APPLYING OF MATHEMATICS METHODS IN DECISION TAKING PROCESS CONCERNING REPLACEMENT OF MACHINES AND EQUIPMENTS USED IN FLEXIBLE MANUFACTURING CELLS

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Abstract: The main problem of establishing equipment replacement decisions rules under specific conditions is to find decision variables that minimize total incurred costs over a planning horizon. Basically, the rules differ depending on what type of production type is used. For batch production organization methods are suitable criteria built on the principle of economies of scale. Proposed models in this chapter are focused on a multiple machine replacement problem in flexible manufacturing cells that is characterized as a flow shop problem.
Keywords: development, manufacturing, production, technological, cost.

1. Introduction

Historically, development of production processes has passed from production structures in automatic rigid flow lines, efficient for mass and wide-range production, to flexible structures, especially efficient in low and medium-range production. Because manufacturing firms has to be flexible towards new market requirements, flexible production forms are increasingly seen as one of the most important manufacturing concepts. Currently, the trend in flexible manufacturing systems is toward small flexible manufacturing structures, called flexible manufacturing cells (FMC). In this sense, two or more CNC machines are considered a flexible cell and two ore more cells are considered a flexible manufacturing system (Groover, 2001).

2. Problem formulation

Flexible manufacturing cell, in generally, allows the processing of pieces which are different in terms of shape and dimensions, in a determined range. This creates prerequisites for the accomplishment of variable products, under high yield conditions. Considerable savings are made because the degree of usage of production means increases, the fabrication time is shortened, the route and duration of transports are reduced, intermediate storage expenses decrease, the area required for production is reduced, the processing process may be systematised, proper conditions for continuous work are created and direct expenses are reduced. However, the real occurrence of failures during the exploitation stage can markedly modify the FMC performances [Corbaa et al, 1997]. For this reason, the time of interruption of FMC due to repair and maintenance has to be analysed and its influence on the processing cost has to be pondered over. In addition, to ensure that manufacturing process is held to be competitive, upgrade or replacement of equipment due to rapid innovations in technology has to be also considered. In these connections often encountered issues in production planning are: Should this equipment be replaced? If not now, then when? Usually, written equipment replacement policy, in which units are scheduled for replacement based on age and expected condition, contains answers on such questions. In this chapter we wish to show several econometric models that could inspire managers for developing their own specific tools when developing an effective equipment replacement policy.

3. Problem solution

1. The problem statement

Theoretically, any equipment replacement decision would be made based on thorough modeling equipment deterioration and projected remaining life. Practically, approaches to equipment replacement decisions are very different from what is recommended. But it is generally accepted that tools for equipment replacement decision create important element of repair/replacement policy. Such a policy provides guidance to production and economic manager regarding when to replace existing equipment or its part; how to conduct the acquisition process; and what should be done with the equipment being replaced. Then, the main importance of developing of equipment replacement decision models in production planning consist in establishing rules for the replacement of old equipment or its part(s) by new. The main problem of establishing the rules is to find decision variables that minimize total incurred costs over a planning horizon (Dehayem Nodem et al 2009). Basically, the rules differ depending on what type of production type is used. For batch production organization methods are suitable criteria built on the principle of economies of scale, where the large fixed costs of production are depreciation-intensive because of huge capital investments made in high-volume operations and are spread over large production batch sizes in an effort to minimize the total unit costs of owning and operating the manufacturing system (Sullivan, 2002 lean). When solving equipment/parts replacement problem within a flexible manufacturing cell, it is necessary to consider the impacts of the replacement decisions on all of the components of the system. Therefore, possible equipment stoppages due to wrong decision results at least in diminishing capacity or stopping the operations in a manufacturing cell. Accordingly, proposed methods by which we model a multiple machine replacement problem that is characterized as a flow shop problem will tend to overcome the above mentioned problem. A parallel flow shop production concept is consisting of a number of production lines. Jobs in such work shop may be composed of a series of works, each of which consists of several similar machines (Jianhua and Fujimoto, 2003). Firstly we will model a simple case multiple machine replacement problem that is characterized for a parallel flow shop environment, in which there is no technological improvement in equipment. Then we will consider the more complicated case where we take account of technological improvement.

2. Related work

Eequipment replacement, as a specific field of knowledge and practice, has been extensively studied in the professional literature from the third decade of the 20th century (Castro et al 2009). An operations research in this domain is frequently classified based on methods used to solve replacement problems, such as: integer programming (Hritonenko and Yatsenko, 2007), dynamic programming (Flynn and Chung, 2004), simulation techniques (Freeman, 1996), Markov decision problems (Love et al, 2000).

Equipment replacement decision approaches related to this work can be divided into two basic types: parallel and series. The difference between these two categories is that in parallel models, the capacity of the system is simply the sum of the capacities of the individual assets and in the series – flow shop models, the minimum capacity assets in the series defines the capacity of the system (Hartman and Ban, 2002). The literature in parallel models is relatively rich. For reading more on those issues, see, for example, Bean et al. (1994). As regards to series models, there is limited work. For instance, Tanchoco and Leung (1987), Suresh (1991, 1992) and Stinson and Khumawala(1987) present various approaches in which machines operate in series.

Equipment Replacements models also can be grouped into further two classes: simple models versus complex computations. By simple models are meant those with a small number of unknown parameters. Under this group of models belongs, for instance, age-based replacement models, where only a small number of observations of time to failure are required to determine a near-optimal value of the critical age for preventive replacement (Baker and Scarf, 1995). The second group of models with a large number of parameters usually possesses the characteristic of high correlations between parameter estimates; this indicates that the available data is unable to distinguish between equally plausible parameter combinations (Scarf, 1997).

Recent discussions in econometric models are highlighted on the question when to use which capital replacement modeling or economic life modeling. By comparison these two approaches, there are certain evidences that capital replacement modeling methods are more application oriented than second ones (Christer and Scarf, 1994; Scarf and Bouamra, 1995).

Pioneering approaches of equipment replacement have mainly addressed the replacement of single machines or systems. Developed methods stated to multi-machine systems have mostly assumed linear production flows, with limited operational flexibility. A multi-period replacement model for flexible automated systems was developed by Lotfi and Suresh (1994). Their model was formulated as a nonlinear integer programming problem and was intended to serve as an analytical approximation along with closed queuing networks.

Obviously, it would be possible to mention more similar works from different authors on that topic, but this was comprehensively provided, for instance, by Fine and Freund (1990) or Cheevaprawatdomrong and Smith (2003).

3. Models development

The further presented equipment replacement decisions models are econometric-based methods. Econometric methods are in generally concerned with using relevant data for modeling relations between economic and business variables. In these methods one problem is the fact that the selection of variables is somewhat subjective. Their role in decision support for the equipment management and replacement consists of finding the adequate moment to change machine-tool in use or its part(s), based on a specified criterion. In the next subparagraphs several mathematical methods regarding the equipment replacement decision are described.

3.1 The OEC and ORC dependence based method

The problem is to choose an optimal replacement policy such that sum of operating equipment cost (OEC) and replacement equipment cost (REC) per unit time is minimized.

In general, the calculation of OC requires the examination of various influencing parameters. Moreover, there is some difference of opinion about whether the wages of equipment operators should be included in the equipment operating cost (Sears et al 2008). In this method the wages are included to this cost. Because we are looking at all costs from cash flow perspective equipment, thus a replacement cost in our approach deals with today's pricing.

The instrumental assumptions of this method imply that at the beginning of every year, data regarding operation and replacement costs is collected on a certain machine. The data shows an increase in the operation cost, because of the damages in certain components of the machine. Some of these components may be replaced, reducing thus the equipment operating cost. The replacement thereof implies costs with the materials and salaries and, hence, such costs have to be compensated through the savings which may be obtained pursuant to the reduction of operating costs. Thus, we want to determine an optimal replacement policy, able to minimize the sum of operating and replacement expenses during the period between two successive data collections.

Let us consider $c(t)$, the operating cost per time unit at the moment *t*, after replacement and c_r, the cost of a replacement. Then, the relation between the operating cost, replacement cost and time is shown in figure no.1.

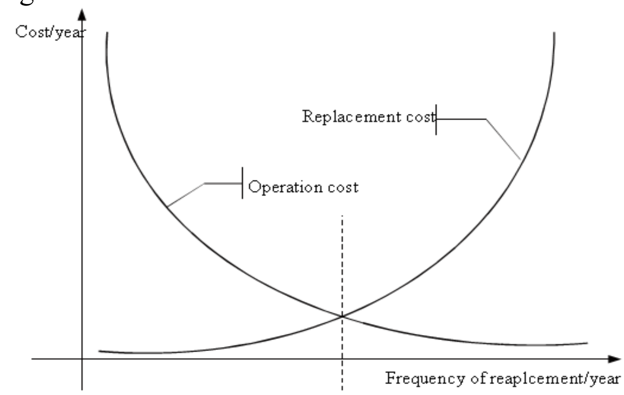

Figure 1 Relation between Operation Cost and Replacement Cost

The replacement policy is presented in figure no. 2, with the following notations: $[0,T]$, the time interval regarding the collection of data on the machine and t_r, intervals when *n* replacements shall occur.

The assumed goal is the determination of the optimal interval between successive replacements, so that the sum of the operating and replacement cost $C(t_r)$ is minimal.

Then, $C(t_r)$ present the replacement cost during the period $[0,T]$, plus operating cost during the period $[0,T]$.

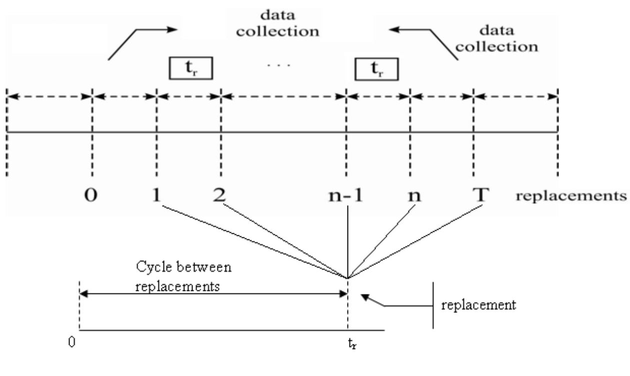

Figure 2 Graphical Representation of the Replacement Policy

Replacement cost by period $[0,T]$ is calculated by

$$\sum C_r = n \cdot C_r$$

The total cost per time unit $C(t_r)$, for the replacement performed at the moment t_r is: $C(t_r)$ i.e. total cost in the interval $(0, t_r)$ related to the length of the interval. The total cost in the interval $(0, t_r)$ is the operating cost plus replacement cost.

$$C(t_r) = \int_0^{t_r} c(t)dt + C_r \approx \left[\frac{\int_0^{t_r} c(t)dt + C_r}{t_r} \right]$$

As we may see, the two different cost calculation methods are similar, because the minimization of $C(t_r)$ is desired, depending on t_r.

Neither of the two methods considers the time required for performing a replacement (figure no. 3)

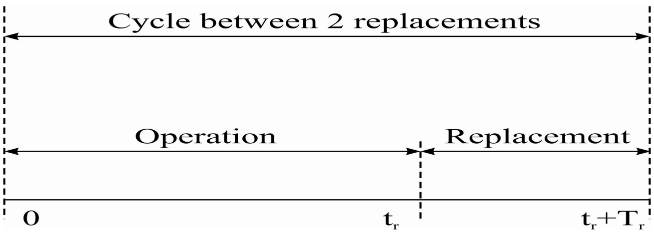

Figure 3 Graphic Representation of the Comparison of Methods

If the time required for performing a replacement is considered, equation 2 becomes:

$$C(t_r) = \frac{\int_0^{t_r} c(t)dt + C_r}{t_r + T_r} \qquad (3)$$

3.2 The method of replacing the equipment at a certain age

We consider that the machine shall be used for a certain number of years. This situation is possible when a certain machine fabricates certain products, according to the production plan. In this case, the goal, in order to minimize the total operating and replacement cost for a fixed time period, consists in the determination of the replacement policy establishing whether: at a certain age of the machine; the latter should be replaced or left to operate continuously.

Let us use *I* to denote the age of the equipment (from the last replacement, with *n* plan periods of proper operation, until the end of the production plan); $c(a)$ to represent the cost of operating the equipment for a plan period, when the equipment has age *a*; *J* to represent the age of the equipment from the moment of the last replacement, having (*n-1*) operating time periods until the end of the production plan; C_r, replacement cost; $C(I, J)$, total cost during the period when the equipment develops from age *I* to age *J*. The proposed goal consists in the determination of a replacement policy, so that the cost of operating and replacing the machine $C_n(i)$, along the following *n* time periods is minimal. When $C_n(i)$ has a minimal value, the smallest cost is defined as $f_n(i)$.

Ten weeks before the end of the production plan, two decisions may be made: continuous use or replacement of the machine. If it is decided that the machine should operate further, the equipment shall have age *4* when a new decision may be made (figure no. 4).

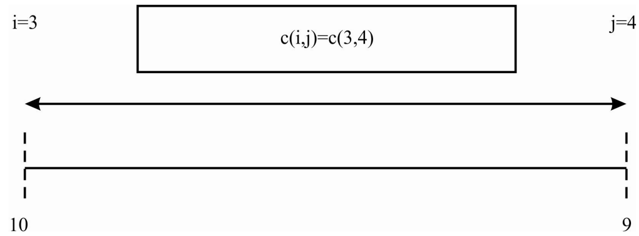

Figure 4 Replacement Policy for the Machine with Age "4"

The total operating cost for the period (10, 9) is:

$$C(3,4) = C(3)$$

If the decision to replace the machine is made, then the total cost for the period (10, 9) shall be:

$$C(3,1) = C_r + C(0)$$

Thus, C_r is the replacement cost, and $c(0)$, the operating cost for a period, when the machine has age *0*.

The optimal replacement policy is graphically presented in figure 5.

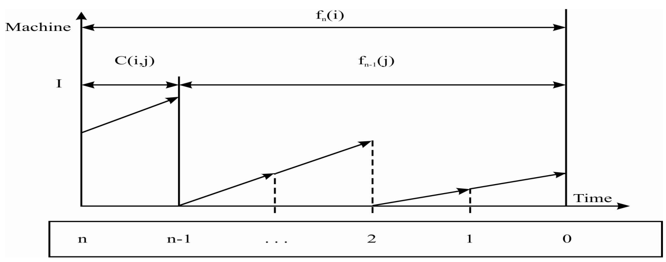

Figure 5 Optimal Replacement Policy

The mathematical model used for identifying this optimal policy has the following form:

Consider: $f_n(i)$, the minimal cost resulting from taking the best decision at the beginning of the period n plus the cost of the best decision taken on the remaining periods (*n-1*); $C(i, j)$ represent the cost resulting from taking the decision at the beginning of the period *n*; $f_{n-1}(j)$, minimal cost by periods (*n-1*) remaining at the moment when the machine has the age *J*.

The cost by the *n* periods is:

$$C(i,j)+f_{n-1}(j),$$

so:

$$f_n(i)=min[C(i,j)+f_{n-1}(j)]$$

with:

- $f_0(i)=0$
- $j=i+1$ or 0.

This equation may be solved by means of dynamic programming methods.

3.3 The method of replacement based on the existence of an equipment in standby

It implies the replacement of assets provided that the manufacturing flow contains a spare asset, and the operating cost increases with the use of the asset existing in production.
In this case, an optimal replacement policy must be determined, combined for the two assets, which shall minimize the total replacement and operating cost for a fixed time period.
The state of the production system at the beginning of a period shall be noted with *I*, where *I* is equivalent to the pair of numbers (*x, y*), where *x* refers to the asset (*A* or *B*) which is generally used, and *y*, to the age of the asset.
Consider: $C_x(y)$, the operating cost for a period; *j*, the state of the production system at the end of a period, where *j* is equivalent to (*x,y*); C_r = replacement cost, considered equal for both assets $C(i,j)$, total cost of the system between the states of the system *i* and *j*. The time required for the replacement of an asset is a period when the replacement decision is made, then the *stand by* asset becomes operative. The proposed goal is the determination of an optimal combined policy for replacement/operation, so that the operating and replacement cost for the following *n* time periods is minimal. Figure no. 6 shows such a policy, where *n=10*, the system is in state *I=(B,2).* At the beginning of period *10*, a decision is made to go on with asset B.

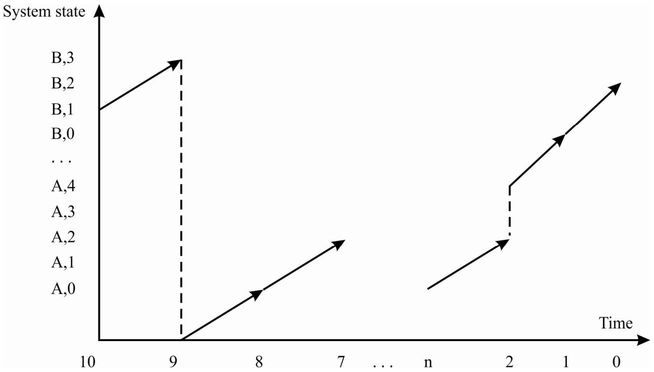

Figure 6 Replacement Policy Given the Existence of the Standby Asset

At the beginning of period *9*, a decision is made to replace asset *B* etc. The total minimal cost for replacement and operation, for the *n* periods is $f_n(i)$.
The cost of the first decision taken at the beginning of the period *n* is $C(i,J)$. At the end of this period, the system is in state *j*, having *(n-1)* operating periods. Then the minimal cost for the remaining period is $f_{n-1}(j)$is:

Total cost = $C(i,j)+f_{n-1}(j)$
and

$$f_n(i)=min[C(i,j)+f_{n-1}(j)].$$

3.4 The method of replacing the equipments based on technological improvement in finished time horizon

Considers that the replacement of an old machine by a new one not always is an exact copy of the old one, but that the latter is better, so that operating and maintenance costs are smaller, efficiency is higher etc. The following model aims at determining the way how the new available machines may be used with a successful purpose, considering that the time period is fixed and finite.
Consider: *n*, the number of operating periods (periods when the machine must operate); $C_{p,i}$, maintenance cost of the current equipment in the period *I* (*i =l,2,...n*); $S_{p,i}$, sales value of the current equipment at the end of the time period; *A*, purchase cost for the new, better equipment; $C_{t,j}$, maintenance cost of the new machine in the *dj* period after installation (*j = l,2,...n*); $S_{t,j}$, sale value of the new equipment at the end of the operating period *j; r* – update factor.
The method aims at determining the value *T* when replacement should be made with the new, better machine (figure no. 7)
T = 0, 1,2,....,n.

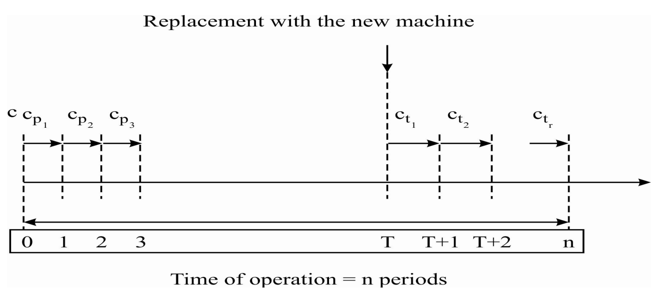

Figure 7 Graphical Calculation of T Value

The total updated cost for the *n* periods when replacement occurs at the end of the *T* period is: $C(T)$, updated cost for the maintenance of the current machine in the period $(0,T)$, plus the updated maintenance cost of the new machine in the period (T,n), plus the updated purchase cost of the new machine, minus the updated sale value of the current equipment at the end of the *T* time period, minus the updated sale value of the new equipment at the end of period *n*:

$$C(T)=\left(C_{p,1}\cdot r^{1}+C_{p,2}\cdot r^{2}+\ldots+C_{p,T}\cdot r^{T}\right)+\left(C_{t,1}\cdot r^{T+1}+C_{t,2}\cdot r^{T+2}+\ldots+C_{t,n-T}\cdot r^{n}\right)+ A\cdot r^{T}-\left(S_{p,T}\cdot r^{T}+C_{t,n-T}\cdot r^{n}\right)$$

Hence,

$$C(T)=\sum_{i=1}^{T}C_{p\,i}\cdot r^{i}+\sum_{j=1}^{n-T}C_{t,j}\cdot r^{T+j}+A\cdot r^{T}-\left(S_{p,T}\cdot r^{T}+C_{t\,n-T}\cdot r^{n}\right)$$

As the sole unknown variable is *T*, the minimisation of $C(T)$ does not raise special further related issues.

Conclusion

Because of the fact that above presented methods are more or less applicable based on specific theoretical preconditions, it might be reasonable to mention wider acceptable view on the given problem. Prior to analysing decisions about equipment replacement in flexible manufacturing cells from practical point of view it is useful recognize two different approaches: deterministic or probabilistic. The decisions to replace a machine in case the fall of a machines follow a probabilistic law are those decisions where the risk is given by the impossibility to exactly determine the moment when such machine falls or the transition moment from proper operating state to non-operating state. Another source of risk is given by the impossibility to determine the state of the equipment when no inspection or other maintenance activity occurs. Let's us consider that there are only two states of the equipment that are always known: a proper operating state or a non-operating state. Then, in order to avoid equipment stoppages in flexible manufacturing cells, the positive replacement decision should come during a proper operating state and accordingly should have a preventive character. In such a way understood preventive replacement for fixed assets implies two conditions:

- *the total replacement cost shall be higher after the fall itself at the moment when the preventive replacement is made;*
- *the replacement of the machine before the fall itself does not affect the chance that the equipment may fall at the following moment*

Therefore, preventive replacement is only justified when the rate of replacement grows. In case the machine is damaged, the specialist in the department should increase the preventive replacement activity. This may be a mistake, as the preventive replacement of machines or their parts is not always justified.

As regards to methods of replacing the equipments based on technological improvement it has to be in addition 'calculated' with known difficulties and problems such as:

- workers' resistance, as they are used to the old machine;
- lack of will to change the work style;
- fear of the unknown, i.e. workers are scared that they will lose their jobs pursuant to the introduction of new technologies or that they won't be able to adjust to the new working requirements;
- lack of support regarding specialized documentation;
- difference of opinions regarding the operation of the equipment.

Therefore, it is not by chance that Dorf and Kusiak (1994) point out that a management attitude toward new manufacturing technology is a crucial factor in determining whether a firm will acquire such technology.

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