Analysis and idealization of strain mechanism of end-plate connections

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Abstract: - The paper deals with problems of an actual behaviour of header plate connections in steel building frames. A traditional model of the joint under examination represents a nominally pinned joint. Such a model is regarded by the author as not quite accurate. That is why he examines the necessity of a general conception of idealization of strain mechanism of this joint type. In order to describe generally the joint behaviour in the loading process, he uses characteristics applied in the theory of semi-rigid connections, i.e. \( M - \phi \) relationship, where \( M \) is bending moment and \( \phi \) is rotation; besides that he defines another deformation component, so-called connection unfolding \( c \), and analogically introduces \( M - c \) relationship.

Key-Words: - steel structures, design, actual behaviour, joints, pinned joints, structural properties of joint, header plate connections

1 Introduction
Joint behaviour has a significant effect on the response of the structural frame and must be included in the global analysis. The proposed paper is directed to influence of connection rotation on extraction of system length of beam.

A typical header plate connection is shown in figure 1. If the connection is not loaded, deformation does not occur (see fig. 1a). If the connection is loaded by bending moment (see fig. 1b), two deformation components occur: a) rotation \( \phi \), b) unfolding \( c \). As can be seen, axis of rotation of the connection does not identical with centroidal axis of the beam. If the rotation \( \phi \) occurs then the unfolding \( c \) occurs too – the unfolding is the horizontal displacement of a header plate from a flange of column. Connection unfolding occurs when the ductile header plate deforms due to tensile forces in bolts. We introduce a quantity \( d_a \) as the distance of the axis of rotation from the centroidal axis of the beam. Unfolding \( c \) depends on the rotation \( \phi \) and on the distance \( d_a \). We measure the unfolding \( c \) along the centroidal axis of the beam.

![Fig. 1 – Schema of strain mechanism of connection](image-url)
2 On experiments

Relationships $M-\phi$ and $M-c$ describe the actual behaviour of joint, where $M$ is bending moment, $\phi$ rotation and $c$ unfolding. The experimental method has been selected as the key method for data collection. Views of experiments are shown in figures 2 and 3. Results of experiments was published in [1], [2], [3]. The table 1 is included some results of the experiments (rotation stiffness $S = M/\phi$, translational characteristics $T = M/c$ and distance of axis of rotation from centroidal axis of beam $d_a = c/\phi$), where the joint configuration is shown in figure 1a.

### Tab 1 – Connection geometry and structural properties of joints

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3 Influence on a structure response

Joint behaviour has a significant effect on the response of the structural frame and must be included in the global analysis.

A typical primary beam in a multi-storey building is shown in figure 4. The primary beam is connected to columns by header plate connections. If the primary beam is not loaded, we call a system length of the beam as $L_b$ (see fig. 4a). If the primary beam is loaded by transverse load (see fig. 4b), then deformations $\phi$ and $c$ occur in both connections; the connections unfolding causes an extraction of system length of the primary beam $L_b + 2c$.

A vertical tie of multi-storey building is shown in figure 5. All primary beams are connected to columns by header plate connections. If the primary beams are loaded by vertical loads, a secondary internal forces occur in vertical bracings. These internal forces are caused by extraction of primary beams.

4 Stiffness matrix of primary beam

We present here a solution using finite element method.

Local vector of end effects is

$$\widetilde{M}_{a,b} = [\widetilde{M}_{a,b}, \widetilde{X}_{a,b}, \widetilde{Z}_{a,b}, \widetilde{M}_{b,a}, \widetilde{X}_{b,a}, \widetilde{Z}_{b,a}]^T$$

(1)

This vector is given by equation

$$\tilde{M}_{a,b} = k_{a,b} r_{a,b}$$

(2)

where $k_{a,b}$ is local stiffness matrix of beam,

$r_{a,b}$ local vector of deformation components.

Forces and deformations are shown in figure 6. The matrix $k_{a,b}$ is

$$k_{a,b} = \begin{bmatrix}
  k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} & k_{1,6} \\
  k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} & k_{2,6} \\
  k_{3,1} & k_{3,2} & k_{3,3} & k_{3,4} & k_{3,5} & k_{3,6} \\
  k_{4,1} & k_{4,2} & k_{4,3} & k_{4,4} & k_{4,5} & k_{4,6} \\
  k_{5,1} & k_{5,2} & k_{5,3} & k_{5,4} & k_{5,5} & k_{5,6} \\
  k_{6,1} & k_{6,2} & k_{6,3} & k_{6,4} & k_{6,5} & k_{6,6}
\end{bmatrix}$$

(3)

The vector $r_{a,b}$ is

$$r_{a,b} = [\phi_{a}, w_{a}, \phi_{b}, u_{b}, w_{b}]^T$$

(4)

We introduce conditions of deformations equality

$$\begin{align*}
\phi_a &= \phi_{a,b} + \phi_a \\
\phi_b &= \phi_{b,a} + \phi_b \\
u_a &= u_{a,b} + c_a \\
u_b &= u_{b,a} + c_b
\end{align*}$$

(5)

(6)

where $\phi_a, \phi_b$ are rotations of joints,

$\phi_{a,b}, \phi_{b,a}$ rotations of ends of beam,

$\phi_a, \phi_b$ rotations of connections,

$u_a, u_b$ horizontal displacements of joints,

$u_{a,b}, u_{b,a}$ horizontal displacements of ends of beam,

$c_a, c_b$ unfoldings of connections.
We use factors of bending and axial ductility of beam

\[ \alpha_{a,b} = \frac{L_b}{3EI_b} + \frac{1}{S_{a,b}} \]  \hspace{1cm} (7) 

\[ \alpha_{b,a} = \frac{L_b}{3EI_b} + \frac{1}{S_{b,a}} \]  \hspace{1cm} (8) 

\[ \beta = \frac{L_b}{6EI_b} \]  \hspace{1cm} (9) 

\[ \delta = \frac{L_b}{EA_b} \]  \hspace{1cm} (10) 

\[ \tau_{a,b} = \frac{1}{T_{a,b}} \]  \hspace{1cm} (11) 

\[ \tau_{b,a} = \frac{1}{T_{b,a}} \]  \hspace{1cm} (12) 

where \( E \) is Young’s modulus, 
\( A_b \) cross-sectional area of beam, 
\( I_b \) the second moment of area of beam, 
\( L_b \) span of beam.

Members of local stiffness matrix are:

\[ k_{1,1} = \alpha_{a,b} \delta - \tau_{b,a}^2 \]  \hspace{1cm} (13) 

\[ k_{4,4} = \alpha_{a,b} \delta - \tau_{a,b}^2 \]  \hspace{1cm} (14) 

\[ k_{1,4} = \beta \delta - \tau_{a,b} \tau_{b,a} \]  \hspace{1cm} (15) 

\[ k_{1,5} = k_{5,1} = -k_{1,2} = -k_{2,1} = -\alpha_{b,a} \tau_{a,b} + \beta \tau_{b,a} \]  \hspace{1cm} (16) 

\[ k_{4,5} = k_{5,4} = -k_{2,4} = -k_{4,2} = \alpha_{a,b} \tau_{b,a} - \beta \tau_{a,b} \]  \hspace{1cm} (17) 

\[ k_{2,2} = k_{5,5} = -k_{2,5} = -k_{5,2} = \alpha_{a,b} \alpha_{b,a} - \beta^2 \]  \hspace{1cm} (18) 

\[ k_{1,3} = k_{3,1} = -k_{1,6} = -k_{6,1} = \frac{k_{1,1} + k_{1,4}}{L_b} \]  \hspace{1cm} (19) 

\[ k_{3,4} = k_{4,3} = -k_{4,6} = -k_{6,4} = \frac{k_{4,4} + k_{1,4}}{L_b} \]  \hspace{1cm} (20) 

\[ k_{2,6} = k_{6,2} = k_{3,5} = k_{5,3} = -k_{2,3} = -k_{3,2} = \frac{k_{1,5} + k_{4,5}}{L_b} \]  \hspace{1cm} (21) 

\[ k_{3,3} = k_{6,6} = -k_{3,6} = -k_{6,3} = \frac{k_{1,3} + k_{3,4}}{L_b} \]  \hspace{1cm} (22) 

Fig. 4 – Extraction of system length of primary beam
5 Worked example of multi-storey building

A worked example of multi-storey building is shown in figure 7. Length of building is 76.5 m, breadth of building is 35.2 m, number of storeys is 4. Value of impose load is 2.5 kN/m²; values of snow load are for the 1st zone and of wind load for the 4th zone. Axial forces in members of vertical bracing are shown in figure 8 – at a) without influence of extraction of system length of beams, at b) with influence of extraction of system length of beams. Difference of values of axial forces does not negligible.

6 Conclusion

The goal of the paper is to present some problems of actual behaviour of header plate connections. The actual behaviour of header plate connection may be expressed by the connection rotation \( \phi \) and the connection unfolding \( c \). The connections unfolding causes an extraction of system length of the primary beam; the extraction of system length of the beam causes a secondary load of vertical bracings. Some results of experimental research of actual behaviour of selected type of connection are proposed. It is explained here using finite element method how the results should be used in a design of steel structure. A worked example of 4-storey building is presented too.
Acknowledgements:
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