The method of positioning source of radio signal Doppler difference -
analytical solution and analysis of method

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Abstract: - The object of this paper is locations and determination of electronics signal by method Doppler difference. The whole project is divided in several parts. The first part is focused on general characterization, operating principles and typology of Doppler difference. The second part deals with various types of method Doppler difference solving. The third part is completely focused on the analytic solving development. The fourth part analyses exactness and solvability of method Doppler difference. And finally the last part solves calculation of locations determination of electronics signal mistake.

Key-Words: - Doppler difference, Electronics signal, Exactness, Solvability, Mistake

1 Introduction
Question and the issue of passive radars and radio surveys is currently the subject of much attention. The advantage of these resources is great scope and great technical and reconnaissance capabilities and because they work only on the principle of capture and signal processing and active radars themselves do not emit any energy, are other means adversary poorly detectable and identifiable. It is these features make passive radio technical resources so interesting for contemporary modern combat. This work focuses on the part of the localization, thus positioning the targets (sources radio signal). Generally, there are several methods of determining the target position. Each method is based on the specific configuration of passive radio systems, types of measured parameters of received signal, etc. But in the literature [1], [2] and [3] are often quoted the following three methods for determining the target position: Triangulation Method, Time difference of arrival method and Method Doppler difference. This work is focused on detailed analysis of possible solutions to the Doppler difference method and the analytical derivation of the Doppler difference method. Doppler difference method, which is also called differential Doppler [2], FDOA (Frequency Difference Of Arrival) [3], or simply abbreviated to DD, is based on measuring the difference frequency signals arriving at several position-receiving stations. It exploits the Doppler effect and is therefore subject to the mutual movement between the target and the receiving unit. The biggest disadvantage of this method is the measurement accuracy Doppler difference frequency, which depends mainly on the nature of the signal. By their nature, it is possible to solve differential Doppler method in several ways, the best known of which are graphic solutions, graphic-numerical solution, an iterative solution for the job and key analytical solutions.

2 Doppler difference
The work individually discussed all the ways to address the Doppler difference method. For easy comparison of results of individual solutions and because it is very challenging analytical calculation was chosen a model situation. The most simplistic element is the choice of topology, i.e. the choice of location receivers and goals. There are several deployment options, the most basic are shown in Figure 1.

![Fig. 1: Topology of receivers and targets](image-url)
Situation a) shows the simplest variant, where the receiving station moving at the same speed and the same direction in one axis. In this situation, we are able to portray izodopplers [1] and select a suitable recruitment sites to determine the target position. In topologies b) and c) occur problems with significant deformations izodopplers and positioning objectives are becoming very demanding and analytically almost intractable task. The situation marked with the letter d) is currently useless option because they are so deformed izodopplers that mathematically cannot be generalized and used in calculations. To measure the position of moving targets using static reception sites, this method is inappropriate because there is a need for at least five simultaneously irradiated sites, a practice very demanding requirement. However, the method can be used to determine the location of static or stationary targets through the moving three receivers located mostly on an airplane. In practice, however, most often used to use two receivers installed on board aircraft, which measures the difference frequency of received signals. From the above facts is selected model situation that is very simplified compared to real use, but still is an analytical calculation of very complex and challenging. The target is compared to three by moving the entrance habitats marked L, R, C, and static. All three receiving stations are moving at the same speed in the same direction in one axis (see Fig.1) and Fig 2). For all calculations in this work was used computing environment Matlab.

Part of a model situation that I have said is the definition of the basic model variables:
- Speed of light: \( c=3\times10^8 \) m/s
- Target coordinates and transmitted frequency: \( x=20000 \) m, \( y=50000 \) m, \( f_0=100\) MHz
- Coordinates of receivers:
  - Receiver C: [0,0]
  - Receiver L: [a,0] \( a=-25000 \) m
  - Receiver R: [b,0] \( b=30000 \) m

All receivers are moving at \( [v,0] \):
\( v=100 \) m/s

Now we only briefly mention the characteristics of the graphic-numerical methods and iterative methods of solution and illustrate the results.

2.1 Graphic-numerical solution

Graphic-numerical method in essence is based on graphical methods [6], but is designed to process computer technology. A big difference from the graphical method is the transition from rectangular to polar coordinate system. The result is the intersection of the two solutions izodopplers. This intersection can be computed by different methods. The disadvantage of this method is its accuracy positioning radio signal sources. This does not mean that it could not be used. It is suitable for example for determining the initial estimate in an iterative method. In our case, this algorithm is included [6] not only numerical output, as well as graphical output that is used to verify the numerical calculation.

As mentioned graphic-numerical method of Cartesian coordinate system moving into the polar coordinate system and therefore follows display the graphic-numerical methods in polar coordinates (fig4).
2.2 Newton iterative method for solving
Newton iterative method is designed to search for roots of polynomials [4], [5]. This is quite widely used for their speed approaching the root. In many cases, to achieve the desired accuracy of just a few iterations because each iteration refines the result by one to two decimal places. This method can be easily explained by the geometrical in nature is calculated by the intersection of the tangent at some point the graph of the axis x. Newton iterative method always gives one result - the root, which is given initial guess $x_0$, because it is always found near the root from the initial estimate. If the derivative in point was zero, as in a local maximum or minimum function stops derivatives. This is one of the drawbacks of iterative methods. The drawback of this method, however, particularly it is positioning accuracy sources, which is dependent on the initial estimate.

3 Analytical methods for the derivation of the Doppler difference
This method is still used in practice and its application possibilities are summarized in [6] based on the results of the analysis error locating targets. The method is based on basic formulas for calculating the Doppler frequency [1], [2]. The disadvantage of this method is certainly the complexity of the calculation, as we can see [6]. The complexity is also related to time-consuming calculation, so I used to calculate the previously mentioned computer program Matlab, which has many mathematical functions. During the calculation, I used the most symbolic function of solving mathematical relationships. Given the extensive calculation and duration of expression, the implementation of the procedure here in the article seemed quite hard to read and so the impression given here is a brief procedure.

$$\text{FOAC} = f_0 + \left( f_0 v/c \right)^* x / \left( \sqrt{x^2 + y^2} \right) \quad (1)$$
$$\text{FOAL} = f_0 + \left( f_0 v/c \right)^* (x-a) / \left( \sqrt{(x-a)^2 + y^2} \right) \quad (2)$$

measured Doppler Difference is:
$$\text{DDL} = \text{FOAL} - \text{FOAL} \quad (4)$$
$$\text{DDR} = \text{FOAR} - \text{FOAC} \quad (5)$$

substitution:
$$L = \text{DDL} * c / (f_0 v) \quad (6)$$
$$R = \text{DDR} * c / (f_0 v) \quad (7)$$

after substitution:
$$L = (x-a) / (\sqrt{(x-a)^2 + y^2}) - x / \left( \sqrt{x^2 + y^2} \right) \quad (8)$$
$$R = (x-b) / (\sqrt{(x-b)^2 + y^2}) - x / \left( \sqrt{x^2 + y^2} \right) \quad (9)$$

substitution:
$$K^2 = x^2 + y^2 \quad (10)$$
$$M = x / K \quad (11)$$

And after putting expression and $K^2$ to compare:
$$K^2 \text{ (from the equation (8))} = K^2 \text{ (from the equation (9))}$$

After substitution of equations (8) and (9) I’ve got a complex expression, from which I expressed during the $M$ variable expression, was necessary to make a number of substitutions [6], I came to a relatively understandable result. The result is a polynomial 22nd instance, which cannot be solved analytically, but there are many mathematical approaches to solving a polynomial of higher degree [5]. In our case, I used the Matlab function to calculate the roots of a polynomial of higher degree. Polynomial 22nd degree is just a partial solution of the analytical method. The computation of roots of Matlab we get 22 roots, of which only one root is real and nonnegative. Extract roots and their graphic representation can be found in [6]. The root of a polynomial substitute into equation (11) from which we get the x and substituting into equation (10) gives y, then the target position. The method of locating sources of radio signal includes not only our own transformation the conversion of differential Doppler plane in Cartesian plane, or map coordinates, but also calculates the positioning error sources. Error positioning method source radio signal is independent of the method of solution. Output value of positioning error sources are in the form of covariance matrix, or error patterns. Determination of target position is the final task of the process of locating sources of radio signals. However in reality can be only determined estimate of the true target position, because measurement of quantities needed to determine the target position is hampered by measurement error. From the physical point of view it is an indirect measurement, and
even each directly measured quantity, which can be used for determining the position is encumbered with measurement error, which is caused by additive noise in the received signals. We consider that the additive noise has a normal probability density function with zero mean and given variance. The calculation error was made just to check in two ways. First, I made mistakes analytical solution, while I checked the result of errors by simulating a large number of measurements loaded with the same error I have calculated the analytical calculation errors. The result of both the error ellipse, which should be approximately the same and the same tilt angle semiaxes. In addition to comparing ellipses can be compared to the numerical values of the covariance matrix, which is the output of both methods.

3.1 Analytical solution positioning error sources

From an analytical solution can be obtained from the error calculating the error ellipse, whose parameters (length of the semiaxes and the angle relative to the origin coordinate system) is proportional to its own numbers, own vector and covariance matrix. The error ellipse is a graphical representation of the accuracy of the method.

The covariance matrix:
\[
C = \begin{bmatrix}
40182 & -14110 \\
-14110 & 26485
\end{bmatrix}
\] (13)

The covariance matrix can calculate your own figures and its own vector, whose values are needed to render the error ellipse.

own number:
\[
\lambda = \begin{bmatrix}
17650 & 0 \\
0 & 49018
\end{bmatrix}
\] (14)

own vector:
\[
v = \begin{bmatrix}
-0.5307 & -0.8475 \\
-0.8475 & 0.5307
\end{bmatrix}
\] (15)

from his own vector angle can be calculated ellipse:
\[
\gamma = \text{atan2}(-0.5307, -0.8475) = -2.5821 \cdot \frac{180}{\pi} \approx -148^\circ
\] (16)

3.2 Simulation source position measurement method of Doppler difference

To verify the accuracy of analytical methods for calculating an issue I have carried out difference Doppler measurements simulating a large number of positions of radio signal sources contaminated additive Gaussian white noise with a frequency of 5Hz. The value of the error, which is burdened by the calculation is the same as in the case of analytical calculation errors by locating sources of DD. And I made back-calculation of the position of sources in the Cartesian coordinate system. For this calculation can be used many methods. In terms of speed and efficiency of calculation, I chose the Newton iterative method. Whose basic problem of determining initial estimates does not care. The starting point of iteration (initial guess) I set very precisely, because the search value is already known from the analytical calculation. The calculation is artificially introduced error of ± 5 Hz, and the process goes through 20,000 cycles. The degree of iteration is set to 8.

the covariance matrix:
\[
C = \begin{bmatrix}
40361 & -14098 \\
-14098 & 26627
\end{bmatrix}
\] (17)

The covariance matrix can calculate your own figures and its own vector, whose values are needed to render the error ellipse.

own number:
\[
\lambda = \begin{bmatrix}
17698 & 0 \\
0 & 49644
\end{bmatrix}
\] (19)
from his own vector angle can be calculated ellipse:
\[ y = \frac{\text{atan2}(-0.5344, -0.8452)}{\pi} \approx -148^\circ \]  

(20)

The following plot estimates 20,000 positions radio signal sources with loaded error 5Hz.

![Fig. 6: Ellipse calculated target positions with an error 5Hz](image)

For visual comparison of the two error patterns is created Fig.7, where the two ellipses are drawn in the same scale. Within the error ellipse (blue) is the target with an accuracy of 40.79%.

![Fig. 7: An error ellipse in shape](image)

4 Conclusion
At the beginning I had to work mainly oriented in the basic concepts, principles, laws and possibilities of solving the positioning method source radio signal Doppler difference. Based on this information, I chose a model situation that is starting while the connecting point of the whole work. This is a simplification of the real situation so that it is possible to calculate the Doppler difference method by several different methods. Despite the complexity of calculations is very high. To compare the results of each possible solution to the input parameter values in all cases. In [6] it is possible to find several possible ways of solving differential Doppler method. The greatest emphasis is however placed on analytical methods for the derivation of the Doppler difference. All previous methods of solution have the disadvantage in its accuracy, and therefore this work focuses on analytical methods for solving the DD, which serves a specific target coordinates. Analytical solution is indeed very accurate, but very difficult to calculate. In the paper, I noted only a very brief description of the very extensive analytical calculation. A detailed calculation can be found in [6]. At the end of the calculation I made an error evaluation method of locating sources of Doppler difference. Here are both results in numerical form and in graphical form. A key element of this chapter is to determine the error patterns and the corresponding covariance matrixes. So far I did not address the detailed analysis of clarity and analytical calculation of polynomial 22nd degrees, which creates additional opportunities continue this project. In the future I would like to carry out the derivation of analytical solution methods for Doppler difference for other topologies, which are listed here.

References: