A Design Method for USBL Systems with Skew Three-element Arrays

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Abstract: - This paper describes a design method for ultra-short baseline (USBL) systems where the location of the underwater object is defined by utilizing of orthogonal and non-orthogonal (skew) basic (three-element) measuring systems. The case of calculation of the Cartesian coordinates of the object in the reference coordinate system bounded up with the USBL system’s carrier is considered. The proposed design method and the algorithm are based on the utilization of multiple rotations of the basic (three-element) USBL receiving arrays (orthogonal and non-orthogonal) around the horizontal and vertical axes associated with the carrier coordinate system. In the article the case of a five-element USBL system is investigated. The proposed construction of the receiving array allows it to obtain different spatial orientations of the basic USBL arrays and to increase the consistency and reliability of object coordinate determination. It is supposed that the spatial orientation of the receiving USBL array is controlled by the measurement of its pitch and roll angles. The coordinate determination algorithm for the proposed USBL system is designed and tested with the assumption that the USBL array can have significant inclination.

Key-Words: - Ultra-short baseline (USBL) system, underwater object, transponder, carrier coordinate system, local coordinate system, skew coordinate system, pitch and roll angles.

1 Introduction
The central problem of the USBL system design is the underwater object location determination with high accuracy in real marine conditions. The principle of operating of the USBL systems is well known and is described in detail in [1,2]. Object position determination with this method is realized by means of the measuring of the distance to the object and its angular position relative to the measuring system location. During the last decades, numerous studies and investigations for improvements in accuracy and reliability of object position determination with use of the USBL systems were realized [3-7]. To improve the reliability of the USBL systems various special signal processing techniques were employed. In particular the chirp signals and greater inter-element array separation [3] were used. Also the acoustic digital spread spectrum [4] and modulated Barker-coded signals [5] were applied. In [6] the USBL system with frequency-hopped pulses was investigated. The problem of improvement in accuracy in the case of instability of the position of the receiving USBL array was studied in detail in [7]. In [8,9] the problem of low precision in coordinate determination for the case of the object found in the plane of receiving bases is studied. In [10,11] the design method of multi-element USBL systems is proposed. The idea of this design method was to increase the accuracy of coordinate determination with use of the supplementary receiving elements of the USBL array. The idea of the design method in the present article is to use not only orthogonal receiving bases of antenna array. It allows the utilization at the most the latent reserves of receiving antenna. The ambiguity of coordinate determination with these non-orthogonal three-element systems can be resolved with the use of other elements of antenna.

2 Problem Formulation
2.1 Basic USBL System
The measuring of the object coordinates is realized as the following. The USBL system transmitter sends an interrogation acoustical impulse in the propagation medium where the object is located at an accessible distance. The object must be equipped with a transponder that receives the interrogation impulse and sends an acoustical impulse in reply. The distance to the object is determined by the measurement of the values of propagation times of the interrogation impulse and the transponder response pulse. The angular position of the object is determined by the measurement of the phase difference of the transponder pulse carrier frequency.
on the receiving array outputs. The minimum number of receiving elements for the USBL system for object coordinate determination is three [1]. To improve the reliability and accuracy of coordinate determination the number of elements of receiving USBL array can be increased. In general the propagation medium is non-homogeneous. Furthermore, in this paper we assume that the propagation medium is homogeneous and the multipath interference is absent.

We consider briefly the principle of coordinate determination for the case of the three-element orthogonal USBL array. Let $\Sigma=(0,x,y,z)$ be the sea-surface associated reference frame with the origin in the point $O$ (see Fig.1). Let the plane $xy$ coincide with the sea surface (it is supposed that the sea surface is not perturbed) and the positive direction of the $z$ axis run down. It is supposed that the carrier and the carrier coordinate system $\Sigma_{\text{carrier}}=(0,x_{\text{carrier}},y_{\text{carrier}},z_{\text{carrier}})$ coincides with the $\Sigma=(0,x,y,z)$ when the carrier does not have pitch and roll inclinations. Also it is supposed that in such the $x$-coordinate axis coincides with the carrier longitudinal axis $L-L'$ (the positive direction coincides with the direction of the straight arrowed line), the $y$-coordinate coincides with the carrier lateral axis $B-B'$, and $z$ axis goes downwards. Now we can define the USBL array orientation in the introduced carrier coordinate system. Let $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ be the local coordinate system for the considered USBL system (with receiving elements 1,2,3). It is assumed that the origin of the USBL coordinate system coincides with the origin of the carrier coordinate system, the angle between the $y$-axis and base 1-2 is 135°, and the angle between the $y$-axis and the base 3-2 is 45°. It is also supposed that the receiving USBL array is rigidly mounted on the carrier hull.

The geometry of the introduced sea-surface associated coordinate system $\Sigma=(0,x,y,z)$ and the receiving three-element USBL coordinate system $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ is presented in Fig.1. The reference grid is shown only for the USBL coordinate system.

In Fig.1 we specify the angles ($\alpha$, $\beta$ and $\gamma$) that define the position of the underwater object (located in point $P$) in the $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ coordinate system. We also assume that the USBL system is equipped with a special unit to measure the pitch and roll angles of the carrier (the pitch and roll angles of the USBL array are the same as for the carrier). Let angles $\xi$ and $\zeta$ be pitch and roll angles corresponding to the receiving USBL antenna (in the figure these angles show the rotations relative to the carrier lateral $B$-$B'$ axis and the carrier longitudinal $L-L'$ axis).

Let that the interrogation impulse has been sent and the reply impulse is being received by antenna. The distance to the object is defined by measuring the propagation times of the interrogation and reply pulses. Time delays on receiving elements define the object’s angular position. The time delays $\tau_{12}$ and $\tau_{32}$ of the signal on the outputs of the receiving elements of the base 1-2 and the base 3-2 (it is supposed that $R>>d$) can be expressed in the following way:

$$
\tau_{12}=\frac{d}{c}\cos\beta \quad \text{and} \quad \tau_{32}=\frac{d}{c}\cos\alpha,
$$

where $c$ is the speed of the sound in the water. With $d/c$ defined as $\tau_d$ we can write the direction cosines $\cos\alpha=\frac{\tau_{12}}{\tau_d}$ and $\cos\beta=\frac{\tau_{32}}{\tau_d}$. The third direction cosine is defined as:

$$
\cos\gamma=\sqrt{1-(\frac{\tau_{12}}{\tau_d})^2-(\frac{\tau_{32}}{\tau_d})^2}.
$$

Cartesian coordinates of the point $P$ in the $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ coordinate system are:

$$
X_{123}=R\cos\gamma, \quad Y_{123}=R\cos\beta, \quad Z_{123}=R\cos\alpha.
$$

To obtain the coordinates of point $P$ in the $\Sigma=(0,x,y,z)$ coordinate system (point $P(X,Y,Z)$ in Fig.1) it is necessary to carry out the corresponding transformation of obtained coordinates $X_{123}$, $Y_{123}$, $Z_{123}$.

Let that the pitch and roll rotations of the receiving antenna take place. The pitch and roll rotations of the elemental three-element USBL antenna are shown in Fig.1. After the first rotation on pitch angle $\xi$ relative to B$'$-B axis the receiving elements are displaced to points $1^\xi$ and $3^\xi$ respectively. After the second rotation on the angle $\zeta$ relatively $L-L'$ axis the receiving elements are displaced to point $1^{\xi\zeta}$ and $3^{\xi\zeta}$ respectively (see Fig.1). With the rotations of receiving bases the corresponding transformations of coordinate systems from $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ to $\Sigma_{123}'=(0,x_{123}',y_{123}',z_{123}')$ and to $\Sigma_{123}''=(0,x_{123}'',y_{123}'',z_{123}'')$ have taken place.

Calculation expressions for the case of the pitch and roll of the three-element receiving antenna with introduced orientation relative to the carrier have been obtained in [8,9]. So we describe the calculation...
A procedure for determining the coordinates of an object in a coordinate system involves using a transformation matrix. The transformation matrix $A$ that represents the coordinates of the object in the $\Sigma_{123}$ coordinate system and vector $p_{123}$ represents the coordinates of the object in the $\Sigma_{123}$ coordinate system can be written as:

$$ \mathbf{B} = \mathbf{B} \left( \mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3 \right) $$

with direction cosines $\eta_{123} = \cos(\chi_{123}, z)$, $\chi_{123} = \cos(y_{123}, z)$, $\nu_{123} = \cos(z_{123}, L)$ (matrix $\mathbf{B}$ transforms vector $p_{123}$ to vector $\mathbf{p}^{\xi\zeta}$). We can write the equation:

$$ p_{123} = A^T p^{\xi\zeta} $$

where $A^T$ is the transpose of matrix $A$.

In order to obtain the coordinates of the object in the $\Sigma = (0, x, y, z)$ coordinate system (we will have the same coordinates of the object in the carrier coordinate system $\Sigma_{\text{carrier}} = (0, x_{\text{carrier}}, y_{\text{carrier}}, z_{\text{carrier}})$ and in the sea-surface associated coordinate system $\Sigma = (0, x, y, z)$ if the carrier is not inclined) it is necessary to make one more rotation of the coordinate system $\Sigma_{123}$ around the axis $z$ on the angle of $135^\circ$ (see Fig.1). For the $\Sigma = (0, x, y, z)$ coordinate system, we have the following direction cosines for the $z$-axis: $\cos(x, z) = 0$, $\cos(y, z) = 0$, $\cos(z, z) = 1$. If we introduce vector $\mathbf{p} = (X, Y, Z)^T$ (vector $\mathbf{p}$ represents the coordinates of the object in the $\Sigma = (0, x, y, z)$ coordinate system) and the transformation matrix $\mathbf{C}$ (matrix $\mathbf{C}$ transforms vector $\mathbf{p}$ to vector $p_{123}$) the final equation to find vector $\mathbf{p}$ will be the following: $\mathbf{p} = C^T A^T B^T \mathbf{p}^{\xi\zeta}$. Thus to find the coordinates in the $\Sigma = (0, x, y, z)$ coordinate system we write the final matrix equation:

$$ \mathbf{p} = C^T A^T B^T \mathbf{p}^{\xi\zeta}. $$

### 3 Problem Solution

#### 3.1 Skew three-element USBL system

We will investigate the proposed design method for the five-element USBL array that has been studied in [8,9]. The construction of the foregoing five-element USBL array is shown in Fig.2. The USBL array has the following orthogonal three-element USBL systems: USBL123, USBL234, USBL341, USBL412, USBL152, USBL253, USBL354. The additional non-orthogonal three-element USBL systems presented in this construction are: USBL152, USBL253, USBL354, USBL451 (in this particular case all of these non-orthogonal three-element arrays form equilateral triangles).

Our proposal is to utilize these non-orthogonal (skew) three-element arrays as a part of the USBL measuring system. We will demonstrate the proposed method in detail only for the case of the skew USBL152 system. For the rest of the skew three-element USBL systems the sequence of necessary coordinate determination algorithm were proposed. In particular the number of transducers of the receiving array was increased to nine. As a result the new USBL array had a larger number of elemental USBL arrays with different spatial orientations and a higher reliability of system was obtained.

The main goal of the present investigation is to check the possibility of obtaining similar results with a smaller number of receiving array elements. If we analyze the construction of the five-element USBL array investigated in [8,9] we can see that besides having orthogonal three-element basic arrays this USBL antenna also has a non-orthogonal three-element basic arrays. We can also see that these non-orthogonal USBL arrays have spatial orientations similar to the orientations of the additional orthogonal three-element USBL arrays of the nine-element array (mentioned above and have been investigated in [10] and [11]).
3.2 Calculation expressions for skew three-element USBL system

We will consider the derivation of the calculation expressions for USBL152 system in detail. Then we will present the expressions for the other skew three-element USBL systems (USBL253, USBL354, USBL451).

For convenience to get the calculation expressions we will firstly consider the case of location of the three-element USBL152 system on a horizontal plane. Let $\Sigma_{152}=(0,x_{152},y_{152},z_{152})$ be the local coordinate system for the considered USBL152 system (with receiving elements 1,5,2). The angle between the axis $x_{152}$ and the axis $y_{152}$ is defined by the expressions:

$$\alpha_1 = \arctg \left[ \frac{1}{\sin \nu} \left( \cos \beta \cos \alpha - \cos \nu \right) \right];$$

$$\beta_1 = \arctg \left[ \frac{1}{\sin \nu} \left( \cos \alpha \cos \beta - \cos \nu \right) \right].$$

The angle $\gamma$ is defined as follows:

$$\gamma = \arcsin \frac{\cos \alpha}{\cos \alpha_1} = \arcsin \frac{\cos \beta}{\cos \beta_1},$$

The covariant coordinates $X_{152_{\text{covar}}}$, $Y_{152_{\text{covar}}}$ and $Z_{152_{\text{covar}}}$ in skew coordinate system $\Sigma_{152}=(0,x_{152},y_{152},z_{152})$ are defined by the expressions:

$$X_{152_{\text{covar}}} = R \cos \alpha_1;$$

$$Y_{152_{\text{covar}}} = R \cos \beta_1;$$

$$Z_{152_{\text{covar}}} = R \cos \gamma.$$

The contravariant coordinates $X_{152_{\text{contr}}}$, $Y_{152_{\text{contr}}}$ and $Z_{152_{\text{contr}}}$ in skew coordinate system $\Sigma_{152}=(0,x_{152},y_{152},z_{152})$ are defined by the expressions:

$$X_{152_{\text{contr}}} = X_{152_{\text{covar}}} - R \sin \alpha_1 \tan \nu;$$

$$Y_{152_{\text{contr}}} = Y_{152_{\text{covar}}} - R \sin \beta_1 \tan \nu.$$
\[ Z_{152, \text{ contr}} = Z_{152, \text{ covar}} = R \cos \gamma. \quad (7) \]

After measuring the coordinates of the object in the skew coordinate system (in this particular case in the \( \Sigma_{152} = (0, x_{152}, y_{152}, z_{152}) \) coordinate system) we must transform these coordinates to regular Cartesian coordinates. We can do this transformation in this stage. It will allow us to make the further coordinate transformations by using the already designed procedures (the rotations of Cartesian coordinate systems around earlier defined axes by pitch and roll angles). So now we define the coordinate system in the Cartesian coordinate system \( \Sigma_{152, 90} = (0, x_{152, 90}, y_{152, 90}, z_{152, 90}) \). The z-axis in the new coordinate system will not change. The Cartesian coordinates in the new coordinate system will be defined with the expressions:

\[
X_{152, 90} = X_{152, \text{ contr}} \cos 345^\circ + Y_{152, \text{ contr}} \cos 285^\circ, \\
Y_{152, 90} = X_{152, \text{ contr}} \cos 75^\circ + Y_{152, \text{ contr}} \cos 15^\circ, \\
Z_{152, 90} = Z_{152, \text{ contr}} = R \cos \gamma . \quad (8)
\]

To obtain the coordinates of point \( P \) in the \( \Sigma = (0, x, y, z) \) coordinate system (point \( P(X, Y, Z) \) in Fig.1) it is necessary perform the corresponding transformations of obtained coordinates \( X_{152, 90}, Y_{152, 90}, Z_{152, 90} \).

Let the pitch and roll rotations of the receiving antenna take place. The pitch and roll rotations of the three-element USBL arrays are shown in Fig.1,2. The difference of the consideration of the rotation of the USBL123 coordinate system from the rotation of USBL152 coordinate system is in the following: in the first case the rotation is realized around the origin that is located in point 2 and in the second case the rotation is realized around the origin that is located in point 5. The first rotation on pitch angle \( \xi \) relative to \( C-C' \) axis the receiving elements are displaced to points \( 1^5 \) and \( 2^5 \) respectively. After a second rotation on the angle \( \zeta \) relatively \( D-D' \) axis the receiving elements are displaced to point \( 1^{55} \) and \( 2^{55} \) respectively (see Fig.2). In our algorithm we will carry out the rotations with orthogonal coordinate systems. With this assumption the corresponding transformations of coordinate systems from \( \Sigma_{152, 90} = (0, x_{152, 90}, y_{152, 90}, z_{152, 90}) \) to \( \Sigma_{152, 90}' = (0, x_{152, 90}', y_{152, 90}', z_{152, 90}') \) and to \( \Sigma_{152, 90}'' = (0, x_{152, 90}'', y_{152, 90}'', z_{152, 90}'') \) have taken place.

Calculation expressions for the case of the pitch and roll of the orthogonal three-element receiving antenna with different orientation relative to the carrier have been obtained in [8,9]. After the transformation of the skew coordinates to the orthogonal coordinates in the \( \Sigma_{152, 90} \) the further procedure of the calculation of the coordinates of the object in the \( \Sigma = (0, x, y, z) \) coordinate system is accomplished in the same way as it is made for the other orthogonal three-element USBL systems. Using a similar sequence of steps as was applied to the orthogonal USBL123 system we will have the final equation for the object’s coordinates in the \( \Sigma = (0, x, y, z) \) coordinate system defined with the USBL152 system:

\[
p_{\text{USBL152}} = C^{-1}_{152, 90} p_{152, 90} F^{-1}_{152, 90} A_{152, 90} p_{152, 90} \quad (9)
\]

where \( p_{\xi \zeta, 152, 90} = \begin{bmatrix} X_{\xi \zeta, 152, 90} \\ Y_{\xi \zeta, 152, 90} \\ Z_{\xi \zeta, 152, 90} \end{bmatrix} \) is the vector that represents the coordinates of the object in \( \Sigma_{152, 90} \) coordinate system; \( B_{152, 90} \) is the matrix that transforms vector \( p_{\xi \zeta, 152, 90} \) to vector \( p_{\xi \zeta, 152, 90} \); \( A_{152, 90} \) is the matrix that transforms vector \( p_{\xi \zeta, 152, 90} \) to vector \( p_{\xi \zeta, 152, 90} \); \( D^{-1}_{152, 90} \) is the matrix that transforms vector \( p_{\xi \zeta, 152, 90} \) to vector \( p_{\xi \zeta, 152, 90} \); \( p_{\xi \zeta, 152, 90} \) is the vector in the vertical oriented coordinate system \( \Sigma_{152, 90} \); \( C^{-1}_{152, 90} \) is the matrix that transforms vector \( p_{\xi \zeta, 152, 90} \) to vector \( p_{\xi \zeta, 152, 90} \).

### 3.3 Calculation expressions for five-element USBL system

In the case of the five-element USBL array, the system consists of ten basic USBL systems with ten different spatial orientations: USBL123, USBL234, USBL341, USBL412, USBL153, USBL254, USBL152, USBL354, and USBL451.

The coordinates of the object are determined individually in each elemental USBL system. The USBL234, USBL341 and USBL412 systems differ from the USBL123 system in their own values of the pitch and roll angles and in their own angles of rotation of each USBL antenna around the z-axis. The coordinate determination for the USBL153 systems is almost the same as for the USBL123 system. The difference is that one additional step is required to reduce the USBL153 system coordinates to a horizontal plane (by rotation the USBL153 system on a 90° plane). We have to do the same with coordinates obtained with the USBL254 system. To accomplish these additional rotations for the USBL153 and USBL254 arrays we introduce for each system the transformation matrix (matrix \( D \)). The USBL253, USBL354 and USBL451 systems differ from the USBL152 in their own direction cosines, in their own values of the pitch and roll angles and in their own angles of rotation of each USBL array around the z-
axis (see Fig.2). So for the USBL\textsubscript{253}, USBL\textsubscript{354} and USBL\textsubscript{451} systems we have to accomplish the all steps that have been implemented for USBL\textsubscript{152} system. In order to distinguish the results of the measured coordinates by different basic USBL systems we introduce the consequent designations for the vectors and transformation matrices for each particular USBL system:

\begin{align*}
\mathbf{p}_{\text{USBL}_{123}} &= C^{-1} A^{-1} B^{-1} p_{123}^{\text{c}}; \\
\mathbf{p}_{\text{USBL}_{234}} &= C^{-1} A^{-1} B^{-1} p_{234}^{\text{c}}; \\
\mathbf{p}_{\text{USBL}_{341}} &= C^{-1} A^{-1} B^{-1} p_{341}^{\text{c}}; \\
\mathbf{p}_{\text{USBL}_{412}} &= C^{-1} A^{-1} B^{-1} p_{412}^{\text{c}}; \\
\mathbf{p}_{\text{USBL}_{153}} &= C^{-1} A^{-1} B^{-1} p_{153}^{\text{c}}; \\
\mathbf{p}_{\text{USBL}_{254}} &= C^{-1} A^{-1} B^{-1} p_{254}^{\text{c}}; \\
\mathbf{p}_{\text{USBL}_{122}} &= C^{-1} D^{-1} F^{-1} A^{-1} B^{-1} p_{122}^{\text{c}}; \\
\mathbf{p}_{\text{USBL}_{253}} &= C^{-1} D^{-1} F^{-1} A^{-1} B^{-1} p_{253}^{\text{c}}; \\
\mathbf{p}_{\text{USBL}_{354}} &= C^{-1} D^{-1} F^{-1} A^{-1} B^{-1} p_{354}^{\text{c}}; \\
\mathbf{p}_{\text{USBL}_{451}} &= C^{-1} D^{-1} F^{-1} A^{-1} B^{-1} p_{451}^{\text{c}}.
\end{align*}

The final stage of the algorithm implies the calculation of the means of the object coordinates in the \(\Sigma=(0,x,y,z)\) coordinate system with reliable data obtained by the elemental USBL systems.

### 4 Conclusion

In this article we have considered the method of design of USBL systems both utilizing of orthogonal and non-orthogonal (skew) basic three-element arrays. A case for the design of a five-element USBL system is presented. The paper focused on the problem of exploiting the latent resources of the receiving USBL array (skew three-element arrays) for accurate coordinate determination in conditions when the receiving antenna can have significant inclinations and the location of the object is arbitrary. The proposed algorithm accomplishes the selection of reliable elemental USBL arrays (orthogonal and non-orthogonal) utilizing the analysis of the values of latitude angles to the object for each elemental array. The presented algorithm is a significantly modified version of the algorithm designed for the five-element USBL system where only orthogonal elemental three-element arrays have been utilized [9]. Algorithm simulation was realized for a variety of the mutual positions of the USBL system and object in a wide range of pitch and roll of receiving arrays (pitch and roll angles \(\xi\) and \(\zeta\) are assumed to be in the range from \(-40^\circ\) to \(+40^\circ\)). For all tested angular receiving array positions and object locations the designed algorithm showed reliable operation.

### References:


