5	16.8492	16.6911	12	15.8886	15.6188	22	16.8493	16.7824
4	14.4048	14.8282	13	14.2455	14.1276	23	16.8493	16.7034
4.5	12.1901	11.9975	14	12.8995	12.8572	24	16.8493	16.8065
5	10.2157	10.3042	15	11.7815	11.7896	25	16.8493	16.7920

It can be seen that the results obtained from both methods are very close. The probabilistic method provides comparative results with Monte Carlo simulation, these proving the accuracy of analytic model, developed in eq. (10).

The proposed model may be used to analyze the effects of different cut-in, rated and cut-off wind speeds on the capacity factor value. From table 2, a wind turbine generator, placed in Iasi location, is expected to operate with a maximum capacity factor of 22.33% for a wind turbine generator characterised by a wind speed parameters set to v_{cut-in}=2.5m/s, v_{rat}=11.5m/s and $v_{cut-off}$=20m/s, respectively. A capacity factor of 22.33% from a 1.5kW wind generator means a mean output power of 0.335 kW or an annual power output of 2934.6 kWh.

The effects of the wind turbine generator parameters on the capacity factor are shown in figure 6. In the same coordinate system is shown the dependence of capacity factor for various wind speeds values around of wind turbine generator parameters (cut-in, rated, cut-off speeds).

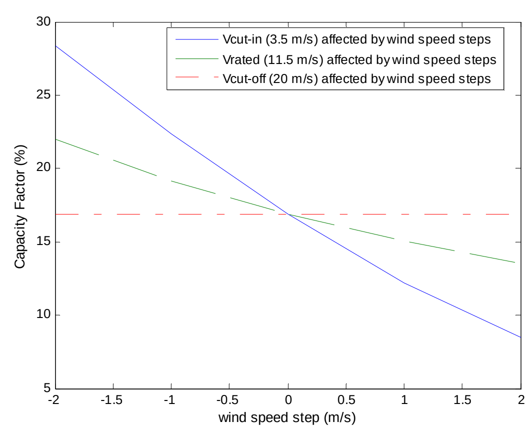


Fig. 6.- Effect of Wind turbine parameters on CF

As can be seen from the figure 6, a certain value of capacity factor can be achieved by action on the two parameters of wind turbine generator. Most important parameter and provide the greatest degree of freedom is cut-in wind speed. It has been shown that the cut-in wind speed has a significant effect on the capacity factor values. The capacity factor values decrease approximately linearly as the cut-in wind speed increases.

The second parameter of wind turbine generator with effect on the capacity factor is the rated wind speed. It has been shown that the rated wind speed has a relatively small effect on the capacity factor values. The rated wind speed growth leads to the capacity factor value decrease, but this effect is less significant than that of the cut-in wind speed

It has been shown that the cut-off wind speed has no effect on the capacity factor values. The cut-off wind speed is a safety parameter and is usually large. For relatively few times the instantaneous wind speed at a particular area will be greater than the cut-off speed. The selection of the cut-off speed parameter is therefore less important than that of the cut-in and the rated wind speed parameters.

4 Conclusion

Integration of wind energy is an important activity in the developing process of the electric power system. Knowing the capacity factor values is a key factor when examining wind energy potential for a wind turbine located in a specific area. The probabilistic methods are the recommended solution for wind integration analysis, since they can take into account the wind power uncertainty.

This paper presents a probabilistic model to evaluate the capacity factor of a wind turbine based on the output power distribution. The results were validated using the Monte Carlo simulations, and the analysis provides that the probabilistic model results are very accurate. The model has the advantage that can be easily implemented in computer programs and require a computing time considerably less than in the case of simulation methods.

The electric energy output of a wind turbine for a specific area depends by many factors. These factors include the wind speed conditions at the area, and the characteristics of the wind turbine generator.
The case studies show that turbine cut-in wind speed has a significant effect on the capacity factor values while the cut-off wind speed has almost no effect. Significant capacity benefits can be obtained by selecting appropriate wind turbine parameters.

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