# Finite Element Analysis for Reactive and Dissipative Rectangular Muffler

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*Abstract:* - The paper describes the pressure-wave propagation in a muffler for an internal combustion engine in case of a lined muffler with rectangular cross section resonator chamber. The approach is generally applicable to analyzing the damping of propagation of harmonic pressure waves. The paper purpose is to show finite elements analysis of both inductive and resistive damping in pressure acoustics. The main output is the transmission loss for the frequency range 50 Hz–3000 Hz.

*Key-Words:* - absorptive muffler, transmission loss, geometry of muffler, glass wool

# **1** Introduction

A silencer is an important noise control element for reduction of machinery exhaust noise, fan noise, and other noise sources involving flow of a gas, so that these devices should classify in specific norms and regulations [1].

In this paper, we will examine two cases of reactive and dissipative muffler, using FEM – finite element method. The detailed design procedures for mufflers and absorbent materials are available in the literature [2], [3] and [7].

There are two basic types of mufflers:

•*Reflective* (or *reactive*) *mufflers* - those that reflect acoustic waves by abrupt area expansions or changes of impedance.

•*Dissipative mufflers* - mufflers based on dissipation of acoustic energy into heat through viscous losses in fibrous materials or flow-related (resistive) losses in perforated pipes.

Reflective mufflers are best suited for the low frequency range where only plane waves can propagate in the system, while dissipative mufflers with fibers are efficient in the mid-to-high frequency range. Dissipative mufflers based on flow losses, on the other hand, work also at low frequencies. A typical automotive exhaust system is a hybrid construction consisting of a combination of reflective and dissipative muffler elements.

The reflective parts are normally tuned to remove dominating low-frequency engine harmonics while the dissipative parts are designed to take care of higher-frequency noise.

In the industry, exhaust systems are typically analyzed with nonlinear 1D gas-dynamics codes. Such codes, however, do not capture 3D acoustic effects such as higher-order duct modes, and the modeling of fibrous materials is not satisfactory. In practice, there is therefore a need to use linear acoustic models of exhaust and intake systems to enable detailed modeling and optimization of the acoustic response, at the cost of neglecting nonlinear effects.

The expansion chamber muffler consists of one or more chambers or expansion volumes which act as resonators to provide an acoustic mismatch for the acoustic energy being transmitted along the main tube.

Reactive silencers consist typically of several pipe segments that interconnect a number of larger diameter chambers or rectangular cross section resonator chamber. These silencers reduce the radiated acoustical power primarily counting on to the impedance mismatch, that is, by allowing the acoustical impedance discontinuities to reflect sound back toward the source.

The noise from an exhaust system consists of three components: pulsation noise and flow generated noise coming from the orifice of the muffler outlet and shell noise coming from the shell of the muffler. Shell noise may be limited by using a stiffer or damped shell, while flow generated noise, such as turbulence and vortex shedding may be limited by minimizing geometrical discontinuities (edges, sharp bends etc) [8].

Minimizing pulsation noise, caused by the valves opening and closing inside the IC engine, is the focus of this paper and this is obtained by designing the internal parts of the muffler in such a way that the most critical part of the frequency spectrum is attenuated.

The use of a silencer is prompted by the need to reduce the noise radiated from a source but in most applications the final selection is based on some trade-offs among the predicted acoustical performance, the mechanical performance, the volume/weight ratio, and the cost of the resulting system.

Most silencers are subject to volume/weight constraints, which also influence the silencer design process. In addition, the initial purchase/installation cost and the periodic maintenance cost are other important factors that influence the silencer selection process.

## 2 Model Definition

The muffler—schematically depicted in Figure 1 consists of a resonator chamber with centred exhaust pipe included at each end (Fig. 1). The radius of input and output tube is 50 mm, both 150 mm length. Resonator chamber is a square cross section of side 200 mm and 600 mm length.

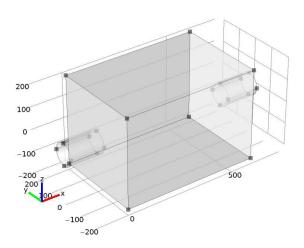


Fig. 1. Geometry of the lined muffler with rectangular cross section

The exhaust fumes enter through the left pipe and exit through the right pipe. In the first version of the model the chamber is empty (Solution 1). In the second version it is lined with 15 mm of absorbing glass wool (Solution 2).

## **3** Domain Equations

This model solves the problem in the frequency domain using the time-harmonic Pressure Acoustics application mode. The model equation is a slightly modified version of the Helmholtz equation for the acoustic pressure, p:

$$\nabla \cdot \left( -\frac{\nabla p}{\rho} \right) - \frac{\omega^2 p}{c_s^2 \rho} = 0 \tag{1}$$

where  $\rho$  is the density,  $c_s$  equals the speed of sound, and  $\omega$  gives the angular frequency.

In the absorbing glass wool, the damping enters the equation as a complex speed of sound,  $c_c = \omega/k_c$ , and a complex density,  $\rho_c = k_c Z_c/\omega$ , where  $k_c$  is the complex wave number and  $Z_c$  equals the complex impedance. For a highly porous material with a rigid skeleton, the well-known model of Delany and Bazley estimates these parameters as functions of frequency and flow resistivity.

Using the original coefficients of Delany and Bazley [2], the expressions are

$$k_{c} = k_{a} \cdot \left[ 1 + 0.098 \cdot \left( \frac{\rho_{a} f}{R_{f}} \right)^{-0.7} \right] - K_{a} \left[ i \cdot 0.189 \cdot \left( \frac{\rho_{a} f}{R_{f}} \right)^{-0.595} \right]$$
(2)

$$Z_{c} = Z_{a} \cdot \left[ 1 + 0.057 \cdot \left( \frac{\rho_{a}f}{R_{f}} \right)^{-0.734} \right] -$$

$$- Z_{a} \cdot \left[ i \cdot 0.087 \cdot \left( \frac{\rho_{a}f}{R_{f}} \right)^{-0.732} \right]$$
(3)

where  $R_f$  is the flow resistivity, and where  $k_a = \omega/c_a$ and  $Z_a = \rho_a c_a$  are the free-space wave number and impedance of air, respectively. You can find flow resistivities in tables. For glass-wool-like materials, Bies and Hansen [3] give an empirical correlation

$$R_{f} = \frac{3.18 \cdot 10^{-9} \cdot \rho_{ap}^{1.53}}{d_{av}^{2}}$$
(4)

where  $\rho_{ap}$  is the material's apparent density and  $d_{av}$  is the mean fiber diameter. This model uses a rather lightweight glass wool with  $\rho_{ap} = 12 \text{ kg/m}^3$  and  $d_{av} = 10 \text{ }\mu\text{m}$ .

#### **4** Boundary Conditions

The boundary conditions are of three types:

➢ At the solid boundaries, which are the outer walls of the resonator chamber and the pipes, the model uses sound hard (wall) boundary conditions:

$$\left(-\frac{\nabla \boldsymbol{\rho}}{\boldsymbol{\rho}}\right) \cdot \mathbf{n} = \mathbf{0} \tag{5}$$

The boundary condition at the inlet involves a combination of incoming and outgoing plane waves:

$$\mathbf{n} \cdot \frac{1}{\rho_0} \nabla \rho + ik \frac{\rho}{\rho_0} + \frac{i}{2k} \Delta_\tau \rho =$$

$$= \left\{ \frac{i}{2k} \Delta_\tau \rho_0 + \left[ 1 - (\mathbf{k} \cdot \mathbf{n}) \right] ik \frac{\rho_0}{\rho_0} \right\} e^{-ik(\mathbf{k} \cdot \mathbf{r})}$$
(6)

In this equation,  $p_0$  represents the applied outer pressure,  $\Delta_T$  is the boundary tangential Laplace operator, and *i* equals the imaginary unit [4]. This boundary condition is valid as long as the frequency is kept below the cutoff frequency for the second propagating mode in the tube.

At the outlet boundary, the model specifies an outgoing plane wave:

$$n \cdot \frac{1}{\rho_0} \nabla p + i \frac{k}{\rho_0} p + \frac{i}{2k} \Delta_T p = 0 \qquad (7)$$

### **5** Results and Conclusions

We apply the required boundary conditions and then perform the meshing for free tetrahedral option with 0,25 x - direction scale.

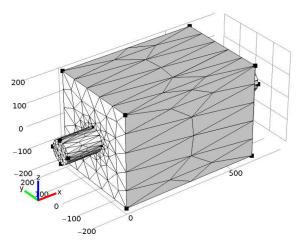


Fig. 2. Meshing of the lined muffler with rectangular cross section

Figure 2 show the 3D meshing of lined muffler with rectangular cross section.

The following equation defines the attenuation (in dB) of the acoustic energy (or transmission loss),  $d_w$  [5, 6, 10]:

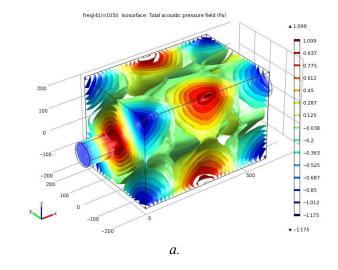
$$d_{w} = 10 \log \left(\frac{W_{o}}{W_{i}}\right)$$
(8)

Here  $w_0$  and  $w_i$  denote the outgoing power at the outlet and the incoming power at the inlet, respectively. You can calculate each of these quantities as an integral over the corresponding surface:

$$W_{o} = \int_{\partial\Omega} \frac{|\boldsymbol{p}|^{2}}{2\rho \boldsymbol{c}_{s}} d\boldsymbol{A}$$
(9)

$$\boldsymbol{W}_{i} = \int_{\partial\Omega} \frac{\boldsymbol{\rho}_{0}^{2}}{2\rho \boldsymbol{c}_{s}} d\boldsymbol{A}$$
(10)

In the Figure 3 is plot the result of acoustic pressure levels for selected geometry of the muffler, absolute pressure (Fig. 3a) for the case of an empty muffler without any absorbing material and total acoustic pressure field (Fig. 3b) with a layer of lining on the chamber's upper and lower walls.



freq(41)=1050 Isosurface: Total acoustic pressure field (Pa)

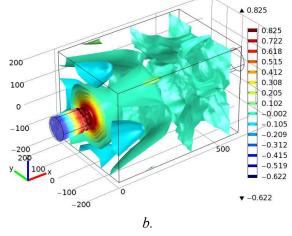
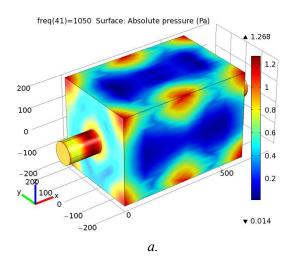


Fig. 4. Isosurface – Total acoustic pressure field

Figure 5 shows the result of attenuation of the muffler, a parametric frequency study for the case of an empty muffler without any absorbing material (Solution 1) and attenuation with a layer of lining on the chamber's upper and lower walls (Solution 2). The plot shows that the damping works rather well for most low frequencies with the exception of a few distinct dips where the muffler chamber displays resonances.

At frequencies higher than approximately 1050 Hz, the plot's behaviour is more complicated and there is generally less damping. This is because, for such frequencies, the muffler supports not only longitudinal resonances but also cross-sectional propagation modes. Not very far above this frequency a whole range of modes that are combinations of this propagation mode and the longitudinal modes participate, making the damping properties increasingly unpredictable.

The glass-wool lining (Solution 2) improves attenuation at on almost all the frequencies studied [9].



freq(41)=1050 Surface: Total acoustic pressure field (Pa)

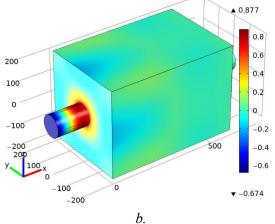


Fig.3. Acoustic pressure levels for at 1050 Hz: a). Absolute pressure; b). Total acoustic pressure field

In Fig. 3b can be seen that there are sharp variations of color - so a small change in pressure and low pressure is present, where we can deduce that the resonance phenomenon does not appear as in figure 3a.

In the Figure 4 is plot the result of isosurface of total acoustic pressure field for selected geometry of the muffler for the case of an empty muffler (see Fig. 4a) without any absorbing material and with a layer of lining on the chamber's upper and lower walls (see Fig. 4b).

For both cases, the graphical representation (Fig. 4) corresponds to frequency of 1050 Hz.

An **isosurface** is a three-dimensional analog of an isoline. It is a surface that represents points of a constant value (in this case it is pressure but it may be and depending on temperature, velocity or density) within a volume of space of muffler; in other words, it is a level set of a continuous function whose domain is 3D-space. For the representation in Figure 5 was set a total of 15 levels.

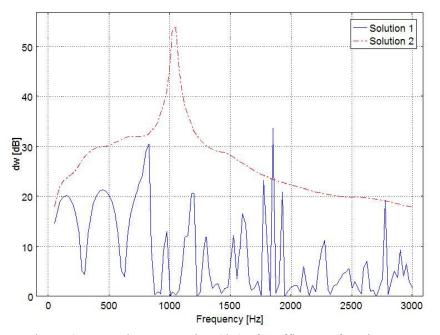


Fig. 5. Comparative attenuation (dB) of muffler as a function

This model uses the Pressure Acoustics physics interface of the Acoustics Module. This interface has the Delany-Bazley [6] coefficients built in. Therefore, the only damping parameter you need to supply is the flow resistivity. The parametric solver provides results for a range of frequencies. The software computes integrals in the power expressions using boundary integration coupling variables, and it plots the resulting attenuation versus frequency.

It should be noted, however, not always possible to use a sound-absorbing material, due to various factors such as high temperature, high speed.

We are still concerned, in the near future, the verification method with finite element modeling results determined in real conditions.

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