Distance protection for smart grids with massive
generation from renewable sources

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Abstract: - It is widely known that Distributed Generation (DG) changes the radial operation of the most common distribution systems. Actually, the system is no longer passive but it becomes active, and bidirectional power flows can appear on the distribution network. In this condition, also the fault currents may be drastically changed causing problems to the protection system, which normally uses overcurrent protection devices in radial distribution circuits. In order to solve this problem, a new protection procedure is proposed, based on distance protection concepts and synchronization of the voltage and current samples coming from the two terminals of the faulted MV line segment. The method involves the circuit equations in the time-domain, and the fault identification algorithm exploits the Recursive Least Square approach. A thorough set of Matlab simulations are carried out, in order to validate the proposed algorithm, which exhibits good performances also in case of high distortion of the acquired voltage and current waveforms. Indeed, it reveals to be robust, precise and very fast. Moreover, the algorithm provides, by product, a very good estimate of the fault resistance so it correctly works also in presence of high fault resistances that can occur in the herein considered distribution systems.

Key-Words: Smart grid protection, Renewable energy sources, Synchronized sampling.

1 Introduction

The great increase in the diffusion of Distributed Generation (DG) requires a substantial evolution of the electrical distribution systems so as to assure high levels of automation, protection and stability. On the other hand, a massive penetration of DG in existing distribution networks can cause problems related to the operation and protection of the distribution system, such as:

- the coordination failure of protective devices and the relay de-sensitization; in some cases over-current protection may even not respond or may take a long time to respond [1];
- the degradation of the current and voltage waveforms [2];
- the difficulty in order to control the voltage profiles due to changes of power injected by distributed generators [2-3];
- the presence of electromechanical transients and dynamic instability phenomena [3].

The above scenario emphasises the need to solve a number of problems in upgrading present distribution networks with the aim to obtain more and more efficient and reliable smart grids in future. In this paper particular attention is paid to improve the protection system by developing a new algorithm for fault-distance estimation. The main techniques adopted on distance protection, which are now mainly used in high voltage transmission lines, are based on the symmetrical component theory (phasor method). Even though these procedures use digital techniques, they actually
depend on concepts derived from analogical formulations and frequency domain approaches [4-11]. As a matter of fact, two particularly important features (usually offered by digital techniques) are ignored:

1) The overcoming of the signal processing, which means a drastic reduction in processing time, since it is possible to operate directly on the acquired samples of voltage and current instead of on their associated signals.

2) The use of more complex, but more precise mathematical schemes, compared with those drawn from simplified circuits based on phasor representation.

Unlike most common distance protection systems, which usually refer to an identification scheme based on stand-alone relays, in this paper a new identification algorithm is proposed, based on the idea that relays co-operate each other. This way, thanks to a doubled stream of data, the algorithm is able to face also the case of a non negligible fault resistance, even if in the common practice only pure metallic faults are assumed. Indeed, according to the proposed algorithm, also the fault resistance is estimated as a by product, with a very good degree of precision.

From a technical point of view, the idea of co-operating relays requires the hypothesis to assume a synchronization mechanism, ensuring simultaneous measurements coming from the different relays installed on the network.

The wide range of simulations (performed in the Matlab framework) show that the algorithm is able to predict the fault distance with a very good degree of precision, in a very fast way (below 10ms).

2 Power system and sample synchronization

In the following, a single MV feeder is considered that is supposed subdivided in a number of segments. Each line segment is equipped with two distance relays, respectively placed at its terminations. It is assumed that each relay is connected to a circuit breaker that exhibits reclosing capabilities. When a fault occurs, the protection system must detect and isolate only the faulted line segment. The assumed MV radial system can feed any kind of load (linear and nonlinear). In addition, the considered power system may present along the line a number of distributed generators (besides the main equivalent source), normally connected to nodes materialized by MV/LV transforming stations. It is also assumed that the distribution system can be operated in islanded mode if local DG generation is more than load demand.

As concerns about sample synchronization, different technologies can be used; for example a typical Global Positioning System (GPS).

3 The digital identification algorithm

Consider a simplified lumped parameter model of a faulted segment of an MV power line connecting bus 2 to bus 3 as in Fig. 1. This figure is taken from a more complete network which will be used as a test system to evaluate the performance of the proposed algorithm in the following Section (see Fig. 2).

The assumed fault is a symmetrical three-phase short circuit.

![Fig. 1 - Electrical scheme for a pair of synchronized relays.](image)

Here and in the following, \( R_i \) stands for the \( i \)-th relay, while \( R_j \) and \( L_j \) stand for the lumped resistance and inductance between nodes \( j \) and \( k \). We assume the fault occurs in node 5, with \( R_f \) the fault resistance.

The total resistance/inductance of the power line are known, \( R_{23} \) and \( L_{23} \) respectively, so that \( R_{35}, L_{35} \) can be expressed in terms of \( R_{25} \) and \( L_{25} \):

\[
R_{35} = R_{23} - R_{25}, \quad L_{35} = L_{23} - L_{25}. \tag{1}
\]

The distance protection problem is solved by identifying the lumped parameters \( R_{25}, L_{25} \), according to which the fault distance is readily obtained, assuming that the line resistance/inductance are frequency-independent and uniformly distributed along the whole line. By denoting with \( R'_{23}, L'_{23} \) the line unit resistance/inductance, and with \( \hat{R}_{25}, \hat{L}_{25} \) the estimated lumped parameters, the fault distance from relay \( R_3 \) computed according to resistance/inductance estimates will be:

\[
S_{R,R_3} = \hat{R}_{25}/R'_{23}, \quad S_{L,R_3} = \hat{L}_{25}/L'_{23} \tag{2}
\]

respectively.
The differential equations describing the scheme of Fig. 1 are:

\[
\begin{align*}
    v_i(t) &= R_c i_i(t) + L_c \frac{di_i}{dt} + R_p (i_j(t) + i_k(t)), \\
    v_j(t) &= (R_c - R_p) i_j(t) + (L_c - L_p) \frac{di_j}{dt} + R_p (i_k(t) + i_l(t)), \\
    v_l(t) &= (R_c - R_p) i_l(t) + (L_c - L_p) \frac{di_l}{dt} + R_p (i_k(t) + i_j(t)).
\end{align*}
\]  

(3)

Voltage/current measurements \((v_3,i_3), (v_4,i_4)\) are acquired by the relays \(R_3, R_4\) respectively, according to the sample interval \(T_s\), starting from time 0, which is the instant when the fault occurs. The idea of synchronized relays is that each sample time \(kT_s\), four new measurements are available, \(v_3(kT_s), v_4(kT_s), i_3(kT_s), i_4(kT_s)\). According to the digital feature of the identification algorithm, the following discretization will be adopted for the time derivative:

\[
\frac{di}{dt} \bigg|_{v_{i,kT_s}} = \frac{i((k+1)T_s) - i((k-1)T_s)}{2T_s};
\]

(4)

therefore, by using the sample notations:

\[
v_{i,k} = v_i(kT_s), \quad i_{j,k} = i_j(kT_s), \quad i, j = 3, 4, \quad k = 0, 1, \ldots
\]

(5)

system (3) is discretized as:

\[
\begin{align*}
    v_{i,k} &= R_c i_{i,k} + L_c \frac{i_{j,k+1} - i_{j,k-1}}{2T_s} + R_p (i_{k,k} + i_{l,k}) \\
    v_{j,k} &= (R_c - R_p) i_{j,k} + (L_c - L_p) \frac{i_{k,k+1} - i_{k,k-1}}{2T_s} + R_p (i_{l,k} + i_{j,k}).
\end{align*}
\]

(6)

Denoting with \( \theta = [R_c, L_c, R_p]^T \) the parameter vector to be estimated according to a finite set of \( m \) pairs of equations of the (6) type, that is \( k = 1, \ldots, m \), the whole set of measurement equations can be put in the more compact form:

\[
\Phi_m \cdot \theta = \Gamma_m
\]

(7)

where matrix \( \Phi_m \in \mathbb{R}^{2m \times 3} \) and vector \( \Gamma_m \in \mathbb{R}^{2m} \) appropriately collect the voltage/current measurements (they are explicitly reported in the Appendix A).

Due to the measurement/discretization noises affecting the equations, there is no exact solution to the linear problem presented in (7). A way to cope with the absence of an \textit{a priori} knowledge upon the noise statistics is the Weighted Least Square (WLS) method, which is a pure deterministic approach strictly based on the mathematical relationship between parameters and data (see e.g. [12] and references therein). The WLS approach provides the estimate \( \hat{\theta}_m \) which minimizes the following index:

\[
\hat{\theta}_m = \min_\theta J(\theta)
\]

\[
J(\theta) = (\Gamma_m - \Phi_m \theta)^T W_m (\Gamma_m - \Phi_m \theta).
\]

(8)

where \( W_m \) is a symmetric, positive definite weight matrix. In other words, the index \( J \) is the square of the norm of the measurement error \( \Gamma_m - \Phi_m \theta \), weighted by the matrix \( W_m \):

\[
J(\theta) = \|\Gamma_m - \Phi_m \theta\|^2_{W_m}.
\]

(9)

In case of \( W_m \) given by a diagonal matrix, i.e. \( W_m = \text{diag}\{w_{1,1}, \ldots, w_{2m}\} \), (that means all independent measurements), the index \( J \) reduces to the following sum:

\[
J(\theta) = \sum_{k=1}^{m} w_{2k-1} \left( v_{i,k} - R_c i_{j,k} - L_c i_{j,k+1} - i_{j,k-1} - R_p (i_{l,k} + i_{l,k-1}) \right)^2
\]

\[
+ \sum_{k=1}^{m} w_{2k} \left( v_{j,k} - (R_c - R_p) i_{j,k} - (L_c - L_p) i_{k,k+1} - i_{k,k-1} - R_p (i_{l,k} + i_{l,k+1}) \right)^2
\]

(10)

The problem stated in (8) is solved by means of the following equation (see e.g. [12]):

\[
\hat{\theta}_m = \Phi_{W_m}^+ \cdot \Gamma_m
\]

(11)

where

\[
\Phi_{W_m}^+ = (\Phi_m^T W_m \Phi_m)^{-1} \Phi_m^T W_m
\]

(12)

is the Moore-Penrose pseudo-inverse of \( \Phi_m \).

In case of partial knowledge of the noises affecting the system, the weighting matrix \( W_m \) may well assume a precise statistical meaning. For instance, suppose to write equation (7) as:

\[
\Phi_m \cdot \theta + \eta_m = \Gamma_m
\]

(13)

where \( \eta_m \in \mathbb{R}^{2m} \) is a Gaussian zero-mean random vector, with known covariance matrix \( \Psi_m \). Then, it can be shown that the Maximum Likelihood estimate of \( \theta \) is provided by the optimal solution (11-12), with \( W_m = \Psi_m^{-1} \) (see [12] for more details). In other contexts, when no \textit{a priori} information is available about the noises and all measurements are equivalent, then matrix \( W_m \) may well be set equal to the identity matrix.

\textbf{Remark 1.} Note that, according to the non-causal discretization adopted in (4), the estimate \( \hat{\theta}_m \)
requires voltage measurements up to the sample time \(mT_s\) and current measurements up to the sample time \((m+1)T_s\). It means that \(\hat{\theta}_m\) is computed at time \((m+1)T_s\). Such a delay in the estimate availability may be exploited for the synchronization of cooperating relays.

A recursive solution to the optimization problem stated in (8) is adopted to implement the fault estimate algorithm (Recursive Least Square (RLS) algorithm). Since the estimate \(\hat{\theta}_m\) clearly depends on the amount of the acquired measurements, more and more accurate estimates are expected at each new sample time, by acquiring new pairs of voltage/current measurements.

**Remark 2.** It has to be stressed that, according to the proposed algorithm, we estimate the lumped resistance/inductance \(R_{25}, L_{25}\), as well as the fault resistance \(R_F\). Of course, by exploiting (2), we have a pair of estimates of the fault distance. Sometimes there are technical considerations which suggest to prefer the resistance-based estimate instead of the inductance-based, or vice versa; some other times, when there is no a priori knowledge to prefer one estimate to another, the mean value of the two estimates provided by (2) is suggested.

### 4 Test system and simulation results

In the following, the most significant simulation results are reported in order to evaluate the validity limits of the proposed method subordinately to the fault resistance and fault distance values.

**A - The test system**

The proposed method has been tested on simulation on a 20kV system as shown in Fig. 2. Though the rated voltage of the system is 20kV, simulations consider a voltage supply of 12kV, which corresponds to the single phase voltage +4%. According to Fig. 2, the fault is assumed to be somewhere on the line segment connecting busses 2 and 3.

The distribution feeder is supplied at one end by the main source \(e_{s1}(t)\), which injects only the base frequency. A short circuit power of 300 MVA, with \(\cos \varphi_{sc} = 0.3\), is assumed, which corresponds to an internal impedance \(Z_i = 1.30\Omega\).

The DG is connected at the other end of the line. It is assumed to be a voltage generator \(e_{s2}(t)\), with an internal impedance of 38.4\(\Omega\), with \(\cos \varphi_{sc} = 0.3\), which includes the step-up transformer of DG. The DG capacity is limited to 2.5 MVA. In order to take into account random disturbances caused by nonlinear devices [9], [13], [14], higher, sub- and inter-harmonics are added to the fundamental (50Hz, 12kV) according to the following model:

\[
e_{s2}(t) = 0.1e_{52,250H}(t) + e_{52,500H}(t) + 0.1e_{52,100H}(t) + 0.2e_{52,1500H}(t) + 0.1e_{52,1170H}(t)
\]

where, for instance, \(e_{52,250H}(t)\) stands for a sub-harmonic at 28Hz added to the fundamental.

A second source of nonlinear disturbances is modeled by the harmonic current generator \(i_{s2}(t)\) (connected on bus 2), which injects the base frequencies (50Hz, 30A) and other harmonics, whose amplitudes and frequencies are defined by the following relation [9]:

\[
i_{550H}(t) = 0.20i_{5,550H}(t) + 0.14i_{5,350H}(t) + 0.0977i_{5,550H}(t) + 0.059i_{5,550H}(t) + 0.053i_{5,550H}(t)
\]

The MV-line consists of a copper conductor of 140mm²-section which exhibits the following unit parameters:

\[
R' = R'_{L1} = R'_{L2} = R'_{L3} = 0.145 \Omega/km
\]

\[
L' = L'_{L1} = L'_{L2} = L'_{L3} = 1.1 mH/km
\]

The two linear loads, which are connected on busses 2 and 3, draw 3MVA, \(\cos \varphi = 0.9\), and 2MVA, \(\cos \varphi = 0.9\), respectively.

Fig. 2 - The examined MV-distribution feeder with the locations of distance relays (\(R_i, i = 1, \ldots, 6\)) and fault resistance (\(R_F\)); \(e_{s1}\) and \(e_{s2}\) are respectively the main equivalent source and the equivalent of the distributed generator.
The adopted sampling frequency is \( f_s = 2.5kHz \), corresponding to a sampling time \( T_s = 400\mu s \), i.e. 2500 samples per second.

### B – The simulation results

Different simulations have been carried out, according to different fault distances and fault resistances. In all the proposed test cases, the fault distances have been computed as the average value of both the estimates coming from the resistance/inductance estimates, according to (2).

With reference to relay \( R_3 \), three cases have been considered for the fault distance:

- \( S_F = 100m \)
- \( S_F = 900m \)
- \( S_F = 1900m \)

In order take into account close, medium and far distances, while five cases have been considered for the fault resistance:

- \( R_F = 0\Omega \)
- \( R_F = 1\Omega \)
- \( R_F = 10\Omega \)
- \( R_F = 50\Omega \)
- \( R_F = 100\Omega \)

Results are shown in Table 1, concerning 15 tests for different values of \( S_F \) and \( R_F \) (columns 2 and 3); columns 4 and 5 report the percentage error (absolute value) of the lumped resistance/inductance estimates, obtained after a given time (in [ms]):

\[
\Delta R(m) = 100 \frac{\hat{R}_{25}(m) - R_{25}}{R_{25}}
\]

\[
\Delta L(m) = 100 \frac{L_{25}(m) - L_{25}}{L_{25}}
\]

Column 6 reports the percentage error (absolute value) of the fault distance estimate.

Tab. 1: Percentage errors (absolute values) for \( R_{25}, L_{25} \) and the fault distance, obtained after a time period expressed in [ms].

<table>
<thead>
<tr>
<th>( S_F ) (m)</th>
<th>( R_F ) (( \Omega ))</th>
<th>( \Delta R ) (% / ms)</th>
<th>( \Delta L ) (% / ms)</th>
<th>( \Delta S_F ) (% / ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>1900</td>
<td>&lt;1.0 / 4.8</td>
<td>&lt;1.0 / 2.0</td>
<td>&lt;1.0 / 3.6</td>
</tr>
<tr>
<td>Test 2</td>
<td>1900</td>
<td>&lt;1.0 / 6.0</td>
<td>&lt;1.0 / 3.6</td>
<td>&lt;1.0 / 3.6</td>
</tr>
<tr>
<td>Test 3</td>
<td>1900</td>
<td>&lt;1.0 / 7.6</td>
<td>&lt;1.0 / 8.8</td>
<td>&lt;1.0 / 5.6</td>
</tr>
<tr>
<td>Test 4</td>
<td>1900</td>
<td>&lt;1.0 / 8.0</td>
<td>&lt;1.0 / 9.6</td>
<td>&lt;1.0 / 5.6</td>
</tr>
<tr>
<td>Test 5</td>
<td>1900</td>
<td>&lt;1.0 / 9.2</td>
<td>&lt;1.0 / 9.6</td>
<td>&lt;1.0 / 8.0</td>
</tr>
<tr>
<td>Test 6</td>
<td>900</td>
<td>&lt;1.0 / 4.8</td>
<td>&lt;1.0 / 2.0</td>
<td>&lt;1.0 / 3.6</td>
</tr>
<tr>
<td>Test 7</td>
<td>900</td>
<td>&lt;1.0 / 6.0</td>
<td>&lt;1.0 / 3.6</td>
<td>&lt;1.0 / 4.8</td>
</tr>
<tr>
<td>Test 8</td>
<td>900</td>
<td>&lt;1.0 / 7.6</td>
<td>&lt;1.0 / 9.6</td>
<td>&lt;1.0 / 5.6</td>
</tr>
<tr>
<td>Test 9</td>
<td>900</td>
<td>&lt;2.0 / 8.0</td>
<td>&lt;1.0 / 5.6</td>
<td>&lt;1.0 / 8.0</td>
</tr>
<tr>
<td>Test 10</td>
<td>900</td>
<td>&lt;2.0 / 8.4</td>
<td>&lt;1.0 / 8.0</td>
<td>&lt;1.0 / 8.0</td>
</tr>
<tr>
<td>Test 11</td>
<td>100</td>
<td>&lt;1.0 / 5.2</td>
<td>&lt;1.0 / 2.0</td>
<td>&lt;1.0 / 4.8</td>
</tr>
<tr>
<td>Test 12</td>
<td>100</td>
<td>&lt;1.0 / 5.6</td>
<td>&lt;1.0 / 2.0</td>
<td>&lt;1.0 / 4.8</td>
</tr>
<tr>
<td>Test 13</td>
<td>100</td>
<td>&lt;2.0 / 9.2</td>
<td>&lt;2.0 / 4.8</td>
<td>&lt;2.0 / 5.2</td>
</tr>
<tr>
<td>Test 14</td>
<td>100</td>
<td>&lt;6.0 / 9.2</td>
<td>&lt;2.5 / 11.6</td>
<td>&lt;2.5 / 8.4</td>
</tr>
<tr>
<td>Test 15</td>
<td>100</td>
<td>&lt;6.0 / 19.2</td>
<td>&lt;6.0 / 12.0</td>
<td>&lt;5.0 / 8.0</td>
</tr>
</tbody>
</table>

For instance, Test 1 says that according to a fault occurring at distance 1900m from relay \( R_3 \), with a fault resistance \( R_F \) equal to zero:

- resistance \( R_{25} \) is estimated with an error percentage (absolute value) definitely lower than 1% after 4.8ms;
- inductance \( L_{25} \) is estimated with an error percentage (absolute value) definitely lower than 1% after 2.0ms;
- the fault distance \( S_F \) is estimated with an error percentage (absolute value) definitely lower than 1% after 3.6ms.

It has to be stressed that the algorithm is very fast, since it allows to estimate the fault distance with a high degree of precision within the first 10ms passed from the fault. And this happens whatever the value of the fault resistance is assumed. For instance, in case of high distances from relay \( R_3 \) (Tests 1-5), the algorithm is able to estimate a fault at 1900m with an error lower than 20m within the first 8ms; in case of medium distances (Tests 6-10), the algorithm is able to estimate a fault at 900m with an error lower than 10m within the first 8ms; in case of small distances (Tests 11-15), the algorithm is able to estimate a fault at 100m with an error lower than 5m within the first 8ms.

For the ease of the reader, Fig. 4 plots the time evolution of the fault distance percentage error as computed from resistance estimates (squares), from inductance estimates (dots), from the mean average of both (circles), when a fault at 900m is considered, with a fault resistance of 10\( \Omega \).

It must be noted that many of these results are obtained with high fault resistances (10\( \Omega \), 50\( \Omega \), 100\( \Omega \)), which usually would not allow acceptable estimates with non co-operating relays.

![Fig. 4 - Time evolution of the fault distance percentage error as computed from resistance (squares), inductance (dots), and both (circles) estimates. \( S_F = 900m, R_F = 10\Omega \).](Image)

Table 2 reports the time instants (in [ms]) when the percentage error (absolute value) of the fault resistance estimate is definitely below 0.01%. Note that in this case the fault resistance estimates are
even faster (times below 6ms) and provide an excellent degree of precision.

Tab. 2: Time instants [ms] when the error percentage (absolute value) of the fault resistance estimate is definitely below 0.01%.

<table>
<thead>
<tr>
<th>$R_f$ [Ω]</th>
<th>$S_f = 100m$</th>
<th>$S_f = 900m$</th>
<th>$S_f = 1900m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.6</td>
<td>5.6</td>
<td>6.0</td>
</tr>
<tr>
<td>10</td>
<td>4.4</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>50</td>
<td>3.6</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>100</td>
<td>3.6</td>
<td>2.4</td>
<td>2.4</td>
</tr>
</tbody>
</table>

5 Conclusions

With reference to the protection coordination of distribution systems with high penetration of DG, GPS synchronized relays are proposed that use a new type of algorithm developed on the basis of distance protection concepts. In order to estimate the line-fault parameters, a particular application of the Recursive Least Square method is implemented, that operates directly on the acquired synchronized samples. As demonstrated by the performed simulations, the proposed technique is robust, very fast and exhibits good performances also in case of high distortion of the voltage and current waveforms. In addition, a parametric analysis is carried out to evaluate the relay time response, assuming both the fault resistance and fault distance as variables. Obtained results show that the proposed protection system correctly works in any condition, allowing at the same time an accurate and timely selectivity between relays.

Appendix

This Appendix reports the explicit computations concerning matrix $\Phi_m$ and vector $\Gamma_m$. According to eq.s (6), matrix $\Phi_m \in \mathbb{R}^{2m \times 3}$ and vector $\Gamma_m \in \mathbb{R}^{2m}$ have the following explicit forms:

$$\Phi_m = \begin{bmatrix} i_{3,1} & \frac{i_{3,2}-i_{3,0}}{2t_s} & i_{3,1} + i_{4,1} \\ -i_{4,1} & \frac{i_{4,2}-i_{4,0}}{2t_s} & i_{3,1} + i_{4,1} \\ \vdots & \vdots & \vdots \\ i_{3,m} & \frac{i_{3,m+1}-i_{3,m-1}}{2t_s} & i_{3,m} + i_{4,m} \\ -i_{4,m} & \frac{i_{4,m+1}-i_{4,m-1}}{2t_s} & i_{3,m} + i_{4,m} \end{bmatrix}$$

(A1)

$$\Gamma_m = \begin{bmatrix} v_{3,1} \\ \vdots \\ v_{3,m} \\ v_{4,1} - R_{23}i_{4,1} - L_{23} \frac{i_{4,2}-i_{4,0}}{2t_s} \\ \vdots \\ v_{3,m} \\ v_{4,m} - R_{23}i_{4,m} - L_{23} \frac{i_{4,m+1}-i_{4,m-1}}{2t_s} \end{bmatrix}$$

(A2)

References:


