Speed Nonlinear Predictive Control of a Series DC Motor for bidirectional operation

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Abstract: -This paper presents a speed nonlinear predictive control scheme for a Series DC Motor. The proposed scheme is capable of overcoming the singularity presented when the current approaches to zero and the inherently non-linear behaviour. The control scheme presented is composed of a speed predictive control and an internal model control. The effectiveness of this control algorithm has been successfully verified through simulations.

Key-Words: - Nonlinear System, Predictive Control, Induction Motor, Speed-sensorless control, Open loop observer.

1 Introduction

In a series dc motor the field circuit is connected in series with the armature circuit, this type of connection produces a torque which is proportional to the square of the current, obtaining a torque that is greater than other type of dc motor. When its speed is reduced by a load, the series motor develops greater torque, because of that, this type of motor is used in traction applications that require high torque at low speed. The series motor is used in a wide variety of power tools and in industry, due to its low cost and high performance. PID controller is the traditional way to control series DC motor. The nonlinear model is linearized about a nominal operating point in order to use linear controllers [1].

The series dc motor is a system with a nonlinear mathematical model [2]. The design of efficient controllers is difficult due to its nonlinear behavior. Nonlinear controllers have been proposed [4-11], using feedback linearization techniques [3]. However, the applicability of the linearization algorithm fails when the system has singular points. This happens when $L_g L_f^{r-1} h(x) \neq 0$ for $x \neq x_0$ but $L_g L_f^{r-1} h(x_0) = 0$. This can be viewed as regions where the relative degree r cannot be defined. Then x_0 is called a singular point of the nonlinear system and the system is known as a nonlinear system with ill-defined relative degree. These systems are also called singular or non-regular nonlinear systems. The series dc motor presents the problem of singularity when the current approaches to zero. To mitigate the problem of singularity, a technique that use feedback linearization on nonlinear singularly perturbed systems is proposed in [12], and the port controlled Hamiltonian Systems Equivalence is used in [13]. However, in all of the approaches mentioned above, the nonlinear model is used to control the velocity only in one direction to avoid the singularity.

This paper presents a speed predictive control scheme for a model of DC motor rotating in two directions. The Nonlinear Continuous Time Generalized Predictive Control (NCGPC) [14, 15] is an alternative nonlinear predictive controller capable of dealing with non-regular or singular nonlinear systems. In [16], the non-regular nonlinear system is treated by using the advantages of the NCGPC. Another of the main advantages of NCGPC control scheme is when $N_u = N_y - r$ .In this case, the NCGPC does not require on-line optimization as the control law is a feedback linearization, thus closed-loop stability is ensured [17]. Therefore, in this paper a nonlinear predictive control scheme for a series cd motor that switches between an approximate tracking control law as in [19] when it is close to singularities and an exact tracking control law when it is away from singularities. The switching control laws are similar to the controller presented in [16, 18].

2 Dynamics of Series DC Motor

In order to develop the controller it is necessary to determine the main characteristics of the Series DC Motor, which can be represented by the model given in [1]

$$L\frac{di}{dt} = -Ri - K_m L_f iw + V$$
$$J\frac{dw}{dt} = K_m L_f i^2 - Dw - \tau_L \quad (1)$$

where

i Armature current
V Input control voltage
w Rotational speed of the motor
τ_L Load Torque
R Equivalent armature resistance
$R = 7.2 ohms$
L Self inductance of the armature
$L = 0.0917 h$
J Equivalent moment of inertia
$J = .0007046 kg - m^2$
D Equivalent viscous friction
$D = .0004$ N-*m-sec/rad*
$K_m L_f$ Motor Constant
$K_T = K_m L_f = 0.1236$ N-*m/Wb-A*

For notational convenience, the variables and constants are renamed as

$$a_1 = \frac{K_m L_f}{J} \quad b_1 = \frac{R}{L}$$
$$a_2 = \frac{D}{J} \quad b_2 = \frac{K_m L_f}{L} \quad x_1 = w, \; x_2 = i \quad (2)$$
$$a_3 = \frac{1}{J}\tau_L \quad b_3 = \frac{1}{L}$$

Then the system described by (1) becomes

$$\dot{x}_1 = a_1 x_2^2 - a_2 x_1 - a_3 \tau_L$$
$$\dot{x}_2 = -b_1 x_2 - b_2 x_1 x_2 + b_3 V \quad (3)$$

The nonlinear model in equation (1) operates only in one direction; therefore, in order to obtain a model of DC motor rotating in two directions some modifications are made to the nonlinear model given by (3).

$$\dot{x}_1 = a_1 x_2^2 sign(x_2) - a_2 x_1 - a_3 \tau_L$$
$$\dot{x}_2 = -b_1 x_2 - b_2 x_1 x_2 sign(x_2) + b_3 V \quad (4)$$

The following figures shows the rotation in two directions, the input voltage varies from 30V, 0V and -30V, it is possible to see that the current and rotor speed respond according to the expected behavior.

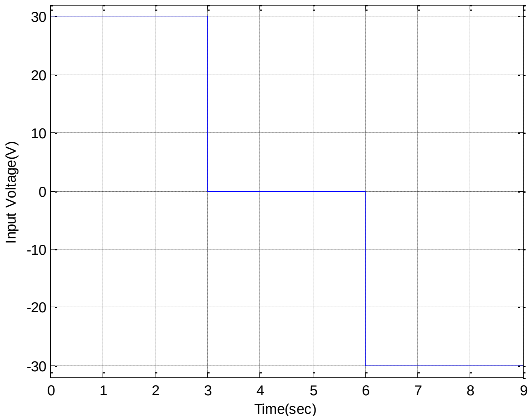

Fig.1 Input Voltage

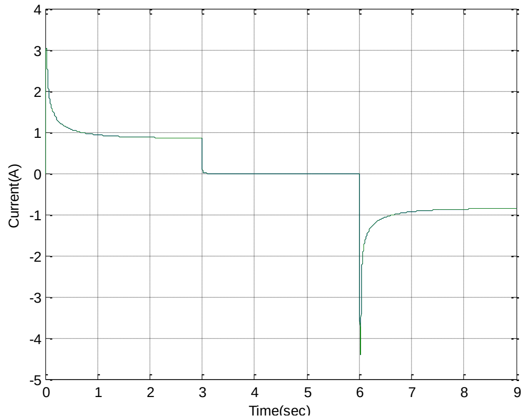

Fig.2 Current

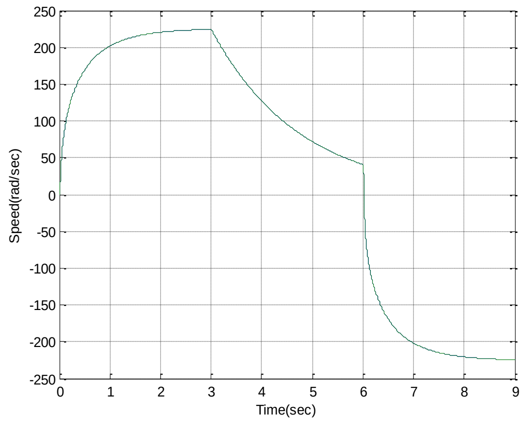

Fig.3 Rotor Speed

3 Predictive Control of the DC Motor

In this section in order to control the non-regular nonlinear systems described by (4), two special cases of NCGPC [11] are developed making use of the advantages of the NCGPC, where the relative degree is well defined or not respectively.

3.1 Case of NCGPC when the relative degree is well defined

In this case, it is assumed that the system described by (1) has stable zero dynamics. The NCGPC is based in taking the derivatives of the output; a process model -open loop observer or internal model depicted in figure (4) - is simulated in parallel in order to get a measurement of the states and from them the derivatives of output. It has a relative degree equal to the process.

$$\dot{x}_m(t) = f_m(x_m) + g_m(x_m)u$$
$$y_m(t) = h_m(x_m) \qquad (5)$$

where f_m, g_m and h_m are differentiable N_y times with respect to each argument, $x_m \in R^n$ is the vector of the state variables, $u \in R$ is the manipulated input and $y \in R$ is the output to be controlled, u and y are the same as the process.

To obtain the predictive controller it was necessary to get the derivatives of output of a process model until $N_y = r$, the relative degree

$$\begin{aligned} y_m^{(1)}(t) &= L_{f_m} h_m(x_m) \\ y_m^{(2)}(t) &= L^2_{f_m} h_m(x_m) \\ &\vdots \\ y_m^{(r)}(t) &= L^r_{f_m} h_m(x_m) + L_{g_m} L^{r-1}_{f_m} h_m(x_m) u(t) \end{aligned} \qquad (6)$$

The output prediction is approximated for a Maclaurin series expansion of the system output as follows.

$$y^*(t,T) = y(t) + \dot{y}_m(t)T + y_m^{(2)}(t)\frac{T^2}{2!} + \cdots + y_m^{(N_y)}(t)\frac{T^{N_y}}{N_y!} \qquad (7)$$

and the reference trajectory is the output of the reference model represented by

$$\dot{x}_r = A_r x_r + B_r w \qquad (8)$$
$$y_r = C_r x_r$$

where $x_r \in R^{n_r}, A_r \in R^{n_r \times n_r}, B_r \in R^{n_r \times 1}, C_r \in R^{1 \times n_r}, w \in R$. In order to define the predicted output of the reference trajectory, a truncated Taylor series is used, obtaining:

$$y_r(t,T) = y_r(t) + \dot{y}_r(t)T + y_r^{(2)}(t)\frac{T^2}{2!} + \cdots + y_r^{(N_y)}(t)\frac{T^{N_y}}{N_y!} \qquad (9)$$

The derivatives are obtained from the reference model simulation.

The cost function is given by

$$J(u^*(t,0)) = [y_r^*(t,T) - y^*(t,T)]^2 \qquad (10)$$

and the minimization results in the following control law:

$$u(t) = \frac{(y_r - y) + (\dot{y}_r - L_{f_m} h_m)T + \cdots + (y_r^{(r)} - L^r_{f_m} h_m)\frac{T^r}{r!}}{L_{g_m} L^{r-1}_{f_m} h_m \frac{T^r}{r!}}$$

(11)

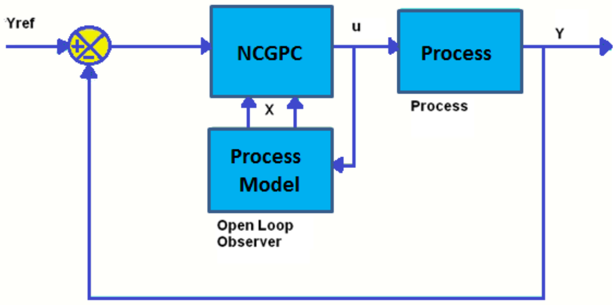

Fig.4 Control Schedule

3.1.1 NCGPC for the Series DC Motor Model when the relative degree is well defined

The bidirectional model of DC Motor given by equation (4) is rewritten

$$\dot{x}_{1m} = a_{1m} x_{2m}^{\ 2} sign(x_{2m}) - a_{2m} x_{1m} - a_{3m}\tau_{Lm}$$
$$\dot{x}_{2m} = -b_{1m} x_{2m} - b_{2m} x_{1m} x_{2m} sign(x_{2m}) + b_{3m} V$$

This model is simulated in parallel in order to get the output derivatives

$$\begin{aligned} y_m &= x_{1m} \\ y_m^{(1)} &= a_{1m} x_{2m}^{\ 2} sign(x_{2m}) - a_2 x_{1m} - a_3 \tau_L \\ y_m^{(2)} &= -2a_{1m} b_{1m} x_{2m}^{\ 2} sign(x_{2m}) - 2a_{1m} b_{2m} x_{2m}^{\ 2} x_{1m} \\ &\quad + a_{2m} a_{3m} \tau_{Lm} - a_{1m} a_{2m} x_{2m}^{\ 2} sign(x_{2m}) + a_{2m}^{\ 2} x_{1m} \\ &\quad + 2a_{1m} b_{3m} x_{2m} V sign(x_{2m}) \end{aligned} \qquad (12)$$

The reference model is represented by

$$\begin{aligned} \dot{y}_t &= y_{t1} \\ \dot{y}_{t1} &= -49 y_t - 14 y_{t1} + 49z \end{aligned} \qquad (13)$$

Thus, taking T=1 the NCGPC is given by

$$u(t) = \frac{(y_t - y) + (\dot{y}_t - \dot{y}_m)T + (y_t^{(2)} - L^2_{f_m} h_m)\frac{T^2}{2!}}{L_{g_m} L_{f_m} h_m \frac{T^2}{2!}} \qquad (14)$$

where

$$\begin{aligned} L^2_{f_m} h_m &= -2a_{1m} b_{1m} x_{2m}^{\ 2} sign(x_{2m}) - 2a_{1m} b_{2m} x_{2m}^{\ 2} x_{1m} \\ &\quad + a_{2m} a_{3m} \tau_{Lm} - a_{1m} a_{2m} x_{2m}^{\ 2} sign(x_{2m}) + a_{2m}^{\ 2} x_{1m} \end{aligned}$$

$$L_{g_m} L_{f_m} h_m = 2a_{1m} b_{3m} x_{2m} sign(x_{2m}) \qquad (15)$$

The relative degree is two, supposing the current is different of zero, in order to have

ISBN: 978-1-61804-051-0

$$L_{g_m} L_{f_m} h_m(x_{2m}) = 2a_{1m} b_{3m} x_{2m} sign(x_{2m}) \neq 0 \qquad (16)$$

3.2 Case of NCGPC when relative degree is not well defined

The CD motor has a relative degree not well defined when $L_{g_m} L_{f_m} h_m(x_{2m}) = 2a_{1m} b_{3m} x_{2m} sign(x_{2m}) \neq 0$ for $x_{2m} \neq 0$ but $L_{g_m} L_{f_m} h_m(0) = 0$. Then the Series DC motor has a singular point in $x_{2m} = 0$. That is, the exact linearization fails when the nonlinear system has a relative degree not well defined. Nevertheless, the NCGPC can deal with this kind of systems by making use of one of advantage of the NCGPC: extending the prediction of the output to values greater that the relative degree of the system. In this case, $N_y = 3$

$$y_m = x_{1m}$$
$$y_m^{(1)} = a_{1m} \underbrace{x_{2m}}_{0}{}^2 sign(x_{2m}) - a_2 x_{1m} - a_3 \tau_{Lm}$$
$$\begin{aligned} y_m^{(2)} &= -2a_{1m} b_{1m} \underbrace{x_{2m}}_{0}{}^2 sign(x_{2m}) - 2a_{1m} b_{2m} \underbrace{x_{2m}}_{0}{}^2 x_{1m} \\ &\quad + a_{2m} a_{3m} \tau_{Lm} - a_{1m} a_{2m} \underbrace{x_{2m}}_{0}{}^2 sign(x_{2m}) + a_{2m}{}^2 x_{1m} \\ &\quad + 2a_{1m} b_{3m} \underbrace{x_{2m}}_{0} sign(x_{2m}) V \end{aligned}$$
$$\begin{aligned} y_m^{(3)} &= -4a_{1m} b_{1m} \underbrace{x_{2m}}_{0} \dot{x}_{2m} sign(x_{2m}) - 4a_1 b_2 x_{1m} \underbrace{x_{2m}}_{0} \dot{x}_{2m} \\ &\quad - 2a_{1m} b_{2m} \underbrace{x_{2m}}_{0}{}^2 \dot{x}_{1m} - 2a_{1m} a_{2m} \underbrace{x_{2m}}_{0} \dot{x}_{2m} sign(x_{2m}) + a_{2m}{}^2 \dot{x}_{1m} \\ &\quad + 2a_{1m} b_{3m} \dot{x}_{2m} V sign(x_{2m}) + 2a_{1m} b_{3m} x_{2m} \dot{V} sign(x_{2m}) \end{aligned} \qquad (17)$$

Around the neighborhood of the singular *point* $x_{2m} \to 0$, similar approximations are made as in [19].

$$y_m^{(3)} = a_{2m}{}^2 \dot{x}_{1m} + 2a_{1m} b_{3m} \dot{x}_{2m} V sign(x_{2m})$$
$$\begin{aligned} y^{(3)} &= a_{2m}{}^2 \left[a_{1m} \underbrace{x_{2m}}_{0}{}^2 sign(x_{2m}) - a_{2m} x_{1m} - a_{3m} \tau_{Lm} \right] \\ &\quad + 2a_{1m} b_{3m} \left[-b_{1m} \underbrace{x_{2m}}_{0} - b_{2m} x_{1m} \underbrace{x_{2m}}_{0} sign(x_{2m}) + b_{3m} V \right] V sign(x_{2m}) \end{aligned}$$

The output derivatives becomes:

$$y = x_1$$
$$y_m^{(1)} = -a_{2m} x_{1m} - a_{3m} \tau_{Lm}$$
$$y_m^{(2)} = a_{2m}{}^2 x_{1m} + a_{2m} a_{3m} \tau_{Lm}$$
$$y_m^{(3)} = -a_{2m}{}^3 x_{1m} - a_{2m}{}^2 a_{3m} \tau_{Lm} + 2a_{1m} b_{3m}{}^2 V^2 sign(x_{2m}) \qquad (18)$$

The reference model is represented by:

$$\dot{y}_r = y_{r1}$$
$$\dot{y}_{r1} = y_{r2}$$
$$\dot{y}_{r2} = -1000 y_r - 300 y_{r1} - 30 y_{r2} + 1000 z \qquad (19)$$

The equation (18) can be used to approximate the series DC motor around the neighborhood of the singular point.

The control law $u = V^2$ is given by

$$u(t) = \frac{(y_r - y) + (\dot{y}_r - \dot{y}_m)T + (y_r^{(2)} - y_m^{(2)})\frac{T^2}{2!} + (y_r^{(3)} - L_{f_m}^3 h_m)\frac{T^3}{3!}}{L_{g_m} L_{f_m}^2 h_m \frac{T^3}{3!}} \qquad (20)$$

where

$$L_{f_m}^3 h_m = -a_{2m}{}^2 a_{3m} \tau_{Lm} - a_{2m}{}^3 x_{1m}$$
$$L_{g_m} L_{f_m}^2 h_m = 2a_{1m} b_{3m}{}^2 sign(x_{2m})$$

It is possible to see that the control law does not have singular points as its denominator is a constant.

3.3 The NCGPC by using switching control

The switching control scheme is summarized as follows:

- $|x_{2m}| > 0.01$. When the system is relatively far from the singular point, the NCGPC becomes an exact linearization as in [3], the predictive output is extended until $N_y = 2$, the controller is given by equation (14)

$$u(t) = \frac{(y_t - y) + (\dot{y}_t - \dot{y}_m)T + (y_t^{(2)} - L_{f_m}^2 h_m)\frac{T^2}{2!}}{L_{g_m} L_{f_m} h_m \frac{T^2}{2!}} \qquad (14)$$

- $|x_{2m}| < 0.01$.When the system is very close to any singular point the predictive output is extended until $N_y = 3$, around the neighborhood of the singular point, $x_{2m} \to 0$, thus, NCGPC is given by equation (20)

$$u(t) = \frac{(y_r - y) + (\dot{y}_r - \dot{y}_m)T + (y_r^{(2)} - y_m^{(2)})\frac{T^2}{2!} + (y_r^{(3)} - L_{f_m}^3 h_m)\frac{T^3}{3!}}{L_{g_m} L_{f_m}^2 h_m \frac{T^3}{3!}}$$

4 Simulation results

Figures (4), (5) and (6) show the effectiveness of the NCGPC control scheme for ill-defined relative degree. The switching control is using two reference models given by equation (13) and (19). In figure (4) the rotor speed which follows adequately the reference trajectory is shown. It must be noted that the rotor speed changes directions meaning the current takes the values $x_{2m} = 0$ and around its neighborhood; that is, when the system is close or in its singular point. Finally, the simulations include an external load indicating that the control system is robust to external perturbations.

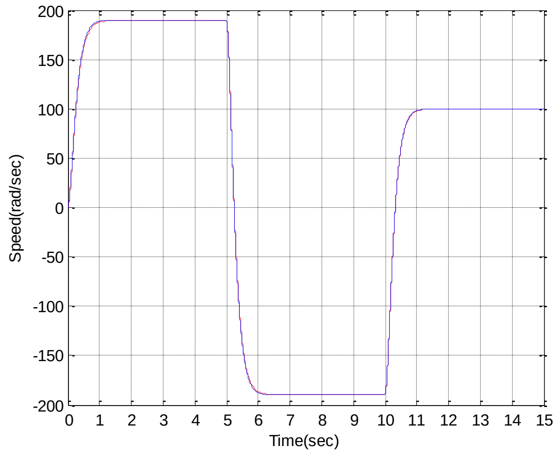

Fig.5 Rotor Speed

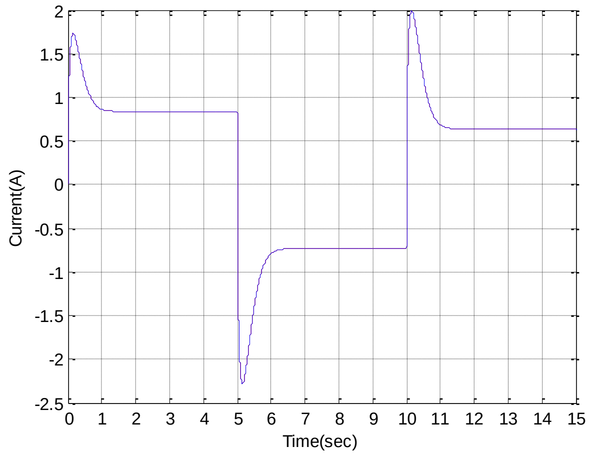

Fig.6 Current

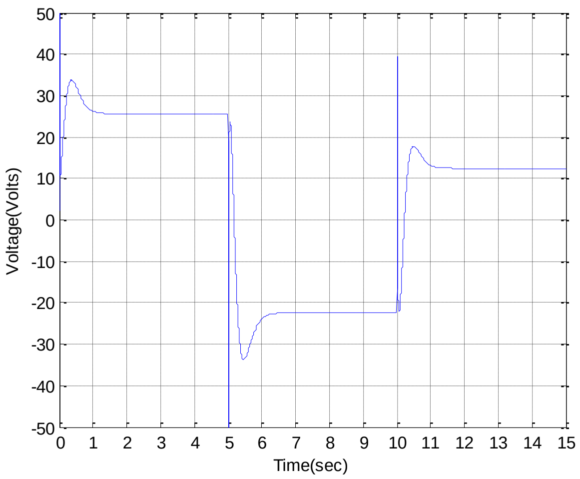

Fig.7 Voltage

5 Conclusions

A switched **NCGPC** in conjunction with an internal model controller for Series DC Motor was designed. The proposed approach is capable of overcoming the singularity of this kind of motors when its armature current approaches to zero by extending the predictive horizon at values larger that the relative degree of the system. Also, the Series DC motor model was modified in such a way that now it is possible to analyze Series DC Motors when its rotor speed moves in both directions. This condition was neglected in previous works in order to avoid the singularity of the model. Also, despite the drastic changes of the reference signal the control system present an excellent performance. Moreover, the armature current is well within realistic values as its maximum value is 2*amp*.

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