Decentralized Adaptive Control and the Possibility of Utilization of Networked Control System

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Abstract: The main objective of this paper is to discuss a possible new concept of using communication and networked control system in decentralized adaptive control which guarantees zero residual tracking errors in all subsystems. It is studied where the on-line coordination or communication take place in the decentralized adaptive control scheme. This concept is an extension to the algorithms reported in the literature.

Key–Words: Decentralized Adaptive Control, Networked Control Systems, zero residual tracking error

1 Introduction

For a limited amount of information available for the individual subsystems in large-scale decentralized systems, an additional communication between subsystems is usually used. Such communication serves as a basis for decision-making process and control. The communication is provided by a communication network. Under this term we understand a set of network devices and technologies which are needed to implement the communication. When using such elements in the control system then it is classified as a networked control system. Thus this term often involves a decentralized approach of control system design.

The theoretical problem of decentralized adaptive control was first introduced by Ioannou [2], and later, with some modifications by Gavel and Šiljak, for example see [1]. Recently, the field of decentralized adaptive control is the subject of intense research, see [8] and it’s references, and [3, 5, 6, 7].

The best that can be achieved using methods of classical robust adaptive control in decentralized adaptive control in the presence of parametric uncertainties is the convergence of errors to some bounded residual set [5]. Despite of this fact it has been shown that the combination of adaptive control and decentralized control allows to take advantage of both fields.

Using additional communication between subsystems is essential in achieving of the convergence of errors to zero instead of some bounded residual set. Although we consider the communication between subsystems, the decentralization is maintained because the subsystems do not exchange information on their actual outputs.

The fundamental idea has been proposed in recent years as follows [5, 6]. Each subsystem has the information of reference signals of all other subsystems. In other words, each subsystem has the signal from the all reference models. This allows to achieve asymptotic convergence of the overall error (the tracking errors of the all subsystems) to zero.

One possibility is that the subsystems receive this information off-line, because it is often available a priori. Therefore there is no on-line communication and the principle of strictly decentralized system is maintained. This idea is referred as the implicit cooperation of subsystems [6]. Slightly different point of view is that the subsystems are coordinated using outputs of the reference models. Again, this can be the off-line coordination and therefore the system is considered as strictly decentralized.

The concept of coordination by means of reference models has been introduced by B. M. Mirkin [4]. Independently of the results obtained by B. M. Mirkin the similar results has been reported by Narendra et al. [6, 7]. The main idea in their work is the compensation of interactions while the information from the other subsystems is replaced by information on their reference signals (the reference model output). It is clear that this is just a different point of view on the same concept.

Important in both works is that the control law contains a high gain feedback term. In the work of
Mirkin are used the results reported for example in [1], and the coordinating term is added as mentioned above. In this case it is essential that the control law signal vector includes an adaptation error signal [1], which is not standard in model reference adaptive control, even in the special applications of this theory, for example see [9] or [10]. Authors Narendra and Oleng’ are closer to the conventions of model reference adaptive control. They use the standard signal vector. However, in the control law is introduced additional feedback term which is a P–controller of tracking error, assuming its high gain.

2 Utilization of Networked Control System

Our main objective in this paper is to discuss a possible new concept of using communication and networked control system in decentralized adaptive control which guarantees zero residual tracking errors in all subsystems.

The aim of the authors Narendra and Oleng’ is to increase the control performance in the decentralized adaptive control systems (eg unacceptably large transient errors). They have proposed to use an additional communication between the subsystems for this purpose. Research is looking for an answers to questions like: What information have to be shared among the subsystem controllers? At what time intervals? What is an indicator of necessity to communicate? It is in this context, where the role of the networked control system obviously appears.

In some cases involving the strictly decentralized adaptive control, having information about the entire trajectories of the other subsystems may be unnecessary and the convergence of errors to zero might be actually attained with much less information [8].

In further work we will consider the on-line coordination or communication instead of off-line coordination of subsystems. Then we need to find answers to similar questions as mentioned above.

First step is to determine where the on-line coordination or communication take place in the decentralized adaptive control scheme. This issue is discussed in the next sections.

2.1 Control algorithm with respect to on-line coordination

In this section we briefly describe a decentralized adaptive control algorithm with respect to on-line coordination which is based on the results mentioned above. As an example we consider a state tracking problem. Consider the large-scale system $S$ with $N$ interconnected subsystems $S_i$ in the form

$$S_i: \dot{x}_i = A_ix_i + b_iu_i + b_i^TF_ix_i$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector and $u_i(t) \in \mathbb{R}$ is the input of $i$-th subsystem $S_i$. The vector $x^T = [x^T_1 \cdots x^T_N]$ is composed of the state vectors of all subsystems with length $n = \sum_{i=1}^{N} n_i$. Matrix $F_i \in \mathbb{R}^{n \times n}$ is known, in the form $F_i = \text{blkdiag}(F_{1i}, \ldots, F_{ji}, \ldots, F_{IN})$ where

$$F_{ij} = \begin{cases} I \in \mathbb{R}^{n_x}, & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

The output of function \text{blkdiag}() is the block diagonal matrix, the arguments are placed on diagonal, while other elements are zero. Vector $g_i \in \mathbb{R}^n$ is constant but unknown. Matrix $A_i$ and vector $b_i$ are constant and unknown. It is assumed that there exist the vector $k_i^* \in \mathbb{R}^n$ and scalar $l_i^* \in \mathbb{R}$, so that $A_{mi} = A_i + b_i k_i^*$ and $b_{mi} = b_i l_i^*$, where $A_{mi}$ is known, asymptotic stable, and $b_{mi}$ is known vector.

A control objective is to force $x_i(t)$ to track the state $x_{mi}(t)$ of a given reference model which is assigned to each subsystem in the form

$$S_{mi}: \dot{x}_{mi} = A_{mi}x_{mi} + b_{mi}r_i$$

where $r_i(t)$ is known and bounded reference signal.

Consider the control law in the form

$$u_i = k_i^T x_i - \gamma_i \left( e_i^T P_i b_i \right) + l_i r_i - h_i^T F_i x_m$$

where $k_i(t)$ and $l_i(t)$ are the estimates of $k_i^*$ and $l_i^*$ respectively. In the first two terms in (4) are used only local signals of subsystem. In the last two term in (4) are used signals $r_i(t)$ and $x_m(t)$ which we will refer to as an external signals or a coordinating signals, where $x^T_m = [x^T_{m1} \cdots x^T_{mN}]$. The adapted parameter of the last term is the vector $h_i(t)$ with same size as $g_i$. In this case the ideal value of vector $h_i = g_i$, for more details see [6]. The parameter $\gamma_i$ is the positive real constant specified bellow. The matrix $P_i$ satisfies Lyapunov equation in the form $A^T_{mi} P_i + P_i A_{mi} = -Q_i$, where $Q_i$ is positive definite, symmetric, arbitrary matrix with corresponding size. The subsystem state tracking error is defined as

$$e_i = x_i - x_{mi}$$

We define the adaptation parameter errors in the form

$$\tilde{k}_i = k_i - k_i^*; \quad \tilde{l}_i = l_i - l_i^*; \quad \tilde{h}_i = g_i - h_i$$


84
Then the control law (4) can be written in the form
\[ u_i = k_i^T x_i + \tilde{k}_i^T x_i - \gamma_i \left( e_i^T P_i b_i \right) + l_i^T r_i + \tilde{l}_i r_i - h_i^T F_i x_m \]
and the equation of subsystem (1) in the form
\[ \dot{x}_i = A_i x_i + b_i u_i + b_i g_i^T F_i x_m + b_i g_i^T F_i e \]
Substituting the equation (7) into the equation (8) leads to
\[ \dot{x}_i = (A_i + b_i k_i^T) x_i + b_i l_i^T r_i + b_i \theta_i^T \omega_i - b_i \gamma_i \left( e_i^T P_i b_i \right) + b_i g_i^T F_i e + b_i g_i^T F_i x_m \]
where the vector \( \omega_i = \left[ x_i^T \ r_i \right] \) and the parameter error vector \( \theta_i^T = \left[ \tilde{k}_i^T \ \tilde{l}_i \right] \) are introduced. Finally subtracting (3) from (9) leads to the error equation in the form
\[ \dot{e}_i = A_{mi} e_i + b_i \theta_i^T \omega_i - b_i \gamma_i \left( e_i^T P_i b_i \right) + b_i g_i^T F_i e + b_i \tilde{h}_i^T F_i x_m \]
Consider the adaptation laws in the form
\[ \dot{\theta}_i = \dot{h}_i = -\left( e_i^T P_i b_i \right) \omega_i \]
(11) and (12) are asymptotic stable with respect to the parameters \( \tilde{k}_i \) and \( \tilde{l}_i \). The outline of the proof is as follows:
Consider the Lyapunov function candidate in the form
\[ V_i = e_i^T P_i e_i + \theta_i^T \dot{\theta}_i + \tilde{h}_i^T \dot{h}_i \]
The time derivative \( \dot{V}_i \) along the trajectory of the considered subsystem is in the form
\[ \dot{V}_i = e_i^T P_i e_i + e_i^T P_i \dot{e}_i + 2 \theta_i^T \dot{\theta}_i + 2 \tilde{h}_i^T \dot{h}_i \]
Substituting equations (10), (11) and (12) to the equation (14) leads to
\[ \dot{V}_i = -e_i^T Q_i e_i - \gamma_i \left( e_i^T P_i b_i \right)^2 - \gamma_i \left( e_i^T P_i b_i \right)^2 - 2 \left( e_i^T P_i b_i \right) \gamma_i^{-1} \left( g_i^T F_i e \right) \]
After completing the squares we obtain
\[ \dot{V}_i = -e_i^T Q_i e_i - \gamma_i \left( e_i^T P_i b_i \right)^2 - \gamma_i \left( e_i^T P_i b_i - \gamma_i^{-1} \left( g_i^T F_i e \right) \right)^2 + \gamma_i^{-1} \left( g_i^T F_i e \right)^2 \]
The second and the third terms in the right hand side of equation (16) are always negative. Therefore \( \dot{V}_i \) satisfies an inequality
\[ \dot{V}_i \leq -\lambda_{min} (Q_i) \left\| e_i \right\|^2 + \gamma_i^{-1} \left\| g_i \right\|^2 \left\| F_i \right\|^2 \left\| e_i \right\|^2 \]
\[ \dot{V}_i \leq -\lambda_{min} (Q_i) - \gamma_i^{-1} \left\| g_i \right\|^2 \left\| F_i \right\|^2 \left\| e_i \right\|^2 \]
where we have used the fact that \( \left\| e_i \right\| \leq \left\| e \right\| \) and \( \lambda_{min} (Q_i) \) is the smallest eigenvalue of matrix \( Q_i \). By selecting a sufficiently large value of parameter \( \gamma_i \) so that
\[ \gamma_i > \lambda_{min}^{-1} (Q_i) \left\| g_i \right\|^2 \left\| F_i \right\|^2 \]
we have \( \dot{V}_i \leq 0 \) along the trajectory of subsystem (10)—(11)—(12). The parameter error vector \( \dot{\theta}_i \) and \( \dot{h}_i \) are bounded, therefore the adapted parameters \( \theta_i^T = \left[ \tilde{k}_i^T \ \tilde{l}_i \right] \) and \( h_i \) are bounded. The tracking error \( e_i \rightarrow 0 \) with time \( t \rightarrow \infty \).
In order to be able to choose the value of parameter \( \gamma_i \), an euclidean norm of vector \( \left\| g_i \right\| \) must be known. If the norm of vector \( g_i \) is not known, we can use the adaptation law for this parameter in the form, see [8]
\[ \gamma_i = e_i^T e_i \]
(20)
The overall Lyapunov function \( V \) of large-scale system \( S \) is simply the sum of sub-Lyapunov functions
\[ V = \sum_{i=1}^{N} V_i \]
Therefore, the overall closed-loop system is stable because \( \dot{V} \leq 0 \), and the zero residual tracking errors in all subsystems are guaranteed.

### 2.2 The control scheme with coordinator

We may remark that the introduction of the matrix \( F_i \) in each subsystem allows to classify the signal \( x_m \) in the control law (4) as an external signal. The matrix \( F_i \) determines which components of the signal \( x_m \) will be used in the control law as an external signal. This allows to transmit one signal to all subsystems. The similar approach is obviously possible in the case of reference signals \( r_i \). The signal \( x_m \) can be generated locally (in every time instant) or it can be received as an external signal (not necessarily in every time instant — stability problems may arise).

Introducing the coordinator is straightforward. The coordinator is formed by the reference models of all subsystems. The same concept is used in [3, 5]. Consequently we consider an one way communication from coordinator to the controllers of each subsystem. Here the communication network (or networked control system) take place.

The scheme of the control algorithm is shown in the Fig. 1.
The reference models are in the form (3), where \( x_{m1}(t) \in \mathbb{R}^2, x_{m2}(t) \in \mathbb{R}^2, \)

\[
A_{m1} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad b_{m1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
A_{m2} = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix}, \quad b_{m2} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}.
\]

The reference signals \( r_1(t) \) and \( r_2(t) \) have the form

\[
r_1 = 10 \text{sign}(\sin(0, 4t)); \quad r_2 = -5 \sin(0, 7t)
\]

The arbitrary parameters of the control algorithm are chosen to be \( Q_1 = Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \). The initial conditions of all differential equations are chosen to be zero, including the initial values of adapted parameters.

The adaptive controller in each subsystem has the form (4) with the adaptation laws in the form (11) and (12).

For the simulation purposes, it is assumed that the reasonable estimates of norms \( \|g_1\| \) and \( \|g_2\| \) are known (\( \|g_1\| = 18.25; \|g_2\| = 13 \)). Therefore we have \( \gamma_1 = 1 + \lambda_{\min}(Q_1)\|g_1\|^2\|F_1\|^2 = 334 \) and \( \gamma_2 = 1 + \lambda_{\min}(Q_2)\|g_2\|^2\|F_2\|^2 = 170. \)

The results of the simulation are in the Fig. 2 and Fig. 3.

### 4 Conclusion and Future Work

This article has discussed the possibility of using the one way on-line communication from the coordinator to the subsystems in the decentralized adaptive control. This concept is an extension to the algorithms reported in the literature. The main idea is to replace the off-line coordination by the on-line coordination.

The zero residual tracking error is attained using information from all of the reference models. If the reference signals are not known a priori, then the information from all of the reference models can not be used off-line. This constraint is relaxed by discussed concept. Moreover, this concept allows to use much less information for attaining the zero residual error.

In our future work, we will study the similar questions as mentioned in the section 2. The stability issues and the performance issues of the discussed decentralized adaptive control scheme will be studied too.

**Acknowledgements:** This work has been supported by Slovak Research and Development Agency through grant APVV-0211-10. It has been supported by the project Req-00048-001 too.
Fig. 2: Simulation results for subsystem $S_1$

Fig. 3: Simulation results for subsystem $S_2$

References:


