Thermoelectricity: from Quadrupole formulation to Space applications

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Abstract: - There is an increasing interest in thermoelectric (TE) devices even if the efficiency is rather low. Many fields such as aerospace, automotive or building applications are concerned. It is then interesting to model the phenomena involved especially into the thermoelectric legs and to try to optimize the design of the thermoelements. In a first part, the quadrupole method is performed to solve easily the transient case. No mesh is required. The whole matrix formulation is given for different initial conditions. In a second part, the focus is put on the design of a thermoelement applied to Radioisotope Thermoelectric Generators (RTGs). The optimization of segmented legs are proposed and explained on a real test case considered by the Russian firm BIAPOS.

Key-Words: - Thermoelectricity, Quadrupole formulation, Radioisotope Thermoelectric Generators, Efficiency, Aerospace applications, Partial differential equation.

1 Introduction

The aim of almost all the devices designers is obviously to achieve high efficiency. Thermoelectricity is not an exception to this rule that is why a lot of studies are made in order to improve the efficiency of thermoelements for instance for space applications [1, 2] or automotive heat recovery [3].

There is of course a great interest to model the behavior of the thermoelement as done for instance in [4] in order to be able to have a rough estimation of the efficiency or the coefficient of performance and also to study the influence of the thermophysical parameters.

In this paper, we propose to develop a technique called the quadrupole method in order to have a semi-analytical solution in terms of temperature and heat flux within the thermoelectric leg.

Then in a second part, we propose the design of a thermoelement applied to Radioisotope Thermoelectric Generators (RTGs) corresponding to a real case from the Russian firm Biapos. The first RTG launched in space was in 1961 aboard the Navy Transit 4A spacecraft : it is the 60th birthday.

2 TE leg modelling

2.1 Equations

Consider a single thermoelectric leg of length $l$ and cross-sectional area $A$. An electrical current $I = JA$ enters uniformly into the element.

The one-dimensional energy balance that describes the thermal behaviour of the leg is the following partial differential equation, written in a synthetic way

\[ \frac{\partial^2 \theta(x,t)}{\partial x^2} - \frac{A}{\lambda} \frac{\partial \theta(x,t)}{\partial x} - \frac{1}{\rho c_p} \frac{\partial \theta(x,t)}{\partial t} = -B \]  

(1)

where \( A = \frac{\sigma J}{\lambda} \),

(2)

\( B = \frac{J^2}{\sigma \lambda} \),

(3)

The temperature is a function of the spatial variable $x$ and the time $t$. The typical property used in transient case is the thermal diffusivity $a$. The relevant material properties are also the density $\rho$, the heat capacity $c_p$, the thermal conductivity $\lambda$, etc.
the electrical conductivity $\sigma$ and the Thomson coefficient $\tau$. The initial temperature of the leg is $T_0$.

To solve this partial differential equation, it is very convenient to apply the Laplace transform which transforms the partial differential equation into an ordinary differential equation in the Laplace domain.

Let introduce $p$ the Laplace variable and note:

$$
L(f) = \tilde{f}(p) = \int_0^{+\infty} f(x,t) \exp(-pt)dt.
$$

The equation (1) becomes

$$
\frac{d^2 \vartheta}{dx^2} - \frac{A}{a} \frac{d\vartheta}{dx} - \frac{p}{a} \vartheta = -\frac{1}{p} - \frac{1}{a} T_0(x)
$$

(4)

The roots of the characteristic associate equation are

$$
\gamma_1 = \frac{A}{2} \pm \sqrt{\frac{A^2}{4} + \frac{p}{a}} = \frac{p}{2\lambda} \pm \sqrt{\frac{r^2 J^2}{4} + \frac{p}{a}}.
$$

(5)

As a consequence, the solution of the equation (4) is:

$$
\vartheta(x) = \xi_1 \exp(\gamma_1 x) + \xi_2 \exp(\gamma_2 x) + \vartheta(x)
$$

(6)

where $\vartheta(x)$ is a particular solution of equation (4) and $\xi_1, \xi_2$ are two constants.

It is although interesting to have the expression of the heat flux going through the thermoelement in order to have a complete modelling of the heat transfer within the leg.

Now, let express the heat flux which is a linear combination of the temperature and the derivative of the temperature (where $\alpha$ is the Seebeck coefficient)

$$
\varphi = \alpha IT - \lambda A \frac{dT}{dx}.
$$

(7)

Considering equations (6) and (7), the Laplace transform of the heat flux is:

$$
\overline{\varphi}(x) = \alpha I \left( \xi_1 \exp(\gamma_1 x) + \xi_2 \exp(\gamma_2 x) + \vartheta(x) \right) - \lambda A \left( \xi_1 \gamma_1 \exp(\gamma_1 x) + \xi_2 \gamma_2 \exp(\gamma_2 x) + \frac{d\vartheta(x)}{dx} \right)
$$

(8)

2.2 Quadrupole method

2.2.1 Principle

From a mathematical point of view, the quadrupole method belongs to the class of analytical unified exact explicit method for solving linear partial differential equations in simple geometries. It relies on classical tools such as Laplace integral transforms (in time) or space integral transforms (Fourier for instance). Steady state or transient problems could be treated in one or multidimensional heat transfer.

This approach was first presented by H.S. Carslaw [5] for the conduction of heat in solids. A precise name was not given to this method first used in engineering electrical applications but as it refers to an equation with four scales or poles then it was naturally called quadrupole method. The treatment of heat transfer problems could follow a similar way the modelling of electrical circuits once a boundary condition is known. The Laplace heat flux (respectively the Laplace temperature) is the analogue of the electrical current (respectively the electrical potential). Then the thermal solution can be expressed in terms of linear matrix relationships between transformed temperature flux vectors at the boundaries of the system. The intrinsic solutions obtained thanks to this formulation have a form which does not depend on these boundary conditions.

Quadrupole formulation has been and is still widely used and developed [6] to solve different and complex thermal problems such as heat transfer in multilayer [7] or coupled conductive radiative transient heat transfer in semi-transparent media with anisotropic scattering [8]. It is particularly well adapted to perform calculations quickly and to be used for instance to inverse data in order to remote to thermophysical parameters such as for instance thermal diffusivity [9] or to make estimations in real time in order to have an adaptive control of the cutting process [10].

To summarize, this method provides a transfer matrix for the medium that linearly links the input temperature–heat flux column vector at one side and the output vector at the other side.
Let consider the Laplace transform of the temperature called $\bar{\theta}_0$ (respectively $\bar{\theta}_L$) at $z = 0$ (respectively at $z = L$) and also the Laplace transform of the heat flux called $\bar{\varphi}_0$ (respectively $\bar{\varphi}_L$).

The quadrupole formulation of the heat transfer is given by the following equation:

$$
\begin{pmatrix}
\bar{\theta}_0 \\
\bar{\varphi}_0 
\end{pmatrix} = 
\begin{pmatrix}
A & B \\
C & D 
\end{pmatrix}
\begin{pmatrix}
\bar{\theta}_L \\
\bar{\varphi}_L 
\end{pmatrix}
$$

(9)

The four coefficients of the matrix are either constant or functions of the Laplace variable. They involve the geometrical and thermophysical properties of the medium. The aim is to determine their explicit expression (see section 2.2.2).

The quadrupole formulation presents the advantage to have a synthetic representation (cf Fig.1)

For instance, to represent a thermal contact resistance or heat loss in Cartesian coordinates the following quadrupoles representations are used (cf Fig.2):

Combining equations (10) and (11), the quadrupole formulation of the problem is directly given by:

$$
\begin{pmatrix}
\bar{\theta}(0) \\
\bar{\varphi}(0) 
\end{pmatrix} = \mathbf{N}^{-1} \begin{pmatrix}
\bar{\theta}(L) \\
\bar{\varphi}(L) 
\end{pmatrix} = \mathbf{N}^{-1} \left( \mathbf{M} \begin{pmatrix}
\xi_1 \\
\xi_2 
\end{pmatrix} + U_p \right) + V_p
$$

(16)

3 TE modelling applied to RTGs

3.1 RTG principle

The first Radioisotope Thermoelectric Generators (RTGs) for space applications were developed in the early 1960s with the beginning of activities on the System for Nuclear Auxiliary Power (SNAP) program in the USA. RTGs work by converting heat from the natural decay of radioisotope materials into electricity [11]. As the temperature gradient is very large at the two sides of thermoelement, the legs must be segmented with different materials [12]. In this way, materials are operating in their most
efficient temperature range [13]. But not only must the figure of merit be considered but also the factor of compatibility of the materials [14, 15].

3.2 Design and optimization
The modeling allows to have an estimation of the efficiency (about 11%) and especially to investigate the influence of the parameters such as the relative current density, the temperatures at the sides of the leg or the thermoelectric properties of the materials. The design and the optimization of the segmented thermoelement for BIAPOS applications allow to summarize for instance in the table 1, the influence of the temperature at the cold side on the efficiency.

<table>
<thead>
<tr>
<th>Value of the efficiency after optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leg P</td>
</tr>
<tr>
<td>20°C-480°C</td>
</tr>
<tr>
<td>12.01%</td>
</tr>
</tbody>
</table>

Value of the relative current density at the hot side which gives the optimal value of the efficiency

\[
\begin{align*}
\nu_6 &= 3.70 \text{ V}^{-1} \\
\nu_7 &= 3.62 \text{ V}^{-1} \\
\nu_8 &= 2.99 \text{ V}^{-1} \\
\nu_9 &= 2.97 \text{ V}^{-1}
\end{align*}
\]

Table 1: Efficiency values after optimization [15].

4 Conclusion
The modeling of thermoelement through the quadrupole method and also for segmented legs applied to space is performed in order to investigate the influence of several parameters and to find the optimal design.

Acknowledgments:

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[2] A. A. Pustovalov, Nuclear thermoelectric power units in Russia, USA and European space agency research programs, International Conference on Thermoelectrics, ICT, Proceedings 1997, 559-562