Numerical optimization of flow-heat ducts with helical micro-fins, using Entropy Generation Minimization (EGM) method.

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Abstract:
An enhancement of heat transfer in heat exchangers channels is always associated with increased pressure drop and energy for pumping - especially when the channel wall is not smooth. Bearing in mind entire channel as a thermodynamic system, there are two irreversible "competing" processes - fluid flow and heat transfer, which cause entropy generation in it. In the literature, this method of analysis was called Entropy Generation Minimization (EGM) or Thermodynamic Design (THD). In this paper are presented theoretical basis and geometry formulation for considering cases and results of numerical simulations for fully developed 3D flow in tubes with micro-fins on the wall. As a parameter, was examined helical angle of micro-fin (referred to the axis tube). This angle was changed in the range from 0 (grooves parallel to the axis) to 90 degrees (grooves perpendicular to the axis). To realize numerical simulation ANSYS-CFX code was used. Before numerical investigation, for tube with helical angle of 30 degrees, verification of calculation code has been carried out. Comparison results of numerical simulations and experiment (obtained from a specially built stands) gave a good correlation, especially for the SST turbulence model. On this basis, the results obtained from simulations, for other angles are also credible. The 3D-chart presents results in dimensionless and normalized coordinates. Generated entropy and its minimum is shown as a function of the helical angle and Reynolds number. Additionally, on the graph, the distributions of the heat transfer coefficient on the grooved surface for studied angles were shown – as an example of visualization capabilities.

Key-Words: Entropy Generation Minimization, numerical simulation, helical angle, micro-fins, channel

1 Introduction

Heat transfer enhancement in any heat exchanger ducts is associated with an increase of the pressure drop required for pumping – particularly when the duct is not smooth. Taking into consideration a whole duct as a thermodynamic system, there are two irreversible "opponent" processes which cause generation of entropy in it – heat transfer and fluid flow. Bejan [1], as one of the first researchers, gave the appropriate theoretical background of this approach and called it Thermodynamic Design (THD) or Entropy Generation Minimization - EGM method. This approach can be used to determine the operating parameters or geometric shapes and sizes thermal devices. One can be also determine the thermodynamic optimum, which is the smallest amount of the lost exergy. However, it requires reliable information about correlation of friction factor - f and Nusselt number - Nu. For smooth tube, basic relations and solution of this problem was given by Bejan [1], whereas for more complicated geometrically channels these coefficients and dependences are very often unknown. As a point of reference for all results obtained from numerical calculations, was a geometry of a smooth pipe.

The purpose of this paper is present results and analysis for ducts with helical angle of micro-fins – using EGM method. This approach bases on computer simulation results which have been obtained for fully developed fluid flow with heat transfer, in elementary, periodic and repeatable section of helical grooved duct. In computer simulations and experiments as a fluid was used the water with an average temperature 310 K. In these studies, the values of mass flow rate were changed, while the wall heat flux per unit length was a constant for all cases – in spite of different area of heat transfer for every β angle. (This effect was achieved by appropriate matching areal heat flux for

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Nomenclature

B	constant, expressed by (eq. 6)	Nu	Nusselt number
Be	Bejan number, expressed by (eq. 2)	Re	Reynolds number
L	duct section length [mm]	Pr	Pandtl number
$\dot{m}$	mass flow rate through the duct [kg/s]	D	outside diameter [mm]
q'	wall heat flux per unit length [W/m]	d	root diameter [mm]
Re_{opt}	value for minimum entropy generation	d_t	inside diameter [mm]
S'_{gen}	total entropy generation rate [W/mK]	d_h	hydraulic diameter [mm]
$S'_{gen,min}$	minimum entropy gener. rate [W/mK]	H	height of grooves [mm]
$S'_{gen,\Delta T}$	heat transf. entropy gen. rate [W/mK]	Tf	thickness of wall [mm]
$S'_{gen,\Delta p}$	pressure grad entr. gen. rate [W/mK]	α	apex angle [deg]
Φ	irreversibility ratio distribution	β	helical angle of micro fins [deg]
grad p	(dp/dx) pressure gradient [Pa/m]	n	number of grooves
f	friction coefficient	T_{av}	average temperature [K]
h	heat transfer coefficient [W/m2K]	$\boldsymbol{u}_\theta$	angular velocity component

varying geometries). Hydraulic diameter for grooved tube is the diameter, which has a smooth pipe with the same cross-section of flow. Smooth pipe is the reference for further calculations of all cases. Helical angle of micro-fins (apart 0 and 90 deg) causes swirling flow in laminar layer between grooves, however direction of velocity in turbulent core outside laminar and transition layer, is always the same and very similar to a smooth pipe – parallel to wall and axis. In tubes, which have parallel micro-fins to axis, i.e. at β = 0 deg, there is no vortex production because grooves do not cause breaking away laminar layer. Transverse grooves to the flow direction, create vortexes in laminar layer and flow may be treat as axially-symmetric. Every development of inside surface of tube causes increase of flow resistance what involves with enlarge pressure drop and power for pumping. On the other side, increasing area of heat transfer and degree of turbulization enhances heat transfer coefficient between wall and fluid. These cross correlations for varied β, in respect to total entropy production, are shown in further part of paper.

The most similar cases have been presented by Ravigururajan and Bergles [2] but they have not used EGM method and their geometries are not comparable because they tested ducts with higher H/d and other shapes (and dimensions) of ribs.

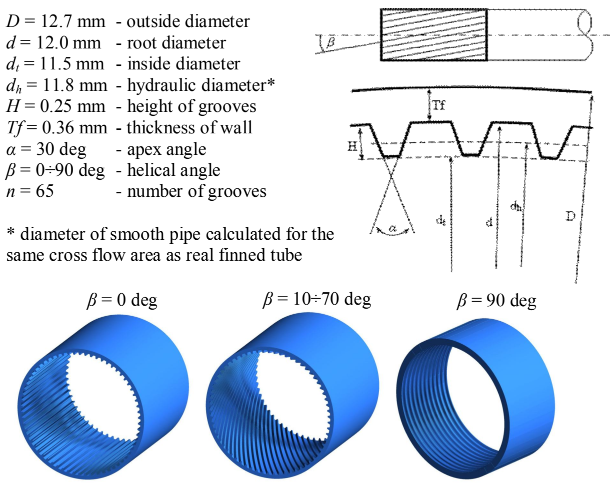

Fig. 1 Geometrical characterization of micro-fin tubes used to analysis.

2 Geometrical model

Fig. 1 shows the characteristic dimensions of geometric and grooves details of analyzed tube, presented in this paper. Summing, took into consideration 9 geometries: 6 with helical angle of micro-fins between 10 and 70 deg and two utmost cases: pipe with parallel grooves (β = 0º) to direction of flow and perpendicular ones (β = 90 º).

3 Mathematical model

As was mentioned above, for the most elementary heat exchanger duct (Fig. 2), basis dependency gave Bejan [1,3]. In this elementary section of pipe, two processes are taking place – heat transfer and fluid flow. Irreversibility of these processes origin from the temperature difference and pressure gradient. Generation of the total entropy S'_{gen} can be written in the following form:

$$S'_{gen} = S'_{gen,\Delta T} + S'_{gen,\Delta P} \qquad (1)$$

The measure of the relative influence of each mechanism on the total entropy is named the Bejan number – Be or irreversibility distribution ratio Φ, and is defined as:

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$$\Phi = \frac{S'_{gen,\Delta P}}{S'_{gen,\Delta T}} \quad \text{and}$$

$$Be = \frac{S'_{gen,\Delta T}}{S'_{gen,\Delta T} + S'_{gen,\Delta P}} = \frac{1}{1+\Phi} \tag{2}$$

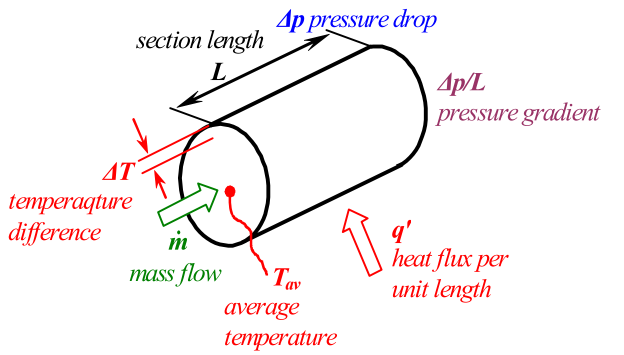

Fig. 2 Elementary section model of smooth tube – forced convection heat transfer

This dimensionless number is equal 0 when the pressure drop dominates and 1 if the heat transfer irreversibility overweighs.

Considering the elementary section of circular smooth tube duct as a thermodynamic system (Fig. 2.), Bejan (1988) derived a formula for the total entropy generation per unit length of duct:

$$S'_{gen} \cong \frac{q' \cdot \Delta T}{T_{av}^{\ 2}} + \frac{\dot{m}}{T_{av} \cdot \rho} \cdot \left(-\frac{dp}{dx}\right) \tag{3}$$

Using the following correlations for the Nu number and the friction factor f:

$$Nu \cong 0.023 \cdot Re^{0.8} \cdot Pr^{0.4} \quad \text{and}$$

$$f \cong 0.046 \cdot Re^{-0.2} \tag{4}$$

and after appropriate transformations, finally, he obtained the following equation [1]:

$$\frac{S'_{gen}}{S'_{gen,min}} = 0.856 \cdot \left(\frac{Re}{Re_{opt}}\right)^{-0.8} + 0.144 \cdot \left(\frac{Re}{Re_{opt}}\right)^{4.8} \tag{5}$$

where:

$$Re_{opt} = 2.023 \cdot Pr^{-0.071} \cdot B^{0.358} \quad \text{and}$$

$$B = \dot{m} \cdot q' \cdot \frac{\rho}{\mu^{\frac{5}{2}} \cdot (\lambda \cdot T_{av})^{\frac{1}{2}}} \tag{6}$$

Fig. 3. shows graphical presentation of (eq.5). This is clearly theoretical graph where the generating entropy S'_{gen} is normalised by minimum generating entropy $S'_{gen,min}$. Exactly the same diagram can be obtaining for a smooth duct from numerical simulations but the calculation of the S'_{gen} requires the use of the (eq.3).

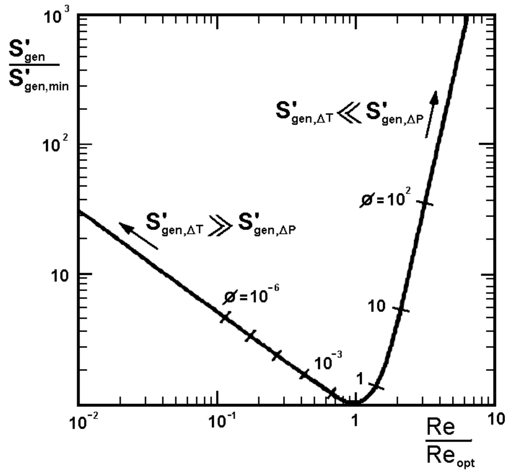

Fig. 3. The relative entropy generation for forced convection heat transfer in a smooth tube from Bejan (1988).

The experiences gained from the numerical and experimental studies showed that, for smooth tube, match better correlation between the pressure drop and mass flow gives the friction factor f, derived by Blassius:

$$f \cong 0.3164 \cdot Re^{-0.25} \tag{7}$$

This formula, gives the friction factor f for turbulent flow in a smooth pipe but for not too high Reynolds number (Re < 10^5). Using this friction factor, have been made much the same transformation of (eq.3) and finally have been obtained the following relationship:

$$\frac{S'_{gen}}{S'_{gen,min}} = 0.856 \cdot \left(\frac{Re}{Re_{opt}}\right)^{-0.8} + 0.144 \cdot \left(\frac{Re}{Re_{opt}}\right)^{4.75} \tag{8}$$

where:

$$Re_{opt} = 1.44 \cdot Pr^{-0.072} \cdot B^{0.3604} \tag{9}$$

and constant B is the same as in (eq.6).

These are very similar relationships as gave Bejan, but for Blassius friction factor. For the optimum condition all ratios presented on (eq.8)

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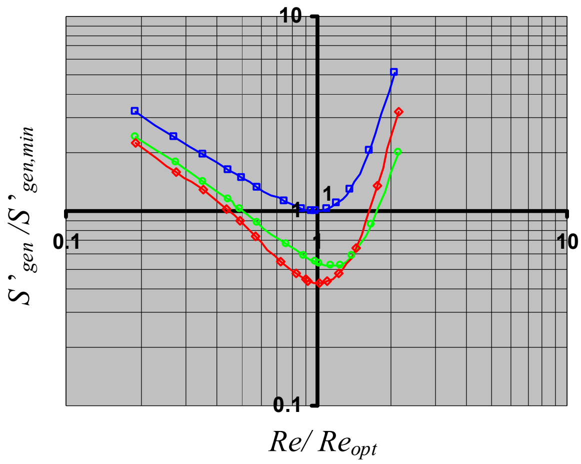

Fig. 4. The relative entropy generation rates – comparison of smooth and micro-fins tubes with 0 and 90 deg angles.

have value of 1 – these contributions are equal to 0.144 and 0.856, respectively and the optimum values of Φ and Be being then 0.168 and 0.856, respectively. Dimensionless entropy generating ratio for micro-fin tubes is related to minimum generating entropy in a smooth tube for the same conditions. It means, the entropy generation S'_{gen} is calculated from numerical simulations results while $S'_{gen,min}$ for smooth pipe, using the same data, from theory. This approaching allows to see all results of studied geometry in respect to smooth duct – and compare they with it.

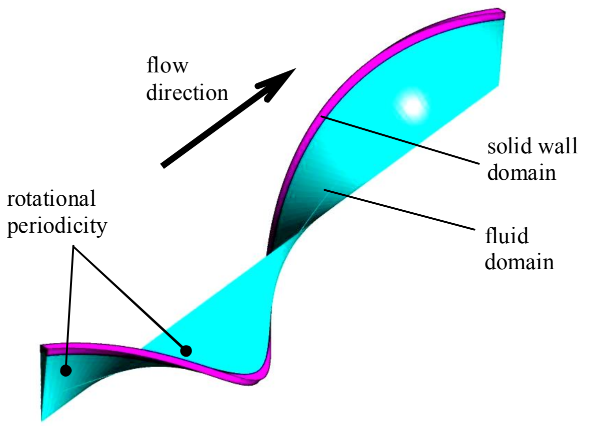

Fig. 5. The elementary and periodic section. One repeatable groove has been used as a computational domain.

The sample plots on Figure 4 show entropy generation rate vs. Reynolds number rate. Three border cases have been chosen to presentation – smooth tube, angle of β=0 and β=90 deg (geometry on Fig. 1). The smooth tube is as if the level of reference for main geometry – micro-fin tube. Minimum generating entropy for pipe with angles 0 and 90 deg is lower than for smooth one. Reynolds number rate, at minimum generating entropy, is a little moved from the one – to the right and left side appropriately.

From point of view of the heat transfer, the values $S'_{gen,\Delta T}$ on the left hand side of the total generated entropy S'_{gen} are dominating and therefore are more interesting for us. It is worth to mention that for whole range of helical angles 0÷90 deg (apart from 80 deg and with 10 degrees step) and for 3000÷150000 Re number range, about two hundreds numerical simulation have been made.

4 Computational model

4.1 Assumptions and methods of simulation

The conditions of fully developed velocity and temperature profiles have been applied – for all simulations. Special geometrical model has been used as a computational domain. Geometries of one axial symmetrically slice with one micro-fin were created, which have two ends (inlet and outlet) twisted about 360 degrees. The length of domain was determined by helical angle β. This way of simulation assumes the conditions of translational periodicity on inlet and outlet of the domain. This setting enforces that velocity and temperature fields are full development (e.g. on the inlet and outlet of the domain, the velocities and temperatures are the same). Since the flow is not axisymmetric, the two lateral sides of the domain have been selected as a rotational periodicity conditions – Fig. 5.

The special procedures in equations of the momentum and energy, named "source terms", have been used. This forces that velocity and temperature fields were periodic during of calculation process. The fluid flow was evoked by pressure gradient in a flow direction. The wall heat flux on inside wall of tube, calculating per unit length, always was set the same q' = 3000 W/m, despite of fact, that area of heat transfer between solid wall and fluid was different for every β angle. In any cases was set the average temperature of fluid equals 303 K (for mentioned temperature of water the Prandtl number is equal about 4.5).

This shape of geometry has been tested and compared to full circle duct at the same longitudinal and radial resolution of mesh. The results of numerical simulation in these two cases were identical, so applying "one groove domain" gave very big benefit in time of calculation. Very similar approach to numerical modelling has been successfully applied, for example: by Ciofalo [4] et al., to simulate the flow with heat transfer using the

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LES turbulence model in channels, to both simple and complicated geometries.

4.2 Test and validation code

Before the main calculations, a lot of validation and verification of numerical simulations have been carried out. Much simple and more complicated geometries have been tested for proper calculations. These results have been presented, e.g. in [5,6,7,8].

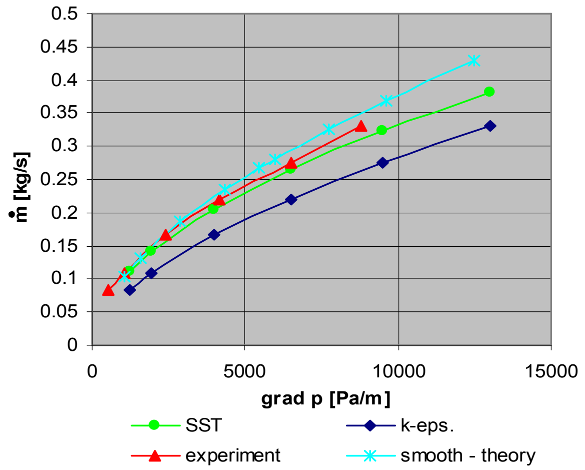

Fig. 6. Comparison of mass flow vs. grad p, – smooth tube, simulations and experiment.

The experimental data have been obtained from a specially designed stand to measure heat transfer and pressure drop in heat exchanger (double pipe type) [9]. Industrial use tube, made by KME Germany AG & Co. KG and labelled "TECTUBE®_fin 12736CV50/65D" have been tested – dimensions are showed on Figure 1, where β = 30 deg. In experiment, the tested range of Re number for flow in a grooved tube, was between 10000÷60000. Generally, mainly for turbulent range of Re number reliable results have been obtained. However laminar and transition range of the Re number also were tested but for the reason that set-up measurement was calibrated only for higher pressure drop and mass flow (linked with Re number), so results for these ranges are not quite trustworthy and have been omitted in this paper.

Figures 6 and 7 show comparison appropriately: mass flow vs. grad p and Nu numbers vs. Re. In numerical simulations the two models of turbulence have been tested: standard k-ε and SST (Shear Stress Transport). Best suited data and good agreement with experimental values have been obtained from the SST model [10] - which can be seen in the Fig. 6 and 7. That is why, this turbulence model has been used to the main calculation. The curve for the smooth tube, calculated from the theoretical equations, is presented as an orientative reference level.

In conclusion, on the basis of good agreement of the results of simulation and experiment for the same geometry dimensions of pipes, and compared it to the theoretical value for the smooth tube, it is assumed that one can obtain a reliable results for tube from silulations – for any helical angle.

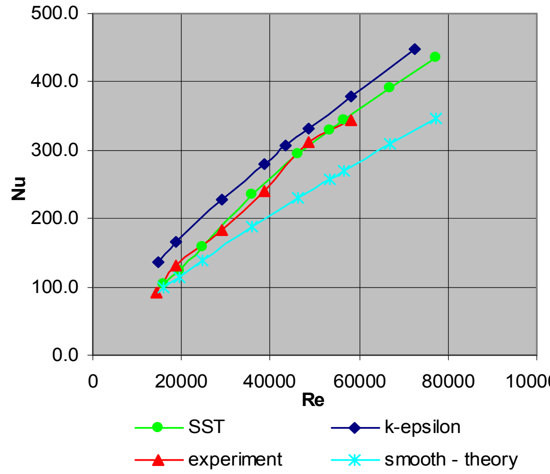

Fig. 7. Comparison of Nu numbers vs. Re, – smooth tube, simulations and experiment.

5 Results

Calculation results of the relative entropy generation rates $S'_{gen}/S'_{gen,min}$, as a function of the Re/Re_{opt} ratio, based on the computer simulation data: temperature, pressure drop and velocity fields, have been presented on Figure 8. This chart shows normalized values obtained from numerical simulations S'_{gen} divided by $S'_{gen,min}$ received for smooth pipe at the same parameters. A localization of curves for particular angles is rather disordered, especially for small angles and 90 deg. This fact means, that it is impossible to predict without simulation e.g. minimum generating entropy in respect to order of individual angles. For higher angles, apart from 90 deg, can be seen some sequence in position of curves. The least value of minimum generating entropy, among considering cases, has tube with 70 deg helical angle, next 60, 50, 40 etc. – for appropriate heat-flow parameters. It is worth to note, that for all pipes with grooved walls, the range level of "minima" is more or less between 0.2 (for 70 deg) and 0.5 (for 10 deg) in reference to 1.0 for smooth tube. This is very important information for practical use, because it is possible significantly decrease of irreversibility production of the total entropy in heat exchangers. It can be also obtain either by changing geometry of flow ducts (from smooth to groove) or/and adjusting

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an appropriate parameters of heat and fluid flow – $\dot{m}$ and q' (eq.4).

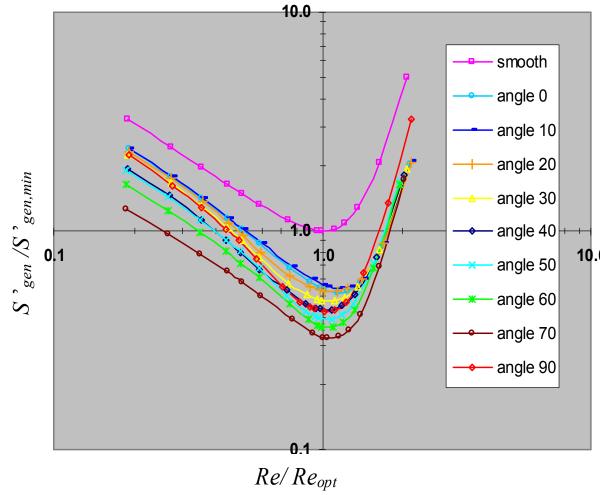

Fig. 8. Comparison of the relative entropy generation rates for tubes with different helical angles β and for smooth tube as a reference.

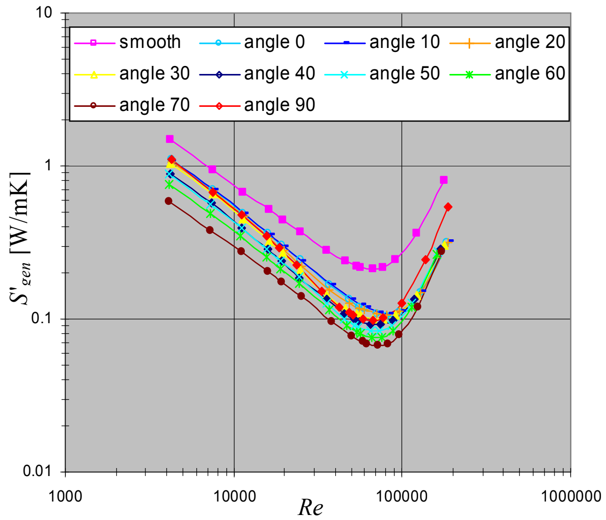

Fig. 9. Comparison of the total entropy generation vs. Re number for smooth tube and for tubes at different helical angles β.

Fig. 9 presents the total entropy generation for examining tubes. On this chart can be seen the characteristics of S'_{gen} vs. Re number at total values. The range of the Re number, for every helical angle β, where the total entropy generation has minimum, is clearly visible.

Looking at the Figure 8 one can see the minimum for every helical angle. Exact value of this is one of the next interesting information for us. On the Figure 10, the smallest values of the total entropy generation for every β angle have been presented. Considering only one, the smallest amount, for respective β angles, one can observe some order – for increasing helical angle the minimal ratio of $S'_{gen}/S'_{gen,min}$ getting smaller, up to 70 deg and afterwards grows up at 90 deg. As mentioned above, the smallest value of the relative entropy generation rates is at 70 deg helical angle. This is amount just about 33% in respect to 100% of the smooth pipe. It means that using this geometry, we can obtain more than ⅔ benefit in reducing generating total entropy.

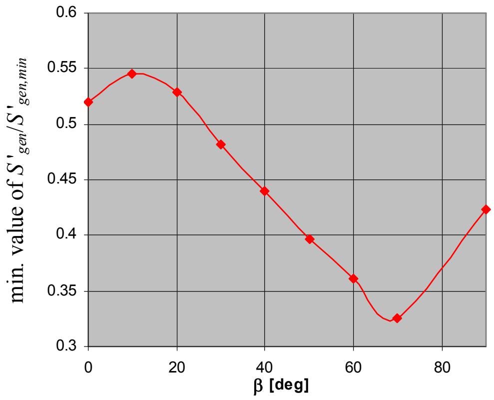

Fig. 10. The minimal value (minimum) of the total entropy generation rate for each helical angle β.

The three-dimensional relation of the ratio $S'_{gen}/S'_{gen,min}$ as a function of helical angle β and flow ratio Re/Re_{opt} is presented on 3D plot in Figure 11. This plot has two axes dimensionless – the ratio of $S'_{gen}/S'_{gen,min}$ and Re/Re_{opt}. Third one represents helical angle and has a unit [deg]. Scales used on the

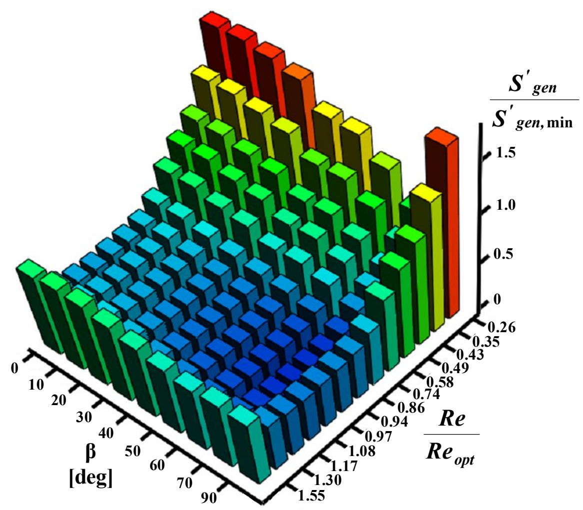

Fig. 11. Three-dimensional relation of the ratio $S'_{gen}/S'_{gen,min}$ as a function of flow ratio Re/Re_{opt} and helical angle β.

bottom surface of this plot are non-linear. Note that the minimal value of the relative entropy generation $S'_{gen}/S'_{gen,min}$ is clearly visible – for the helical angle 70 deg and by $Re/Re_{opt} = 1.08$.

Computer simulation allows "to see" what happens inside of tube during heat-flow process. The example of this is showing the distribution of

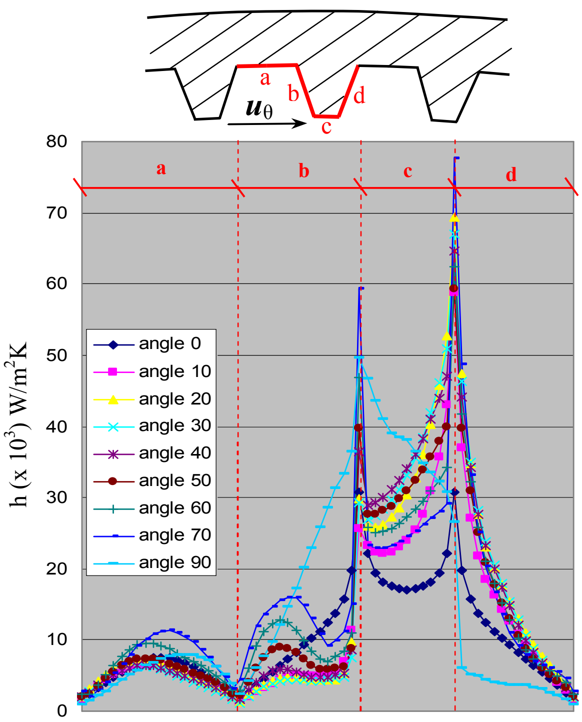

Fig. 12. Heat transfer coefficient on the surfaces of groove for every angle and $Re \approx 35000$.

the heat transfer coefficient on micro surfaces of grooves. Normally, from experiment only average value of this coefficient for entire surface wall is measured – without distinguish the particular surfaces of furrow and their edges. Figure 12 shows heat transfer coefficients on the micro surfaces of individual groove for every helical angle. As we expected, for 0 deg, the layout of h on the surface of fin is symmetrical because the flow is completely parallel to direction of grooves. For helical angles from 10 to 70 deg the layout of h is non-symmetrical because the angular velocity component $\boldsymbol{u}_\theta$ occurs. It is worth to note, that the highest values of the heat transfer coefficient there are on the top of grooves (e.g. surface **c** on the Fig. 12) but particularly on the edges (between **b** - **c** and **c** - **d** surfaces). For angle β of 90 deg layout of h is a bit different, especially on the **c** surface. The micro-fins in this geometry are perpendicular to main stream of flow and for this case, the edge between **b** and **c** surface is a leading edge – just there is the highest value of the heat transfer coefficient.

Fig. 12 is only one sample graph. It can be also shown e.g. heat flux and/or temperature field distribution. Having this data, the complex thermal analysis of that object could be done.

6 Conclusions and discussion

EGM – is the method taking into consideration two opposite irreversibility processes: heat transfer and pressure drop, and "seeks" minimal value entropy generating between them. It can be applicable practically for each heat-flow case with one or more variables parameters.

In this paper, the numerical results and analysis for tubes with micro-fins have been presented. In studies, the following parameters were changed: helical angle of micro-fin (for geometry) and mass flow, linked with Re (for operating conditions). In theory, there are many parameters which can be study additionally for optimizing this kind of tube, e.g. pitch, shape or height of grooves (for geometry parameters) and/or heat flux, max. or average temperature of fluid or wall temperature (for exploational conditions). It is possible to do, but in practice the addition of each of the following parameter to investigate, involves augmentation of time simulations (about one order of magnitude for each parameter). In spite of mentioned fact, the EGM method seems to be quite "good criterion" of optimization of the thermal devices.

The main purpose of this paper is to present application of the EGM method to the heat exchangers with micro-fin tubes. Analogous approaches have been adopted to other problems of thermal design, also based on computer simulations results, e.g. [14, 15]. The common aspect of all works in this area is destroying the least exergy for heat-flow engineering system by finding an optimum operating and geometric conditions.

There are many works analyzing energy problems in a very complex way. For example Bejan [1,3] and Szargut [11-13] describe the exergy in using to analyze of unrenewable natural energy resources – their depletion and ecological cost. This approach is popular to analyze, in particular, complete industrial power plants. Such considerations can be very often helpful in improving of the efficiencies many thermal and chemical apparatuses in the whole plant. Notwithstanding, in many cases these efficiencies can be analyzed practically using only results from computer simulations and the technique discussed in this article. Presented work brings a small contribution in this field.

6.1 Example of calculation of exergy losses

Let's considering 2 pipes: smooth and grooved with β = 70 deg. Both have the same hydraulic diameter and work at identical conditions. For these pipes one can calculate the difference of the exergy losses and, additionally, *Be* number for all range of Re number. From the Gouy-Stodola theorem, exergy losses due to two irreversible processes are expressed by product $T_{av}\,S'_{gen}$. Example for Re=11125 and T_{av}=303K: (a)

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smooth tube S'_{gen}=0.77 W/mK; $T_{av}S'_{gen}$=233 W/m and (b) tube with 70° helical angle S'_{gen}= 0.27 W/mK and $T_{av}S'_{gen}$=81.8 W/m. The difference of the exergy losses in this case is equal to 151.5W/m. This is about 5% of the considered heat transfer rate value in relation to q'=3000W/m. Let's call this savings of exergy losses as ΔB_{lost}. In the same way we can calculate ΔB_{lost} for all range of the Re number and plot it on the chart. Fig.13 shows two characteristics: ΔB_{lost} vs. Re and *Be* vs. Re for tube with 70° helical angle of micro-fins – where ΔB_{lost} is a percentage difference between smooth and grooved tube. The shape of ΔB_{lost} curve is similar to S'_{gen} and we can see that the biggest savings of exergy losses are at small values of the Re numbers.

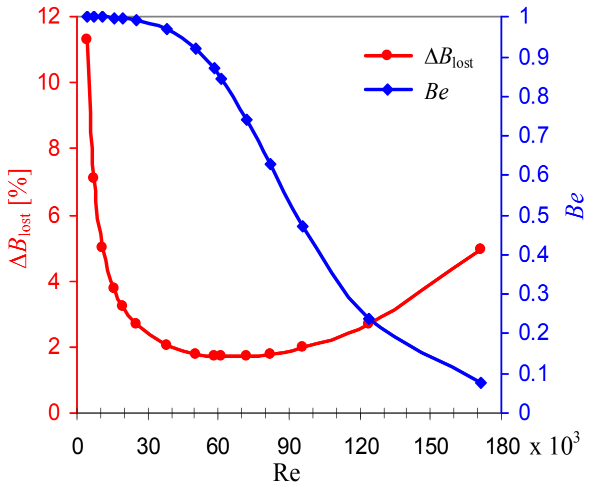

Fig. 13. The exergy saving losses and *Be*-number vs. Re-number, for tube with 70° helical angle.

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