Abstract: The main purpose of the presented paper is to give basis usable to increasing the reliability of designed structures. The article deals with an analysis of the influence of initial imperfection on stress and load-carrying capacity of a steel strut under axial compression. The influence of the initial imperfection on the observed output is quantified applying the so-called stochastic sensitivity analysis. Initial imperfections are defined by their statistic characteristics, obtained from experimental research in the Czech Republic. Real experiments are simulated on computers. The statistical evaluation of variables hard to measure, like stress, can be advantageously carried out. Basic types of stochastic sensitivity analysis are listed and the suitability of their application to the analysis of stability problems of steel structures is discussed. Results can be used for the specification of those input variables which should be, in production, checked with increased accuracy. For dominant variables, it is possible to aim at decreasing their variability, which is inevitably given by production inaccuracy. The numerical simulation method Monte Carlo is used in presented studies.

Key-Words: sensitivity analysis, stochastic, reliability, imperfections, stability, buckling, strut, steel.

1 Introduction
Requirements on the load-carrying capacity and serviceability of building structures are generally met with two kinds of uncertainties, i.e.: indeterminateness due to vague or inaccurate definitions of performance requirements and standards, and randomness due to the natural variability of fundamental variables [1].

The first type of uncertainties is the vagueness. A vague (subjective) uncertainty is brought about by human involvement in the process of design, implementation and utilization of structures. Great majority of phenomena in structures uncertainties are constricted by vague expressions, which we work with, using the natural language. Fuzzy sets can be applied to analyze this type of structures uncertainties [2], [3].

The second type of uncertainties the analysis of which is the subject of this article is the randomness. The randomness is present everywhere, where the analyzed phenomenon is accurately defined and the frequency of the occurrence or non-occurrence is examined. Probability theory and mathematical statistics are applied to the analysis of randomness. Both the probability theory and mathematical statistics have a large usage in technical science, especially when concerning the serial manufacturing of products. Hot-rolled steel beams are most frequently used for steel structures. As a result of manufacturing inaccuracies, struts are not perfect but embody initial imperfections [4], [5] with which they differ from the presumptions, assumed in static calculation.

In the presented article, the input imperfections that most influence the observed output are quantified by means of the stochastic sensitivity analysis. When analyzing the ultimate limit state, the output variable is the static ultimate load-carrying capacity and fatigue resistance. The fatigue resistance is determined by the change of stress state, brought about by the change of load [6]. By means of sensitivity analysis, it is relevant to study the influence of initial imperfections on stress state for working load.

2 Basic Methods of Sensitivity Analysis
The sensitivity analysis studies the relationships between information flowing in and out of the model [7]. Stochastic sensitivity analysis methods enable the quantification of the influence of the variability of input variables on the variability of the output variables. Variability is thought of as a statistic characteristic, i.e., it is determined by a dispersion of random realizations around the mean value. Fundamental methods that can be applied are the methods based on observing the correlation coefficient between the input and output. The Spearman's rank-order correlation coefficient can be used for this purpose; see, e.g., [8], or any other method, which can be used to evaluate the stochastic influence between the input and output [9-12]. The advantage of
applying the Spearman’s rank-order correlation coefficient is that it is easily applicable with numerical simulation methods of type Monte Carlo.

Latin Hypercube Sampling method (LHS), which gives, at the same number of simulations, more accurate estimates of statistic characteristics of output variables, will be used in the presented article [13].

3 Computation Model

The load-carrying capacity of an axially compressed strut was identified as the force \( N_{d} \), under which the plasticization of the strut in the most stressed section starts. It is assumed that the plasticization starts when the axial stress exceeds the yield strength \( f_{y} \). Initial curvature and buckling are assumed in the direction perpendicular to the web. Initial imperfection in the form of the sine function was introduced. The maximum stress mid-span of a strut loaded by axial force \( N \) in its elastic state may be determined according to [14] as:

\[
\delta_{x} = \frac{N}{A} + \frac{N \cdot e_{0}}{W_{z}(1 - N / N_{cr})} = f_{y} \quad (1)
\]

where \( A \) is the cross-sectional area of profile IPE 200, \( N \) is axial strut force, \( N_{cr} \) is buckling force, \( W_{z} \) is the section modulus of profile IPE 200 to the \( z \) axis, \( e_{0} \) is the initial amplitude of the sinusoidal curvature of the strut axis.

The analysis subject is the steel strut the system length of which is equal to its critical length, \( L = L_{cr} = 2.1 \text{ m} \). The non-dimensional slenderness according to EUROCODE3 is 1.0.

4 Input Random Variables

Height of section \( h \), thickness of flange \( t_{2} \), width of section \( b \) and thickness of web \( t_{1} \) were assumed as random. It was assumed that the mean values of these variables correspond to their nominal values. Statistical characteristics were therefore introduced with variation coefficients, determined according to the results of experimental research [15-17]. The amplitude of maximum initial deflection \( e_{0} \) was considered as a random variable with log-normal distribution, of mean value corresponding to a third of one thousandths of the strut length. The standard deviation has been designated in such a way that random variable \( e_{0} \) is found in the interval \((0; L/1000)\) approximately with 95 percent probability. The log-normal distribution is “aggressive” as it causes a higher failure probability in reliability calculations than if we used, e.g., the Gaussian distribution with mean value of zero and standard deviation selected so that the random variable \( e_{0} \) is found with a 95 percentual probability in the interval \((-L/1000; L/1000)\). Another significant variable the statistical characteristic of which is subject to long-term control is the yield strength. Statistical characteristics in Tab. 1 were recently evaluated from experimental results obtained from the dominant Czech manufacturer [15], [17].

The last random variable that can influence the observed output is Young’s modulus \( E \). The variability of this quantity takes into account the influence of the abnormalities of physical mechanical properties of the material, especially inhomogeneity. The Gaussian distribution with mean value of 210 GPa and standard deviation 10 GPa was assumed [18].

Table 1: Statistic characteristics of the input variables

<table>
<thead>
<tr>
<th>Random variables</th>
<th>Type of dist.</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional depth</td>
<td>Gauss</td>
<td>200 mm</td>
<td>0.89 mm</td>
</tr>
<tr>
<td>Cross-sectional width</td>
<td>Gauss</td>
<td>100mm</td>
<td>1 mm</td>
</tr>
<tr>
<td>Web thickness</td>
<td>Gauss</td>
<td>5.6mm</td>
<td>0.234 mm</td>
</tr>
<tr>
<td>Flange thickness</td>
<td>Gauss</td>
<td>8.5mm</td>
<td>0.41 mm</td>
</tr>
<tr>
<td>Flange yield strength</td>
<td>Gauss</td>
<td>297.3 MPa</td>
<td>16.8 MPa</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>Gauss</td>
<td>210 GPa</td>
<td>10 GPa</td>
</tr>
<tr>
<td>Initial crookedness</td>
<td>Log-norm.</td>
<td>0.7 mm</td>
<td>0.8 mm</td>
</tr>
</tbody>
</table>

5 Statistical Analysis

With use of computation (1) we gradually determine the statistical characteristics of random variables \( \sigma_{x} \) and the load-carrying capacity \( N_{d} \). Stress will be evaluated for load corresponding to 60 % of the average static load-carrying capacity \( N_{d} \), which can be determined from (1) as 575.83 kN. The standard deviation equals 63.232 kN. The histogram for the random load-carrying capacity obtained for a 10000 simulations is shown in Fig. 3.

Fig. 1 Histogram of load-carrying capacity
According to standard EN1990 design, the load-carrying can be determined as 0.1 percentile. The Gaussian distribution was used to approximate the histogram of random load-carrying capacity. The design load-carrying capacity determined from the Gaussian distribution has a value of 380.42 kN and approximately equals the design load-carrying capacity of 400 kN determined acc. to EUROCODE 3.

The histogram of maximum axial compressive stress mid-span of the strut for working deterministic load of 345.5 MPa is shown in Fig. 2. The random observations are on Fig. 3. The average value was determined from (1) as 149.5 MPa and standard deviation 21.5 MPa. Relatively high values of skewness 4.2209 and kurtosis 37.963 (!) were found. Statistical characteristics were evaluated using 10000 simulations of the LHS method.

6 Sensitivity Analysis Results

The sensitivity analysis was evaluated by means of the Spearman’s correlation coefficient between the input and output. The Spearman rank-order correlation can be defined as:

\[
r_i = 1 - \frac{6 \sum (m_{ji} - n_j)^2}{N(N^2 - 1)}, \quad r_i \in [-1,1]
\]

where \(r_i\) is the order representing the value of random variable \(X_i\) in an ordered sample among \(N\) simulated values applied in the \(j\)th simulation (the order \(m_{ji}\) equals the permutation at LHS), \(n_j\) is the order of an ordered sample of the resulting variable for the \(j\)th run of the simulation process (\(m_{ji} - n_j\) is the difference between the ranks of two samples). If the coefficient \(r_i\) had the value near to 1 or -1, it would suggest a very strong dependence of the output on the input. Opposite to this, the coefficient with its value near to zero will signalise a low influence; see Fig. 4 and Fig. 5.

Results in Figs. 3, 4 give an idea of how the variability of input variables defined in tab. 1 influence the variability of output variables: load-carrying capacity and stress state. A positive value of the correlation coefficient signifies a positive influence of input variable on the output variable, whilst a negative value entails a negative influence.
6.1 Problems and Applications

It is necessary to note that dependence and correlation are not synonymous. The method is based on the assumption that the random variable having effect on the response variable most considerably (either in a positive or a negative sense) will have a correlation coefficient higher than the other variables. However, the dependence and the correlation are not synonyms [5]. A correlation implies dependence but not vice versa. Dependencies described via correlations are useful for practical numerical computations [19].

Nowadays, many sensitivity analysis techniques are available [7]. The coherent concept of sensitivity analysis enabling an analysis of the influence of arbitrary subgroups of input factors (doubles, triples, etc.) on the monitored output was worked out by the Russian mathematician Ilja M. Sobol’ [20], [21]. Examples and description of the applications of simulation methods Monte Carlo, LHS, Important Sampling, Response Surface methodology and other ones on solution of the Sobol’ sensitivity analysis using linear or non-linear computational models are given, e.g., in the book [7]. The variance of the load carrying capacity of a structure is influenced by the variance of input imperfections. One important distinction between the Sobol’ sensitivity and the classical one is that the Sobol’ sensitivity analysis detects interactions of input quantities through the second and higher order terms, while classical sensitivity methods give only derivatives with respect to single variables [7].

It is evident that the implementation of probabilistic methods in the reliability assessment presents a number of problems. Steel structures are composed of thin members and hence the problem of stability can prove to be one of the most important constituents of the safety [22]. With the development of the algorithms optimizing problems of structures, these procedures can contribute to qualitative improvement of the safety and reliability analysis [23], [24]. Models as a constituent of scientific methods present a subject of epistemiological debate [1]. Compared with other construction disciplines [25], statistical data are available for steel structures [26]. A computational model may be understood as the formulation of simplified descriptions of a system derived from the most important realities with regard to the aim and purpose of the model [27]. Mechanisms of failure of thin walled structures are described, e.g., in [28]. The contributions, experiences, development of knowledge, modern approaches and questions dealing with the problems of safety, optimization procedures and reliability studies of contemporary computational mechanic were discussed, e.g., in [29], [30]. In order for the mathematical models applied to the reliability analysis to provide realistic information on the reliability of real structures, it is necessary that input random quantities are obtained from experimental research on ample samples [31-33].

7 Conclusion

Results displayed in Figs. 4 to 5 reflect that the dominant variable for all observed outputs is the amplitude of initial curvature of strut axis $e_0$. The flange thickness also influences the load-carrying capacity. These results confirm and supplement the results of studies [34] of the load-carrying capacities of struts where the load-carrying capacity of strut of non-dimensional slenderness 1.0 was most sensitive to the influence of the initial imperfection.

For shorter struts, yield strength and the flange thickness have dominant influence on the load-carrying capacity. Ultimate limit state is reached in shorter struts when the stress in the most stressed section equals its yield strength [5]. On the contrary, the load-carrying capacity of longer struts is limited by loss of stability. The yield strength is not a dominant variable; variables which prevent buckling are crucial [5]. The load-carrying capacity of very slender struts limitly approaches the critical Euler force. The increase of Young modulus and second moment of the area lead to the increase of Euler force and thereby also of the load-carrying capacity.
Whilst the variability of the second moment of area can be decreased in the manufacturing process by decreasing the variability of geometric characteristics of cross section, Young’s modulus cannot be directly influenced by the manufacturing process.

The variability of this variable takes into account the heterogeneity of the material, i.e., in an attempt of a homogeneous structure, we contribute to increasing the reliability of struts of higher slenderness. For struts of lower slenderness, we should, in production, make efforts to decrease the standard deviation of the yield strength and concurrently, it is possible to request the increase of the mean value of the yield strength.

The histogram in Fig. 1 is compared with two increase of the mean value of the yield strength. For struts of lower slenderness, we should, in production, make efforts to decrease the standard deviation of the yield strength and concurrently, it is possible to request the increase of the mean value of the yield strength.

The design load-carrying capacity was determined acc. to EN1990 in the first case, and acc. to the procedures of the design standards EUROCODE 3 in the second case. The difference between both values is approximately 5%. If the value determined as 0.1 percentile, with respect to the statistical characteristics of given products, sufficiently coincides with the design value given by the design standards EUROCODE 3 (which is taken from the deterministic characteristics), it is possible to make presumptions on the reliability of design and input variables. The next step could be the verification of reliability by probability calculation with inclusion of load action variability. At present, it is actual to direct these studies to the verification of misalignments of reliability acc. to standards EN1990. The calculations of this nature are very valuable; however, they have limited accuracy because there is relatively little statistical information about load (permanent, long-term, short-term) in comparison with, e.g., geometric and material characteristics of mass-produced steel products.

Even though the static load-carrying capacity is one of the most often reliability-based evaluated variables for constructions under the influence of repeated loading, the fatigue damage is a more common phenomenon. Fatigue cracks are initiated and propagated on the structure, and eventually lead to the collapse of the structure. In this respect, it is relevant to analyze the stress state in places in which crack propagation happens most frequently. Sometimes the term fatigue limit state of structures is used. A typical example is, e.g., a bridge construction loaded, e.g., by up to 400 tram crossings daily. Another limit state is the serviceability limit state, which is most frequently assessed by comparing the maximal deflection with the reference value. In reality, deflection is also a random variable which is influenced by a series of imperfections that inevitably have a random character. As it was shown by the statistic analysis, random stress and deflection in mid-span of strut have very high skewness and kurtosis. The cause of these phenomena needs to be analyzed further.

Acknowledgement
The article was elaborated within the framework of projects of GAČR 103/08/0275 and AV ČR IA201720901. In this undertaking, theoretical results gained in the project GAČR 105/10/1156, were also partially exploited.

References:


