Leak Detection in Oil and Gas Transmission Pipelines

TEIMURAZ DAVITASHVILI, GIVI GUBELIDZE, INGA SAMKHARADZE
I.Vekua Institute of Applied Mathematics
Iv. Javakhishvili Tbilisi State University,
I.Vekua Institute of Applied Mathematics of Tbilisi State University, 2 University Str., 0186, Tbilisi,
Georgia
GEORGIA
tedavitashvili@gmail.com  http://www.viam.sci.tsu.ge

Abstract: - In the present paper the problem of disclosing of the location and expenditure of accidental gas escape from the main gas pipe-line is studied. For solving the problem a reverse task of non-stationary gas flow in complicated main gas pipe-line is investigated. The analytical expressions which are finding location and expenditure of accidental gas escape in the main gas pipe-line with branches are obtained. For learning the affectivity of the method was created quite general test, the manner of the solution had been known in advance. Calculations have shown efficiency of the suggested method.

Key-Words: - leak detection, pipeline, oil and gas, mathematical modelling.

1 Introduction

As known, the Caspian Sea region has abundant oil and gas reserves but a limited accessible market for the products. Today TRANsport Corridor Europe-Caucasus-Asia (TRACECA) is considered as a corridor, which is complementing other existing routes. EU considers Georgia as a partner in the development of the transport networks between the Black Sea and Central Asia because of its geopolitical position. Georgia participates in 23 from 35 projects elaborated by the regional program TRACECA [5]. Using railways, highways, oil-gas-pipelines, strategic row materials – oil, gas, coal, cotton, and non-ferrous metals are conveyed across Georgia from central Asia and Azerbaijan towards the Black Sea and Mediterranean Sea ports. At Present there are already functioning the following routes of oil transportation via the territory of Georgia: Baku(Azerbaijan)-Tbilisi(Georgia) – Ceyhan(Turkey) (BTC) oil pipeline route; Baku-Supsa(Georgia) oil pipeline; Tengiz deposit(Kazakhstan)-Baku-Batumi(Georgia) route-- fulfils a transportation of oil by railway; Baku-Poti(Georgia) route-- fulfils a transportation of oil by railway; Baku-Kulevi(Georgia) route-- fulfils a transportation of oil by railway; the South Caucasus gas pipeline; Mozdok (Russia)-Tbilisi - Yerevan(Armenia) gas pipeline; Chmi(Russia)-Tbilisi-Saguramo(Georgia) gas pipeline. According to the experience of European transit countries the transit of oil and gas causes great losses regarding the ecological situation thus counteracting the intended political and economical benefits [5]. The leaks caused by damage of pipelines are usually very dangerous. Intensive leaks can stimulate explosions, fires and environment pollution, which can lead to the ecological catastrophe. In this case there can be an enormous economical loss. Although it seems that small leaks are not so dangerous, but in practice it is important to carry out special actions preventing such kind of leaks as well, because the spilt oil can damage the corrosion-resistant cover of pipeline and can cause the corrosion processes [2, 4, 8]. This may outgrow in intensive leaks with above-mentioned results. That is why the determination of damage place in pipelines in time is the significant problem [3,7,8]. There are many scientific articles denoted to the problem of the leak-detection modelling [1-4, 6-13]. The pipeline damage character is determined by the leak intensity. At the same time the smaller the intensity of leak, the harder the discovery of damage is and much harder to find the place of leak.
The methods of leak measurement differ from each other by several features\[2\]: the determination of the minimum leak value; the precision (fixation) of the leak place; the working regimes (constant, periodic or episodic). At the same time leak controlling methods can be dividing into two types: contact method (directly on pipeline) and remote control method (which implies moving controlling device across the pipeline trace) [3-6]. According to the above mentioned classification we can list the following methods: visual method of leak detection – detection of oil and gas on ground surface using painting gases; radioactive isotopes etc.; hydrodynamic method – launching the special material in pipeline; acoustic method and mathematical modelling. From existing methods the mathematical modelling with hydrodynamic method is more acceptable as it is very cheap and reliable and has high sensitive and operative features. It is also significant that in this case the defined leak detection placement is possible to control by remote satellite and the earth surface appliances.

2 Problem Formulation

Suppose that there is a complex (with several outshoots) main gas pipeline. The solution of the problem of disclosing the location of an accidental gas escape from the main pipe-line for gas’ stationary flow using mathematical methods is known not only for simple pipe-line but also for the complicated one. In conditions of stationary usage of gas even flow is stationary in the main pipe-line. But from the moment of accidental gas escape non-stationary process is in progress. After some periods new stationary situation is formed. But it’s important to know (detect) the location and intensity of accidental gas escape at the complicated pipeline even in non-stationary flow, with the purpose of reduction of gas lack loss and in ecological way too. So investigation the problem of disclosing of the location and amount of accidental gas escape from the main gas pipeline is one of the urgent task of the present days.

2.1 Mathematical statement of the problem

Let’s consider a linear mathematical model of non-stationary flow in gas pipeline, which is described by the following parabolic partial differential equation \[3,7,8,9\]:

\[
\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + \sum_{k=1}^{m} M_k \delta(x - x_k) + q \delta(x - x^*) \sigma(t - t_0),
\]

\[0 < x < L, \quad t > 0,\]

(1)
is integrated by the following initial and boundary conditions,

\[u(x, 0) = Q(x), \quad 0 \leq x \leq L,\]

(2)

\[\frac{\partial u}{\partial x} = g_1, \quad \text{when } x = 0, \quad t > 0,\]

(3)

\[\frac{\partial u}{\partial x} = g_2, \quad \text{when } x = L, \quad t > 0,\]

(4)

where \(u = u(x,t)\) is function which describes distribution of the pressure along the pipeline, \(a^2\) is a constant and its value is defined by hydraulic parameters of the pipeline, namely \(a^2 = c^2/2a_1\), \(c\) is acoustic speed in gas, \(a_1 = fV_s/D\), \(f\) is hydraulic friction of gas, \(V_s\) is an average velocity of gas in pipeline, \(D\) is diameter of main gas pipeline, \(m\) is number of branching, \(M_k\) (\(k = 1...m\)) are expenditures of gas in the off-shoots, \(x_k\) (\(k = 1...m\)) are coordinates of off-shoots points, \(t_0\) is a moment of the beginning of the gas escape in the pipeline, \(q\) is expenditure of the accidental gas escape. \(x^*\) is coordinate of the beginning of the escape, \(\delta(\cdot)\) is Dirac delta-function, \(\sigma(\cdot)\) is Heaviside function, \(L\) is length of the main pipeline, \(g_1\) and \(g_2\) are expenditures of gas at the entrance and at the ending of the main gas pipeline, respectively, \(Q(x)\) is function which describes distribution of the initial pressure and it may be represented as:

\[Q(x) = \frac{S - R}{L} x + R,\]

where \(R\) and \(S\) are values of the pressures at the entrance and at the ending of the main gas pipeline.
3 Problem Solution

With the help of Green function it is possible to write (find) solution of the problem (1)-(4) which depends on \( x^* \) and \( q \), as the parameters. For parameters \( x^* \) and \( q \), it is necessary to determine two extra conditions. Now suppose that on the basis of the result of measure at time moment of time \( T \) the following equalities are existed:

\[
U(0, T) = \overline{R}, \quad (5)
\]
\[
U(L, T) = \overline{S}. \quad (6)
\]

It is known [9], that a solution of (1)-(4) is represented by the following formula:

\[
U(x, t) = \int_0^L \int_0^L G(x, \xi, t - \tau) \omega(\xi, \tau) d\xi d\tau, \quad (7)
\]

Where \( G(x, \xi, t - \tau) \) is Green function and it has the following form:

\[
G(x, \xi, t - \tau) = \frac{1}{L} \left[ 1 + 2 \sum_{n=1}^{\infty} \cos \frac{n\pi x}{L} \cos \frac{n\pi \xi}{L} \exp \left( - \frac{n^2 a^2 \pi^2}{L^2} (t - \tau) \right) \right]
\]

where

\[
\omega(x, t) = Q(x) \delta(t) - a^2 \delta(x) g_1 + a^2 \delta(L - x) g_2 + \sum_{k=1}^{m} M_k \delta(x - x_k) + q \delta(x - x^*) \sigma(t - t_0)
\]

Taking into account the last formula (7) will take the following form:

\[
U = \int_0^L G(x, \xi, t) Q(\xi) d\xi - a^2 g_1 \int_0^L G(x, 0, t - \tau) d\tau + a^2 g_2 \int_0^L G(x, L, t - \tau) d\tau + \sum_{k=1}^{m} M_k \int_0^L G(x, x_k, t - \tau) d\tau + q \int_0^L G(x, x^*, t - \tau) d\tau.
\]

If we leave only the first member from the sequence of Green’s function and take into account (8), then (5) and (6) then we will get:

\[
\overline{R} = \frac{2q}{Lp} \left( 1 - \exp(-p(T - t_0)) \right) \cos \left( \frac{\pi x^*}{L} \right)
\]
\[
+ \frac{T - t_0}{L} q - \frac{a^2 g_1}{L} \left[ T + \frac{2}{p} \left( 1 - \exp(-pT) \right) \right]
\]
\[
+ \frac{a^2 g_2}{L} \left[ T - \frac{2}{p} \left( 1 - \exp(-pT) \right) \right] + \frac{R + S}{2} + \frac{4(R - S)}{\pi^2} \exp(-pT) + \frac{2}{Lp} \left( 1 - \exp(-pT) \right) \sum_{k=1}^{m} M_k \cos \left( \frac{\pi x_k}{L} \right) + \frac{T}{L} \sum_{k=1}^{m} M_k , \quad (9)
\]

\[
\overline{S} = -\frac{2q}{Lp} \left( 1 - \exp(-p(T - t_0)) \right) \cos \left( \frac{\pi x^*}{L} \right)
\]
\[
+ \frac{T - t_0}{L} q - \frac{a^2 g_1}{L} \left[ T + \frac{2}{p} \left( \exp(-pT) - 1 \right) \right]
\]
\[
+ \frac{a^2 g_2}{L} \left[ T + \frac{2}{p} \left( 1 - \exp(-pT) \right) \right] + \frac{R + S}{2} - \frac{4(R - S)}{\pi^2} \exp(-pT) - \frac{2}{Lp} \left( 1 - \exp(-pT) \right) \sum_{k=1}^{m} M_k \cos \left( \frac{\pi x_k}{L} \right) + \frac{T}{L} \sum_{k=1}^{m} M_k , \quad (10)
\]

where

\[
p = \frac{a^2 \pi^2}{L^2}.
\]

(9) and (10) represent the system of two equations for defining unknown variables \( x^* \) and \( q \).
\[ q = \frac{L(R + S - R - S) + 2a^2T(g_1 - g_2) - 2T\sum_{i=1}^{m} M_i}{2(T - t_0)} \]

\[ x^* = \frac{L}{\pi} \arccos\left\{ \frac{1}{2q(l - e^{-p(T-t_0)})} [p(T - t_0)q + \frac{Lp}{2} (R + S) - LpS + \frac{4Lp(R - S)}{\pi^2} e^{-pT} + pT \sum_{i=1}^{m} M_i + pa^2T(g_2 - g_1) + 2(a^2g_1 + a^2g_2 - \sum_{i=1}^{m} M_i \cos \left( \frac{\pi k_i}{L} \right) (1 - e^{-pT})] \} \right\} \]

It’s clear that:

\[ 1 - e^{-pT} > 0. \] \hspace{1cm} (12)

Let’s put a sign note:

\[ K = \frac{\overline{R}pT}{2((\alpha + 1)R - 2R)(1 - e^{-pT})}. \]

We will receive \( x^* = \frac{L}{\pi} \arccos[K(1 - \alpha)] \),

Where because of (11), (12) \( K < 0 \).

Let’s consider the following cases:

1) When \( \alpha = 1 \), then \( x^* = L / 2 \) and so place of escape is the middle point, it is like real fact.

2) When \( 0 < \alpha < 1 \), then \( k(1 - \alpha) < 0 \) and \( \frac{\pi}{2} < \arccos[K(1 - \alpha)] < \pi \). So \( x^* > \frac{L}{2} \). Which is reality, in these conditions \( S < \overline{R} \) and the point of escape is between the centre and the right ending.

3) When \( \alpha > 1 \), then \( S > \overline{R} \), \( K(1 - \alpha) > 0 \),

\[ 0 < \arccos[K(1 - \alpha)] < \frac{\pi}{2} \] and that’s why \( x^* < \frac{L}{2} \).

This fact describes reality; in this case the point of escape is between the centre and the left ending.

### 3.1 One general test

Below, as an example is presented one general test for learning the efficacy of the created method. Suppose that the ending of the pipeline are locked, i.e. \( g_1 = g_2 = 0 \), and offshoots are locked too, \( M_j = 0 \), \( j = 1 \ldots m \).

Then pressures at the ending points are equal \( S = R \). We suppose that after time moment \( T \) (from the moment of time gas escape), between the endings of pressure we have the following relationship

\[ \overline{S} = a\overline{R}. \]

Under these conditions, we have,

\[ q = \frac{L}{2T} [(\alpha + 1) \overline{R} - 2R]. \]

Let’s note that in the equation (1) \( q < 0 \). In case of escape from the pipe, this implies that:

\[ (\alpha + 1) \overline{R} - 2R < 0. \] \hspace{1cm} (11)

Under above mentioned conditions we will have:

\[ x^* = \frac{L}{\pi} \arccos\left\{ \frac{p\overline{R}T}{2((\alpha + 1)R - 2R)(1 - e^{-pT})} (1 - \alpha) \right\}. \]

### 3.1.1 Another test

With the purpose of illustrate above mentioned method below is constructed one more test, where solution of the task is known ahead. In the Fig.1 break-line ABC is corresponding to the initial distribution of the pressure in the pipeline (where B is known point of the branching), while break-line ADEF is corresponding to the pressure distribution in the pipeline at the moment of time \( T \) (D is point of branching and E is point of gas escape). Following to the Fig.1 we have \( R = 90; S = 50; \overline{R} = 90; \overline{S} = 24; L = 30 \). Also using Fig.1 it is possible to draw up for the AD, DE, and EF lines corresponding formulas. Namely formula \( u = k_1x + b_1 = \overline{3}x + 90; k_1 = -3 \) is corresponding to AD line,
$u = k_1 x + b_2 = -2x + 80; \quad k_2 = -2$; is corresponding to DE line and formula $u = k_1 x + b_3 = -1.6x + 56; \quad k_3 = -1.6$; is corresponding to DE line. Fig.1 represents one non-stationary test for gas pressure distribution in the horizontal, one branching pipeline.

As it is known gas’ currents in pipeline can be presented by expression $a^2 \frac{\partial u}{\partial x}$, if we assume that $a^2 = 1$, then for the point of branching in the pipeline we get $M_1 = K_2 - K_1 = 1$. For the conditions of the above presented accidental gas escape the solution of the issue was $x^* = 20; \quad q = 0.4$;

Now consider the problem following to Fig.1. Taking into account the boundary conditions $a^2 \frac{\partial u}{\partial x} \bigg|_{x=0} = g_1 = 3; \quad a^2 \frac{\partial u}{\partial x} \bigg|_{x=30} = g_2 = -1.6$; and initial condition $Q(x) = \frac{S - R}{L} x + R$, for our case we have $Q(x) = -\frac{4}{3} x + 90$.

Under those conditions at the time moment $T=300$ ADEF break-lines represent the solution of the following equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + M_j \delta(x - x_j) + q \delta(x - x^*)$$

If we admit that in the last formula $x^*$ and $q$ are unknown variables then using the boundary conditions $u(0, T) = 90$, $u(30, T) = 24$ and above mentioned equalities we will be able to compute approximate values for the $x^*$ and $q$.

$q \approx 0.3$ (instead of 0.4) and $x^* \approx 22$ (instead of 20). The obtained results give us confidence in efficient of the method. Also on the bases of the data obtained from the observations we have calculated values of the coordinate of the gas escape $x^*$ and expenditure of the accidental gas escape $q$ for the main gas pipeline (length 30 km) with three branches, which were distanced from the beginning of the pipeline on 10km, 15km and 20 km, respectively. We have performed two types of experimental calculations. In the first version we had supposed that the first branch was locked and gas expenditures at the second and third outshoot were equal to $-0.3836$(kg/s), $-0.1639$(kg/s), respectively and gas pressure at the ending of main pipeline was equal to $1.88 \cdot 10^6$(N/m$^2$). In the second version we had supposed that the third branch was locked and gas expenditures at the first and second outshoot were equal to $0.1639$(kg/s), $-0.3836$(kg/s), respectively and gas pressure at the ending of main pipeline was equal to $1.5 \cdot 10^6$(N/m$^2$). Other parameters for the both of them were identical. Namely they had the following values: $L=3 \cdot 10^4$m, $g_1 =9$(kg/s), $g_2 =-0.8$(kg/s), $a^2=3 \cdot 10^5$(m$^2$/s), $R=3.12 \cdot 10^6$(N/m$^2$), $S=2 \cdot 10^6$(N/m$^2$), $\overline{R} =2,12 \cdot 10^6$(N/m$^2$), $T=300$(s), $\beta=3.28 \cdot 10^3$(1/s). These data we have obtained from the some experimental observations performed at the Georgian gas pipelines. Some results of calculations are presented in the Table 1.

<table>
<thead>
<tr>
<th>Var.</th>
<th>$\bar{S}$</th>
<th>$x_1/ x_2$</th>
<th>$x^*$</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$1.88 \cdot 10^6$</td>
<td>$1.5 \cdot 10^4$/2.10$^4$</td>
<td>2.13-$10^4$</td>
<td>-3.8016</td>
</tr>
<tr>
<td>II</td>
<td>$1.5 \cdot 10^6$</td>
<td>$10^4$/1.510$^4$</td>
<td>1.43-$10^4$</td>
<td>-6.453</td>
</tr>
</tbody>
</table>

The analysis of the data represented in table 1 give to us to conclude that the values of coordinate $x^*$ and gas expenditure $q$ in the main pipeline are reasonable. Namely in the first version the coordinate $x^*$ of the accidental gas escape in the main pipeline was located on the distance 1300m right side of the third outshoots and in the second version 1430m away.
from the entrance point of the pipeline, which are closer to the experimental results. Also gas expenditures of the accidental gas escape in the main pipeline satisfied general gas balance equation.

4 Conclusion

As foreign experience with pipelines shows, the main reasons of crashes and spillages (and fires as a consequence) are the destruction of pipes as a result of corrosion, defects of welding and natural phenomena (earthquakes, landslides, floods etc). Also terrorist attacks and sabotage may occur. The probability of crashes for oil pipeline transport rises with the age of the pipelines in service, and with the extent of their network. For example, 250 ruptures, which were accompanied by spillages of the transferred products, occurred every year from 1973 to 1983 in the US pipeline network with a total length of about 250,000 km. In West Europe it has been found that 10 – 15 leakages happen every year in a pipeline network of around 16,000 km length, resulting in a loss of 0.001% of transferred products. It is obvious that leak detection in oil and gas pipelines has economical, ecological and sociological meaning. Mathematical modelling with hydrodynamic method is very cheap, reliable and high sensitive method for discovering placement of accidental gas escape. We have created a new method for discovering placement of accidental gas escape at the main pipeline with several gas offshoots in case of gas non-stationary flow. For the verification of the proposed method we created quite general test, the manner of the solution had been known in advance. Comparison of the solutions has shown the affectivity of the following method. The results of calculations on the basis of observation data have shown that the performed simulations were much closer to the results of observation.

References:

Acknowledgment

The research has been funded by the Grant of the Georgian National Science Foundation #GNSF/ST09-614/5-210.