Vibration analysis for the rotational magnetorheological damper

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Abstract—Comfort, reliability, functionality performance which provide a longer life cycle requires thorough understanding and analysis of the vibrations, this is a general rule for most of the static and dynamic functionality applications. Vibrations is an extremely important issue to consider when designing various systems.

The hysteresis in the dampers is very important issue when characterizing a damper, it is a very complex phenomena but very important to consider. The hysteresis equations of Bouc-Wen, Lugre, and Dahl have been modeled and simulated in Matlab/Simulink. We have manipulated the different parameters in the models and analyzed their effects on the outcome. The hysteresis models of Bouc-Wen, Dahl and LuGre have been analyzed and compared analytically to really show the difference in the models. At last the Bouc-Wen model was implemented together with the SAS (Semi Active Suspension) system in the laboratory. The model parameters were tuned manually to try to fit the response of the system.

In this paper a predefined methodology has been applied for determining the hysteresis loop parameters using the data collected for vibration analysis under predefined test specifications. The following data has been used later to regenerate the vibration signal, so on get as closer to the real signal.

In the coming work, advanced method will be used to determine the exact parameters for the hysteresis loop as well as using the inverse hysteresis to improve the performance of the vibration suspension in the Semi Active Suspension system.

The behavior of MR dampers can be presented with different mathematical models. The Bouc-Wen model was found to be model to both illustrate the MR damper and recreate the behavior of the SAS system.

Keywords: Magnetorheological damper, Bouc-Wen, Lugre, Dahl, SemiActive Suspension

I. VIBRATION ANALYSIS

The hysteresis identification can be a complex process, understanding the static and the dynamic vibration in the system enable facilitating the process of the hysteresis identification. In this work different vibration responses of the MR damper. When using fluid dampers the exerted force, or torque, will respond differently to vibrations with respect to the system’s natural frequency and the exerted vibration on the damper. We will discuss the static response compared to the dynamic response using the models discussed in III.

A. Static vibration

Static vibration on the MR damper is interpreted as when the damper is working without the mechanical dynamics of the system. The damper’s exerted force, or torque, is then a function fully described by the current and velocity. To illustrate the effects of the vibration, the Matlab Simulink models are used to simulate the maximum torque of the damper with respect to frequency when the displacement is forced in a sine motion. The result using the Bouc Wen model is shown in Figure 1

![Fig. 1. Vibration response with respect to frequency using the Bouc Wen model. Displacement is forced in a sine motion creating a frequency dependent torque.](image)

When using a dynamic model, here Bouc Wen, someone can observe high torque even at low frequencies.

B. Dynamic vibration

Considering the dynamics in a system, it is important to have knowledge of its vibrational response. The system will have a natural frequency and applying a harmonic excitation near the system’s natural frequency can create a highly unstable system. We use the semi-active damper to control and mute these critical vibrations. To analyze, the mechanical system is built up using

$$T = I\ddot{\theta} = -\left(k_s\dot{\theta} + T_{mr}\dot{\theta} + T_{in}\right)$$  \hspace{1cm} (1)

The model is shown in Figure 2 with fictitious values. The system has a natural frequency of
To make a resonance plot, it is required to give the input torque in Figure 2 a frequency, measure the maximum displacement and redo the process with a new frequency. The result is shown in Figure 3.

II. TYPES OF FRICTION

The MR damper can be modeled with different types of mathematical models. Each model describes different aspects of friction and/or dynamic properties of the MR damper. In this section, some friction aspects together with the models will be later simulated, compared and analyzed.

A. Hysteresis

Hysteresis is a dynamic friction phenomena which represents the history dependence of physical systems.\[1\] We get a model for the systems nonlinear behavior at low velocities. Figure 5 show an example of hysteresis. The force versus velocity curve is not coincide for increasing and decreasing velocities.

B. Coloumb friction

Note how the peak is shifting towards higher frequency ratios as the current is increased. For frequency ratios higher than 2.2, the best result may come with lower or no current applied.

In Figure 4 the actual result retrieved from the SAS system is shown. The frequency increases linearly from low (wheel slowly turning) to high. At around 30 seconds, the system is near the its natural frequency (~ 1.6Hz) resulting in maximum displacement change. Passing this critical frequency will again calm the system.

The Coloumb friction model represents the static relationship between friction forces and velocities. This model cannot reproduce friction characteristics that depend on time.\[4\]
C. Viscous friction

Viscous friction is static and linearly dependent to the sliding velocity.

D. Stiction

If someone combine static, Coulomb and the viscous friction model, someone will get a model of stiction. A stiction model includes the threshold force that is needed to start a movement between two bodies, the constant Coulomb friction and the velocity dependent viscous friction.

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G. Strubeck effect

The Strubeck effect appears when the friction is decreasing and later is increasing with increasing velocities starting at zero velocity. This effect is seen in for example bearings. The oil thickness is building up in the beginning, causing the drop in the friction force.

H. Stick-slip motion

If a mass $m$ is pulled with a spring at a constant speed $v_p$ along a surface; the mass will have a periodic motion where the mass varies between sticks and slips.

I. Zero-slip motion

$$m \cdot a = F_d - F$$

The applied force $F_d$ is smaller than the stiction force. This occurs when a masses is dragged along a surface with a low constant velocity while the friction keeps the mass from slipping.
Fig. 11. Zero-slip motion[5]

III. MODELS

A. The Bingham model

Fig. 12. Bingham mechanical model[2]

\[
F_{mr} = F_c \cdot \text{sign}(\dot{x}) + c_0 \dot{x} + F_0
\]  

\(\dot{x}\) : Piston velocity  
\(F_c\) : Frictional force  
\(c_0\) : Damping constant  
\(F_0\) : Offset value (Constant force)

From equation 4 it can assumed that the shape of the Bingham force profile will be equal to the coloumb plus viscous friction as seen in Figure 13. The friction force \(F_c\) equals the starting point of the graph on the y-axis. The damping constant \(c_0\) equals the linear relationship \(\frac{\Delta F}{\Delta \dot{x}}\) which appears when the velocity \(\dot{x}\) is not zero.

The Bingham model is clearly linear and since the MR damper is highly nonlinear, this model will not be an area of focus and detailed study.

B. Dahl friction model

The Dahl's-model was made for the purpose of simulating with friction. It’s a simple dynamic model which captures many properties such as the hysteresis and zero slip displacement, but does not capture the Stribeck effect or stiction [6]. The friction represented by this model depends only on displacement. This relatively simple model has a typical hysteresis curve as shown in Figure 14

\[
\frac{dF}{dx} = \sigma_0 \left(1 - \frac{F}{F_c} \text{sign}(v)\right)
\]

\(\sigma_0\) : Stiffness  
\(F_c\) : Columb friction  
\(v\) : Velocity

Figure 14 shows the force versus displacement hysteresis of the basic Dahl model. We see that the graph is different when the displacement is increasing compared to when its decreasing.

The Dahl model has been modified to show some of the properties of the MR damper[7]:

\[
f_{mr} = k_x \cdot \dot{x} + [k_{wa} + k_{wb}] \cdot v \cdot w
\]

\(\dot{x}\) : Velocity  
\(v\) : Control voltage  
\(w\) : Hysteric variable  
\(k_x, k_{wa}, k_{wb} \) and \(\rho\) : Parameters

C. LuGree friction model

\[
\frac{dz}{dt} = v - \sigma_0 \cdot \frac{|v|}{g(v)} \cdot z = v - h(v) \cdot z
\]

\(F = \sigma_0 \cdot z + \sigma_1 \cdot \dot{z} + f(v)\)
For constant velocity, the steady state force $F_{ss}$ is [4]:

$$F_{ss} = g(v) \text{sgn}(v) + f(v) \quad (10)$$

$$g(v) = F_c + (F_s + F_c)^{-v/v_0} \quad (11)$$

$v$: Velocity between the surfaces in contact

$z$: Internal friction state (average bristle deflection)

$f(v)$: The viscous friction (velocity dependent)

$g(v)$: Approximation of the Stribeck effect and Coulomb friction

$F_s$: Stiction force (static friction)

$F_c$: Coulomb friction

$\sigma_0$: Stiffness

$\sigma_1$: Microdamping

$v_s, \alpha$: Constants

The LuGre model is an extension of the Dahl model.[4] $v_s$ is a constant which determines how quickly $g(v)$ approaches the friction force $F_c$. The LuGre model is very advanced and captures many aspects of friction.

D. The Bouc-Wen model

$$f_{mr} = c_0 \cdot \dot{x} + k_0 (x - x_0) + \alpha \cdot z \quad (12)$$

$$\dot{z} = -\varphi \cdot |\dot{x}| \cdot z \cdot |z|^{n-1} - \beta \cdot \dot{x} \cdot |z|^{n} + \kappa \cdot \dot{x} \quad (13)$$

$c_0$: Damping constant

$\dot{x}$: Piston velocity

$k_0$: Spring constant

$\alpha, \beta, \varphi, \kappa$: Parameters

$z$: Bouc-Wen hysterisis constant

The Bouc-Wen model is the most commonly used model to capture the behavior of non-linear hysteretic systems such as the MR damper.[8] This model can produce a variety of hysteretic patterns.[2]

IV. Testing hysteresis-models on the SAS system

In this section, the zooming and investigating if the Dahl and the bouc-wen can fit the MR damper in the suspension system in the lab. We have not been focusing on the LuGre model because this is just an extension to the Dahl model and much harder to tune because of the many parameters. The SAS system is simplified and modeled in matlab/simulink.

V. SAS model

Since a static vibration analysis with no motion of the wheel, in this situation, it is assumed the wheel to be stiff and with no damping.

The spring represents the physical spring on the model. The damper represents the constant damping in the MR damper plus other external things as friction and so on. The last part is representing the force from the nonlinear damping in the MR damper. $\theta_1$ and $\theta (\theta = \theta_2 - \theta_1)$ is measured through a sensor on the model and plotted on the computer.

VI. Equations for system with Bouc-Wen and Dahl

Model-equations with Bouc-Wen:

$$T = J\ddot{\theta} = -k\theta - c(i)\dot{\theta} - \alpha(i)z \quad (14)$$

$J$ is redundant:

$$\dot{\theta} = -k\dot{\theta} - c(i)\dot{\theta} - \alpha(i)z \quad (15)$$

$c(i)$ and $\alpha(i)$ are defined in the following:

$$c(i) = c_1 + c_2 \cdot i \quad (16)$$

$$\alpha(i) = \alpha_1 + \alpha_2 \cdot i \quad (17)$$

The Bouc-Wen non linearity is a function of $z$. $Z$ can be calculated from the differential equation:

$$\dot{z} = -\gamma|\dot{\theta}|z|z|^{n-1} - \beta\dot{\theta}|z|^{n} + \delta\dot{\theta} \quad (18)$$

This equation was modeled in matlab/simulink and then merged together with the rest of the system.

On the left side, the bouc-wen model (green system) for calculating det parameter $z$. The yellow system is the contribution from the active damping, dependent on the current.
The white blocks illustrate the physical system with a linear spring and damper. The purple block multiplied by z is the passive nonlinear damping.

Model-equations with Dahl:
\[ T = J\ddot{\theta} = -k\theta - k_x \cdot \dot{\theta} + (k_{wa} + k_{wb} \cdot v) \cdot w \quad (19) \]

\[ J \text{ is redundant:} \]
\[ \ddot{\theta} = -k\theta - k_x \cdot \dot{\theta} + (k_{wa} + k_{wb} \cdot v) \cdot w \quad (20) \]
\[ \dot{w} = \rho \cdot (\dot{\theta} - |\dot{\theta}|) \quad (21) \]

The orange blocks represents the dahl-model for MR damper and the white blocks represents the SAS-model.

VII. EXPERIMENTAL TESTING

The experimental tests done using the SAS test setup as shown in figure(19) when pressing the "arm" down to an initial \( \theta(0) = \theta_0 = -5\text{deg} \) and then drop. The arm would then oscillate around equilibrium.

After trying to fit the model by changing the parameters for the hysteresis models, it is shown that the Bouc-Wen actually is a better fit than Dahl. Fitting the Dahl model was more difficult. The problem was that the nonlinear damping leads to a varying frequency, which the Dahl has a bigger problem with than Bouc-Wen. The Bouc-Wen is more adjustable because of the nonlinear terms with the exponential n-parameter.

Next, the same test with 1 amp into the MR damper is performed.

The results with 0A and 1A on the MR damper gave us these curves:

First, trying to fit the dahl and bouc-wen into the passive system with no current.

Bouc-Wen(passive) | Dahl(passive)
---|---
\( n = 0.099 \) | \( \rho = 10 \)
\( \gamma = 1.0 \) | \( k_{wa} = 0.6 \)
\( \beta = 873 \) | \( k_1 = 109 \)
\( \delta = 700 \) | \( c_1 = 0.7 \)
\( c_1 = 0.7 \) | \( k_x = 0.15 \)
\( \alpha_1 = 1 \) | 
\( k_1 = 109 \)

When looking at the Figure(22), it is shown that the Bouc-Wen model has a much better fit than the Dahl.

Seen from Figures(23) and (24) the hysteresis of Dahl is a bit larger than the Bouc-Wen hysteresis(Figure(23)). When the current is set to one, the Changes are increasing dramatically. The Bouc-When which is the most correct model for this MR...
damper has increased to about two times of the Dahl model. This is mainly because the linear damping in the used Dahl model, does not have a linear damping coefficient proportional to the current such as our Bouc-Wen.

VIII. RESULTS

These experiments show us that Bouc-Wen is more adaptable to our system. Even though the Dahl hysteresis is quite similar to the Bouc-Wen, it could not fit the system as good. With no current on the damper the differences were quite small, and could probably been even smaller if someone had used a specific scientific method to estimate the parameters. When the current was set to one, the Dahl model could not manage to increase the linear slope of the hysteresis. This problem could easily be solved by adding a term for the linear damping dependent on current. Like, \( k_i \cdot i \).

A source of error in addition to inaccurate parameters could be that the neglect of the tire damping makes our simplified model too inaccurate.

IX. CONCLUSION

The determined parameters using different models has been utilized to determine the dynamic hysteresis loop, the process was done using different models which simulate the hysteresis implemented in a SAS system.

- The methodology of identifying the hysteresis parameters has been implemented using two different models: Dahl and Bouc-Wen.
- The Semi Active Suspension system has proven to be efficient for studying the rotational MR damper behavior, thus including the vibration and hysteresis analysis.
- The identified parameters which has been harvested from the passive and active vibration data has been used to determine the hysteresis dynamics in different models.
- The Dahl, Lugre and Bouc-Wen have all an adaptable hysteresis, but the Bouc-Wen is found to be the best model for illustrating the MR damper. The Dahl is also pretty good in the passive part but it needs to be modified to fit the active properties. This may be done by adding a voltage dependent damping coefficient similar to the Bouc-Wen and Lugre models.

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