Static Deflections Analysis of Micro-Cantilevers beam under Transverse Loading

Bai Yanping , Hao Yilong

***Abstract*—**Micro-cantilever beams configuration with various size are designed to study static deflections of beams under transverse loading. Linear force-displacement relationship only suit for small displacements. For larger displacements, non-linear terms will appear in the force-displacement relationship. It is difficult to find the closed-form solutions for non-linear force-displacement relationship. The neural network method is a more precise approximation for nonlinear behavior. Because the original data contain noise terms, the data should be processed via wavelets analysis. The simulated results show mean of goodness of fit is 0.9998 for neural networks approach.

***Keywords*—** Artificial neural network, Cantilever beam, Micro-electro-mechanical system, Wavelet transformation.

I. Introduction

Micro Electro Mechanical System (MEMS) is a newly rising interdisciplinary technology. The recent focus of the MEMS world on optical applications of micromachined devices has pushed the field out of surface micromachining technology[1]-[3]. The mechanical design of MEMS is one of the frontiers of mechanical engineering[4]. Micro mechanical cantilever-based sensors have also been widely used for a variety of applications in telecommunications, as well as in biomedicine. Reference [5] gave a micro accelerometer configuration with four suspended symmetric beams and a central proof mass. The PZT thin film on each flexural beam is patterned into two transducer elements, thus eight piezoelectric transducers are arranged on four beams symmetrically to form the sensing devices in the structures, as shown in Fig. 1. Recently, micro accelerometers using piezoelectric thin film have drawn much research interest due to the miniaturization trend of electronic devices, their low cost and their suitability for batch manufacturing. Research has focused on the device's measurement capabilities and structural analysis and modeling to increase the sensitivity of the devices.

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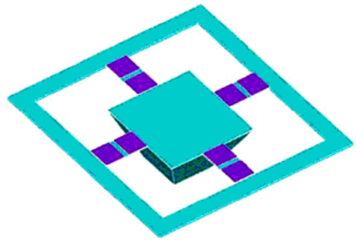

Fig.1 A 3D view of the piezoelectric thin film microaccelerometer

II. Structural Analysis of Cantilever Beam

We consider an array of polysilicon cantilever beams configuration. These beam arrays, which are designed by Fan Wei at the Institute of Microelectronics of Peking University, contain cantilever beams of four, eight, twelve suspended symmetric with various lengths and widths and a central proof mass. An example array is shown Fig. 2. These beams were designed to study static deflections of Micro-cantilever under transverse loading. When the central mass is subjected to vertical vibration (force), the suspending beams can produce bend. One issue critical to understanding beams is understanding how they bend under different loadings. Fig. 3 illustrates the concept being described. The most common method to determine this involves the Euler-Bernoulli equation (1), as in [6]:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} \tag{1}$$

Where x = direction along the neutral axis.
y = direction along the transverse axis.
E = Young's modulus.
I = area moment of inertia,

$$I = \frac{h^3 b}{12} \tag{2}$$

$M(x)$ = the bending moment in the beam, which is usually a function of x.
b = beam width, h = beam thickness, L = beam length, n=beam number.

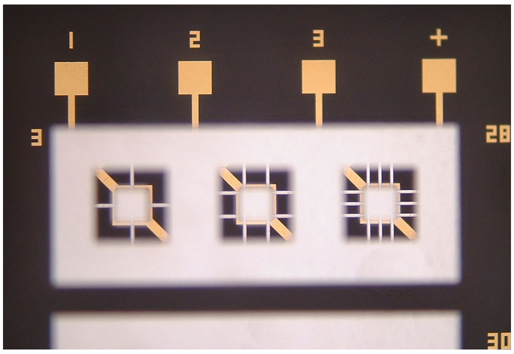

Fig.2 An array of Micro-cantilever beams configuration

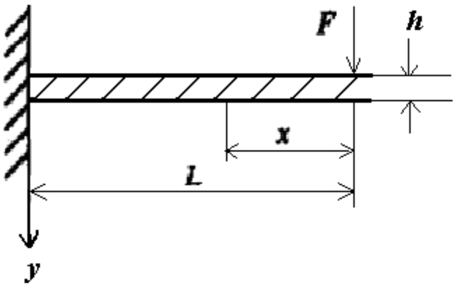

Fig.3 Configuration of a cantilever beam under transverse loading

For a cantilever beam, which is one of the most structural beams in MEMS, with the boundary conditions of $y(L)$=0 and $y'(L)$=0 and a force, F, applied at one end, the equation yields:

$$y(x) = \frac{F}{EI}\left(\frac{x^3}{3} - \frac{L^2 x}{2} + \frac{L^3}{3}\right) \tag{3}$$

Since Equation (3) describes a linear force-deflection relationship at a fixed point x, it is essentially describing a spring reacting to an applied load. This means that it is possible to extract a spring constant, k, from this expression. Evaluating y(x) at a specific point will determine the spring constant. For the specific point x=0, we have following formulation from equation (3):

$$y_{\max} = y(0) = \frac{FL^3}{3EI} = \frac{4FL^3}{Eh^3 b} \tag{4}$$

Rearranging this equation yields:

$$\frac{F}{y(0)} = \frac{Eh^3 b}{4L^3} = k \tag{5}$$

While this expression is useful for predicting displacement under a given load , there are some limitations to it that must be understood. Hooke's law only applies for small displacements. For larger displacements, non-linear terms will appear in the force-displacement equations, as in [6]. The degree to which this equation applies thus depends largely upon how large force is applied to the structure. Often, to simplify the development of devices, designers will construct structures that will operate solely within the linear regime. However it is important to understand that the linear force-displacement equation is only a first order approximation of the actual relationship between force and displacement.

III. Size Influence of Cantilever Beam Specimens

We study an array of Micro-cantilever beams configuration showed in Fig. 2. These beam arrays contain cantilever beams of four, eight, twelve suspended symmetric beams with various lengths and widths and a central proof mass. When the central mass is subjected to vertical force, the suspending beams can produce bend. We investigate the relationship of force and deflection on the mass under different loadings and analysis how large force applied to the structure can get the linear force-displacement relationship. Fig. 4 gives the experiment results.

For a set of experiment data of force and deflection, we approximate the data with small deflection by least-squares linear fitting curves algorithm. Fig. 4 show the relationship of force and deflection is linear when vertical deflections are about less than 800nm. The mean of absolute errors of least-squares linear fitting is less than 5 nm for each data. Component dimensions for each configuration are marked in Fig. 4.

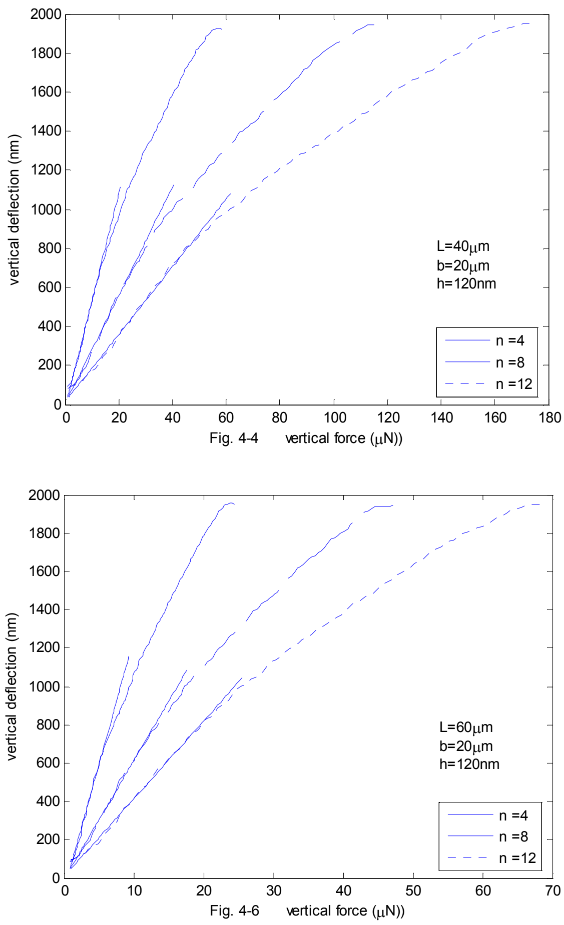

Fig.4 Configuration of a cantilever beam under transverse loading

IV. Modeling of Neural Networks

Because linear force-displacement relationship only suit for small displacements, for larger displacements, non-linear terms will appear in the force-displacement relationship. It is difficult to find the closed-form solutions for non-linear force-displacement relationship. The Artificial Neural Network(ANN) method may be a more precise approximation for nonlinear behavior. With neural network approximation methods, we approximate the unknown nonlinear process with less-restrictive models[7]. The goal is to find an approach or method that forecasts well data generated by often unknown and highly nonlinear processes, with as few parameters as possible[8]. The linear model may be a very imprecise approximation to the real world, but it gives very quick, exact soluteions. The neural network may be a more precise approximation, capturing nonlinear behavior, but it does not have exact solutions. Most of MEMS have nonlinear and complex models. So it is difficult or impossible to detect the faults by traditional methods, which are model-based.

In this paper, A two-layer feed-forward back-propagation network is created with a five-element input and LOGSIG neurons in the hidden layer, as in [9]. Back propagation algorithm is a training algorithm with teachers, whose training procedures are divided into two parts: a forward propagation of information and a backward propagation (BP) of error. The numbers of each layer's neurons in the networks are 5-30-1. The five components of input examples are beam width, beam thickness, beam length, beam number and vertical force. The component of output example is vertical deflection. The network has several constant parameters to be chosen: the learning ratio is lr=0.01, increasing training ratio, lr_inc =1.05, decreasing training ratio, lr_dec =0.7; the momentum factor mc =0.9; max epochs is 1000. The residue is used of the sum of squared errors, goal=0.05.

Because the original data we collected contain noise terms, the original data have obvious false data that can't be used directly. So the data should be processed via wavelets analysis. We decompose the data into discrete wavelet transform coefficients, from which we can then reconstruct our original series. The basic idea is to modify the elements of the discrete wavelet transform coefficients to produce, from which an estimate of the signal can be synthesized. According to the wavelet denoising, wavelet decomposition and wavelet reconstruction, the nonstationarity existed in the data are extracted and separated by wavelet transformation. The ANN model, trained with 45 training datasets, converged with mean square error (MSE) values of 5E-2 . The MSE has been chosen as a performance index for each training method, as shown in Fig. 6. When the network begins to overfit the training data, the error on the validating dataset will typically begin to increase, thus resulting in the termination of the training process.

The simulating results are showed in Fig. 5. The mean of absolute errors of is 6.46nm. The mean of goodness of fit (R2 statistics)The pure linear approximation are compared with the ANN model. The linear models have closed-form solutions for estimation of the regression coefficients. The linear model may be a very imprecise approximation to the real world, but it gives very quick, exact solutions. The neural network may be a more precise approximation, capturing nonlinear behavior, but it does not have exact solutions. The simulating results are showed in table 1. The mean of goodness of fit (R2 statistics) is 0.5589 and 0.9956 for linear and neural network. From table 1, we can see that neural network approximation is superior to the linear . This results show the neural networks approach does better than the linear approach in terms of accuracy and parsimony. This results show the neural networks approach does this good for nonlinear behavior.

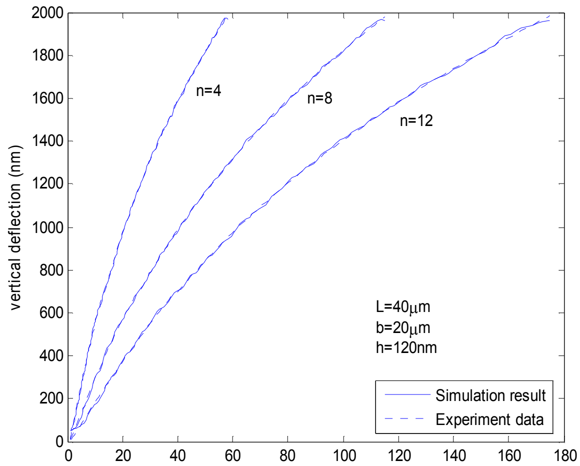

Fig. 5-4 vertical force (μN))

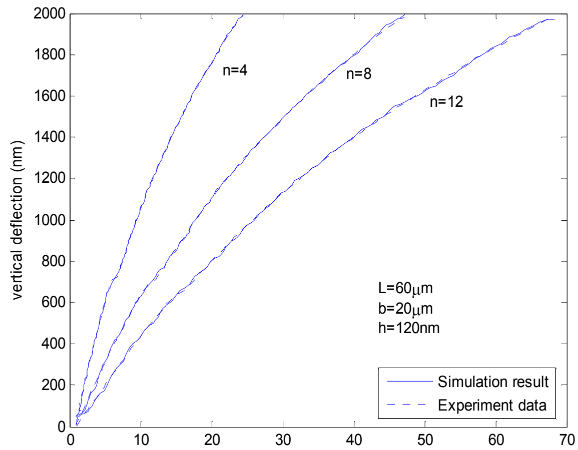

Fig. 5-6 vertical force (μN))

Fig.5 The simulation results of neural network approach

TABLE I
GOODNESS OF FIT (R^2) TESTS FOR THE SET OF DATUM OF APPROXIMATION METHODS

Data	Linear	Neural network
Data11	0.3949	0.9953
Data12	0.4865	0.9976
Data13	0.6119	0.9984
Data21	0.2614	0.9952
Data22	0.3234	0.9941
Data23	0.5245	0.9981
Data31	0.4542	0.9950
Data32	0.4706	0.9939
Data 33	0.4975	0.9957
Data 41	0.7296	0.9991
Data 42	0.7988	0.9999
Data 43	0.7169	0.9999
Data 51	0.5654	0.9998
Data 52	0.6178	0.9999
Data 53	0.6252	0.9999
Data 61	0.5847	0.9994
Data 62	0.5121	0.9998
Data 63	0.5634	0.9998
Data 71	0.6274	0.9996
Data 72	0.7214	0.9999
Data 73	0.7983	0.9999
Data 81	0.4525	0.9997
Data 82	0.5593	0.9998
Data 83	0.5593	0.9998
Data 91	0.4767	0.9992
Data 92	0.4868	0.9998
Data 93	0.6141	0.9998
Data 101	0.3639	0.8593
Data 102	0.5941	0.9987
Data 103	0.7122	0.9999
Data 111	0.7122	0.9999
Data 112	0.5416	0.9990
Data 113	0.4797	0.9995
Data 121	0.5852	0.9908
Data 122	0.3788	0.9993
Data 123	0.5352	0.9994
Data 131	0.4208	0.9989
Data 132	0.4967	0.9997
Data 133	0.6815	0.9997
Data 141	0.5894	0.9994
Data 142	0.5893	0.9997
Data 143	0.5900	0.9999
Data 151	0.5333	0.9995
Data 152	0.6681	0.9999
Data 153	0.6586	0.9999
Average	0.5589	0.9956

V. CONCLUSION

In this paper, we present a theoretical model and neural networks modeling of an array of micro-cantilever beams configuration, which contain 4-24 symmetric beams with various lengths and widths and a central proof mass. These beams were designed to study static deflections of Micro-cantilever under transverse loading and size influence. The experiment results takes into account the effect of device geometry and elastic properties of the specimens, and agrees well with the results obtained by the theoretical model for small deflection. When vertical deflections of specimens are about less than 800nm, the relationship of force and deflection are linear. The mean of absolute errors of least-squares linear fitting are less than 10 nm for each data. This study shows that the vertical deflection increases with increase in the beam length for a fixed beam width and thickness and the vertical deflection decreases with increase in the beam width for a fixed beam length and thickness with 4-24 beams number, respectively. The results of this study can be applied to sensitivity analysis of piezoelectric microaccelerometer.

On the other hand, because linear force-displacement relationship only suit for small displacements, for larger displacements, non-linear terms will appear in the force-displacement relationship. We create a two-layer feed-forward back-propagation network with a five-element input and LOGSIG neurons in the hidden layer. With neural network approximation methods, we approximate the unknown nonlinear process of force-displacement relationship. The pure linear approximation are compared with the ANN model. The simulating results are showed that the mean of goodness of fit (R2 statistics) is 0.5589 and 0.9956 for linear and neural network approximation. The results show neural network approximation is superior to the linear. The neural networks approach does this good for nonlinear behavior.

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