Digital Channel Modeling and Capacity Calculation for Multi-Relay Multi-Hop Wireless Sensor Networks

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Abstract: - This paper presents a new approach for the channel capacity calculation of a multi-relay multi-hop communication system. For the channel model based on Markov chains theory, the exact formula for the channel capacity, which is expressed as a function of packet error rate and number of relays, is derived. The expression of the overall packet error rate is derived in its closed form. The presented theory is supported by numerical analysis.

Key-Words: - Multi-relay network, Multi-hop network, Digital channel, Channel capacity, Multi-state channel.

1 Introduction

Multi-relay communications can be used to mitigate fading and increase energy efficiency in wireless communication systems and networks. In order to increase the communication range in these networks, it is possible to implement multi-hop communications. This paper is dedicated to the mathematical modeling and analysis of a network that includes both multi-relay and multi-hop communications. Primarily, the procedure of digital channel modeling and its capacity calculation are presented in this paper.

In the process of design of wireless networks, one of the basic requirements is to determine the required capacity of their transmission channels. The procedure of capacity calculation for network design, when the network has a star configuration, is well understood inside the research that has been conducted in wireless cellular networks. This procedure can be applied in the wireless sensor networks if the base station communicates directly with the individual nodes, i.e. the network has a star configuration. However, this procedure cannot be directly applied in the network that uses multiple relays and multiple hops inside the network structure, i.e., in the network that we will call multi-relay multi-hop network (MR-MH). Thus, the problem of determining the network capacity in a multi-relay multi-hop network was the main subject of the research work that produced results presented in this paper.

The research results presented here are based on the idea that the MR-MH communication channel can be modelled using the theory of digital channels. A review of the multi-way channels that were developed until 1976 is presented in [1]. In the same reference the problem of modelling a multi-relay channel is presented. In particular we are interested in the capacity calculation of the MR-MH networks. We found the capacity theorems for the Gaussian degraded, reversely degraded and feedback relay channels in [2] and the capacity calculation of a general wireless network with n nodes that are randomly located in a region of area 1 m^2 in [3].

The processing of the signal at relay nodes can follow various strategies. Cooperative communication methods were investigated and capacity theorems proved for relay networks that use either compress-and-forward or decode-and-forward relay strategies [4]. It was assumed that the Gaussian noise and Rayleigh fading are present in the communication channel. The upper and lower capacity bounds for cooperative diversity are derived in [5]. The systems that use decode and forward strategies in relay networks are analysed in [6] –[9] and the exact expressions for the probability of error and channel capacity are derived over independent and non-identical Rayleigh fading channels and Nakagami channel. A survey of digital channel models and the representative models of channels with memory can be found in [10]–[12].

In this paper we will assume that the decode-and-forward procedure is applied at the relay nodes and at each relay in each hop of the network. This procedure was analysed in [13] to find the outage probability in Rayleigh fading channel. For this configuration the packet error rate of a multi-relay multi-hop channel will be calculated as a function of the number of relay channels and packet error probabilities at all relays and hops.

The paper is organised in this way. Section 2 presents the mathematical model of the channel for multi-relay multi-hop network. Numerical analysis is presented in Section 3 and conclusions in Section 4 of the paper.

2 Mathematical Model of Multi-Relay Multi-Hop Channels

2.1 Channel Model

A block schematic of a multi-relay system, which is composed of a source node, a destination node, n relay nodes, R_1 to R_n, and H hops in each relay channel is presented in Fig. 1. The corresponding multi-relay channel of this system consists of a direct channel and n relay multi-hop channels. The direct channel is the channel between the source and the destination. Each channel, which includes a channel between the source and a relay node, the channels between all the relays and the channel between the last relay and the destination, is called the multi-relay multi-hop (MR-MH) channel.

Suppose that any packet of bits that is sent by the source is always received by the destination and all relays inside the first hop. A packet received at each relay is retransmitted according to the decode-and-forward procedure. Thus, each packet is error checked at the relays and retransmitted only in the case of correct reception and disregarded otherwise. The main aim of this paper is to derive the expression for the capacity of this multi-relay multi-hop channel.

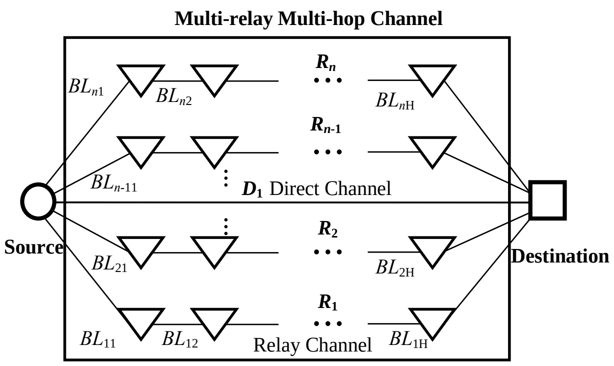

Fig. 1 Multi-relay communication system.

The mathematical model of the multi-relay multi-hop channel will be developed using the theory of Markov chains. A MR-MH channel is defined by the set of its states and the steady-state and transitional probabilities. We suppose that the messages are transmitted in the form of packet. Thus the main probability measures will be the packet error rates in the channel. We define that, the channel is the non-cooperative (S_0) or in a number of cooperative (S_i) states, i = 1, 2, …, N. The channel is, by definition, in the cooperative state if the signal at destination is received from the direct channel D_1 as well as from one or more relay channels from the set of all relay channels (R_1, R_2, …, R_n).

The number of cooperative states is equal to all possible combinations of the direct channel and any number of relay channels. Therefore, the number of states is equal to $N = 2^n$, where n is the number of relay channels. It is important to note that the number of states is higher than the number of relays.

2.2 Two-state model of MR-MH channel

In the case when a multi-relay channel is composed of a direct (D_1) and a single relay (R_1) channel with H hops, it can be represented by a two-state model as shown in Fig. 2. The cooperative state is denoted by S_1 and the non-cooperative state by S_0. For the sake of simplicity these states are also denoted in the binary form with 0 and 1, respectively.

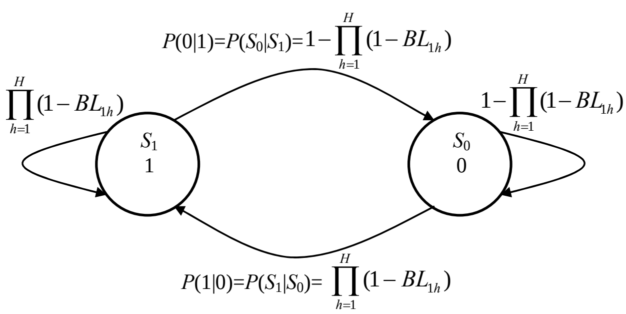

Fig. 2 A single-relay H-hop channel model.

The model is completely described by the steady-state probabilities and conditional probabilities. The channel steady-state probabilities of being in state 0 and 1 are related according to this expression

$$P(0) + P(1) = 1 . \tag{1}$$

The probability of taking a new state depends only on a single previous state, thus the sequence of

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states can be represented by a Markov chain and transitions between the states are defined by four conditional probabilities

$$P(0\mid 0)=P(0\mid 1)=1-\prod_{h=1}^{H}(1-BL_{1h}),$$
$$P(1|1)=P(1|0)=\prod_{h=1}^{H}(1-BL_{1h}), \qquad (2)$$

where BL_{1h} is the packet error rate at relay R_1 and hop h. The steady-state probabilities can be calculated as

$$P(0)=\sum_{i=0}^{1}P(i)P(0\mid i)=1-\prod_{h=1}^{H}(1-BL_{1h}) \qquad (3)$$
$$P(1)=\sum_{i=0}^{1}P(i)P(1\mid i)=\prod_{h=1}^{H}(1-BL_{1h}). \qquad (4)$$

The overall probability of packet error in the single-relay channel can be calculated as

$$\begin{aligned} BL_e &= \sum_{i=0}^{1}P(i)P(e_B\mid i) \\ &= (1-\prod_{h=1}^{H}(1-BL_{1h}))\cdot BL_{\overline{C}}+\prod_{h=1}^{H}(1-BL_{1h})\cdot BL_C \end{aligned} \qquad (5)$$

where $P(e_B\mid i)$ is the probability of packet error e_B when the channel is in state i and $BL_{\overline{C}}$ and BL_C are the probabilities of error in non-cooperative and cooperative states, respectively.

The transition procedure inside the state diagram in Fig. 2 can be represented by a binary stochastic process defined by two states. Let us define that the Packet Error Process is a time series of gaps of packets, where a gap is defined as a series of k correctly received packets $\bar{e}_p$ after one packet e_p is erroneously received. We will use this notation for a gap: ($\bar{e}_p, \bar{e}_p, \ldots, \bar{e}_p, e_p = e_p\bar{e}_p^{\,k}$).

Let us defined the Packet Error Gap complementary distribution function $G(k)$ as the probability of receiving a gap of k packets, which may be expressed as

$$G(k)=P(\bar{e}_p^{\,k}\mid e_p)=A\cdot J^k+(1-A)\cdot L^k, \qquad (6)$$

where $\bar{e}_p$ denotes a correctly received packet and e_p denotes the received erroneous packet. We can find the expressions for J, L and A from the initial conditions for the function $G(k)$ as follows

$$J=\prod_{h=1}^{H}(1-BL_{1h})\cdot(1-BL_C)+(1-\prod_{h=1}^{H}(1-BL_{1h}))\cdot(1-BL_{\overline{C}})=1-P_e$$

$L = 0$ and $A = 1$. Thus, the distribution $G(k)$ for our case is

$$G(k)=J^k=(1-BL_e)^k. \qquad (7)$$

where BL_e depends on the packet error rates on all hops and the packet error rates in cooperative and non-cooperative states. The capacity of the single-relay single-hop channel may be expressed in this form

$$C=1+BL_e\cdot\sum_{i=0}^{1}G(k)\{[1-\frac{G(k+1)}{G(k)}]\log_2[1-\frac{G(k+1)}{G(k)}]+\frac{G(k+1)}{G(k)}\log_2\frac{G(k+1)}{G(k)}\} \qquad (8)$$

If we insert (7) into (8), and calculating the sum of geometric series inside the summation sign, we can find the capacity as a function of the overall probability of packet errors in the single-relay multi-hop channel, expressed as

$$C=1+BL_e\cdot\log_2 BL_e+(1-BL_e)\cdot\log_2(1-BL_e) \qquad (9)$$

where the overall packet error probability BL_e depends on the probabilities that define the channel: the packet error rates in cooperative and non-cooperative state and the transitional probabilities between these states. Therefore, in practical applications we can control the capacity of the single-relay multi-hop channel by controlling these probabilities.

In order to show which procedure need to be followed to find the capacity of a multi-relay network with n relays we will demonstrate how to find the capacity for 2-relay network.

2.3 Two-Relay *H*-hop Network Capacity

In the case when a multi-relay system has 2 relays it can be represented by a 2^2-state model as shown in Fig. 3. Let us denote the channel states by both decimal and binary subscripts: the non-cooperative state is then S_0 and the cooperative states are S_1, S_2 and S_3, or in binary form all the states are denoted as 00, 01, 10 and 11, respectively. The model is completely described by four steady state probabilities and related transition probabilities. The sum of these steady-state probabilities is one.

Because the packet errors occur independently in all relay channels and in all hop-channels, the transitional probability of staying in state S_0 is equal to the probability of state S_0, which can be calculated as a conditional probability

$$P(S_0 \mid S_0) = \left[\sum_{h=1}^{H} BL_{1h} \prod_{j=0}^{h-1} (1 - BL_{1j}) \right] \cdot \left[\sum_{h=1}^{H} BL_{2h} \prod_{j=0}^{h-1} (1 - BL_{2j}) \right] \tag{10}$$

where the initial value for (1- BL_{2j}) is zero, i.e. (1- BL_{20}) = 0. Similarly, we can find the transitional probabilities of other states as

$$\begin{aligned} P(S_1 \mid S_i) &= \left[\sum_{h=1}^{H} BL_{1h} \prod_{j=0}^{h-1} (1 - BL_{1j}) \right] \cdot \left[\prod_{h=1}^{H} (1 - BL_{2h}) \right] \\ P(S_2 \mid S_i) &= \left[\prod_{h=1}^{H} (1 - BL_{1h}) \right] \cdot \left[\sum_{h=1}^{H} BL_{2h} \prod_{j=0}^{h-1} (1 - BL_{2j}) \right] \\ P(S_3 \mid S_i) &= \left[\prod_{h=1}^{H} (1 - BL_{1h}) \right] \cdot \left[\prod_{h=1}^{H} (1 - BL_{2h}) \right] \end{aligned} \tag{11}$$

These probabilities are equal to the probabilities of the steady-states of the model.

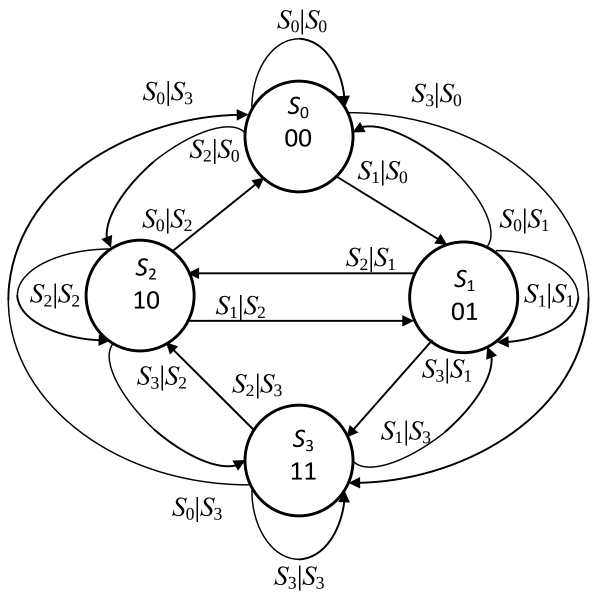

Fig. 3 Two-relay four-state channel model.

The overall probability of packet error in the multi-relay multi-hop channel is

$$BL_e = \sum_{i=0}^{3} P(S_i) \cdot P(e_p \mid S_i) = \sum_{i=0}^{3} P(S_i \mid S_i) \cdot P(e_p \mid S_i), \tag{12}$$

where $P(S_i)$ is the steady state probability of state S_i. For the sake of analysis, suppose that the probabilities of packet errors in all cooperative states are equal. Then the overall packet error rate can be expressed as a function of the packet error rates in cooperative BL_C and non-cooperative states $BL_{\overline{C}}$ as

$$BL_e = P(S_0) \cdot PL_{\overline{C}} + PL_C \sum_{i=1}^{3} P(S_i) = P(S_0) \cdot PL_{\overline{C}} + PL_C \cdot (1 - P(S_0))$$

We may insert (10) and (11) into the last equation to get the overall packet error probability in the two-relay multi-hop system as

$$\begin{aligned} BL_e = {} & PL_{\overline{C}} \cdot \left[\sum_{h=1}^{H} BL_{1h} \prod_{j=0}^{h-1} (1 - BL_{1j}) \right] \cdot \left[\sum_{h=1}^{H} BL_{2h} \prod_{j=0}^{h-1} (1 - BL_{2j}) \right] \cdot \\ & + PL_C \cdot \left[\sum_{h=1}^{H} BL_{1h} \prod_{j=0}^{h-1} (1 - BL_{1j}) \right] \cdot \left[\sum_{h=1}^{H} BL_{2h} \prod_{j=0}^{h-1} (1 - BL_{2j}) \right] \end{aligned} \tag{13}$$

Using this expression the capacity for this channel can be calculated in the similar manner as for the two-state model.

2.4 Analysis of a special case

Let us analyse a special case when all packet error probabilities are equal, i.e.

$$\begin{aligned} BL_{11} &= BL_{12} = \cdots = BL_{1H} = BL_1 \\ BL_{21} &= BL_{22} = \cdots = BL_{2H} = BL_2 \end{aligned} \tag{14}$$

The probability of steady states can be calculated from (10). By developing this expression and adding the terms the final result is

$$\begin{aligned} P(S_0 \mid S_i) = P(S_0) &= \left[\sum_{h=1}^{H} BL_1 \prod_{j=0}^{h-1} (1 - BL_1) \right] \cdot \left[\sum_{h=1}^{H} BL_2 \prod_{j=0}^{h-1} (1 - BL_2) \right] \\ &= \left[\sum_{h=0}^{H} BL_1 \cdot (1 - BL_1)^{h-1} \right] \cdot \left[\sum_{h=1}^{H} BL_2 \cdot (1 - BL_2)^{h-1} \right] \\ &= [1 - (1 - BL_1)^H] \cdot [1 - (1 - BL_2)^H] \end{aligned}$$

By developing this expression and adding the terms the final result is

$$\begin{aligned} P(S_1 \mid S_i) &= [1 - (1 - BL_1)^H](1 - BL_2)^H \\ P(S_2 \mid S_i) &= (1 - BL_1)^H [1 - (1 - BL_2)^H] \\ P(S_3 \mid S_i) &= (1 - BL_{1h})^H \cdot (1 - BL_{2h})^H \end{aligned} \tag{15}$$

The overall packet error rate in (13) can be expressed in simplified form as

$$BL_e = [1-(1-BL_1)^H]\cdot[1-(1-BL_2)^H]\cdot PL_{\overline{C}} + PL_C\cdot(1-[1-(1-BL_1)^H]\cdot[1-(1-BL_2)^H]) \quad (16)$$

If we insert (7) into (8), and calculating the sum of geometric series inside the summation sign, we can find the capacity as a function of the overall probability of packet errors in the two-relay multi-hop channel, expressed as

$$C = 1 + BL_e\cdot\log_2 BL_e + (1-BL_e)\cdot\log_2(1-BL_e) \quad (17)$$

where the overall packet error probability depends on the probabilities that define our two-relay multi-hop channel to be multi-state channel because the capacity depends on these probabilities: packet error rate in cooperative and non-cooperative state and the transitional probabilities between these states. Therefore, in practical applications we can control the capacity of the two-relay multi-hop channel by controlling these probabilities.

3 Numerical Analysis of a Multi-Relay Multi-Hop Channel

We will analyse the special case introduced in Section 2.4 assuming that the packet error probabilities in all relay channels are equal to each other, i.e., $BL_i = BL$ for i = 1 and 2. This would be the case when the relay nodes are chosen in such a way to receive the same power from the source node and from the neighbouring relays, which guaranties that the received signals at relays are approximately of the same level. In this case the overall packet error rate in can be expressed in simplified form as

$$BL_e = [1-(1-BL)^H]^2\cdot BL_{\overline{C}} + BL_C\cdot(1-[1-(1-BL)^H]^2) \quad (18)$$

The main interest in this research is to understand how the capacity of a multi-relay multi-hop channel depends on the packet error probability. The smaller this probability is the less average energy of the source and relays is required.

The dependence of the capacity on the packet error rate in non-cooperative state $BL_{\overline{C}}$, the number of relays n =1 and 2 and the number of hops H = 1 and 2 as parameters, is presented in Fig. 4. When packet error probability in the non-cooperative case $BL_{\overline{C}}$ decreases the channel capacity increases for all cases, because the channel spends some more time in cooperative state. By spending more time in cooperative states the system reduces the probability of receiving erroneous packets at the destination and the capacity increases. Because the probability of packet error in the capacity formulae exponentially decreases with the number of relay, it is obvious that the capacity of the multi-relay systems will increase when the number of relays increases. Also, when the number of hops increases the capacity of the channel decreases.

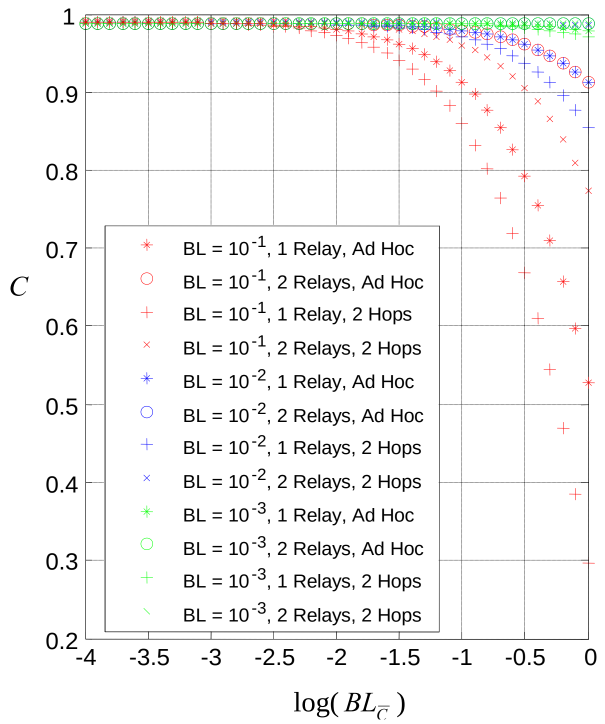

Fig. 4 The channel capacity versus the packet error probability in non-cooperative state for the number of relays and the number of hops as parameters.

We are interested to know the capacity of the multi-relay channel for the fixed values of the overall probability of packet error in this channel. Furthermore, we may ask ourselves how to specify the packet error rate to maximise the channel capacity. The answer to this question is offered by plotting the dependence of the capacity C as a function of P_{en} for the BL values and the number of relays and the number of hops as parameters.

The proposed model and theoretical derivatives for probability of overall packet error rate of a multi-relay channel are developed to serve as practical tools in the design of wireless multi-relay networks. For required the overall probability of packet error and the channel capacity, the model allows us to specify the number of relays and the number of hops for the optimum probability of packet error to minimise the overall power consumption in the network.

4 Conclusion

A multi-relay multi-hop channel can be represented by a multi-state digital channel model that is based on Markov chains theory. The exact expression for the overall probability of packet errors and the channel capacity are derived and numerically analysed. Based on the presented theory and procedures the channel models can be developed and related capacity can be derived for multi-rate multi-hop networks.

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