Multifractal analysis of magnetotelluric data

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Abstract: - We investigated the multifractal structure of the daily time series of apparent resistivity and phase of the magnetotelluric field measured in a site in southern Italy from the 9 of July 2007 to September 2011. The magnetotelluric parameters were computed in two orthogonal directions corresponding to 25 different sounding periods from about 0.75 s to 238 s. Our results indicate that the apparent resistivity and the phases are multifractal series. The multifractal degree of the phases approximately decreases with the increase of the sounding period; while that of the resistivity shows a more complex relationship with the sounding depth.

Key-Words: - Multifractal detrended fluctuation analysis; magnetotellurics

1 Introduction

The analysis of the temporal fluctuations of geophysical processes allows to understand their underlying dynamical mechanisms [1]. The power spectral density $S(f)$, which is defined in terms of the Fourier functions, describes the frequency distribution of the power of a signal and is generally used for investigating the time dynamics of a process. For white noise processes (uncorrelated and independent), the power spectrum is approximately flat for any frequency; no memory phenomena characterizes such processes. For long-range correlated processes the power spectrum behaves as a power-law function of the frequency $f$, with the power-law exponent (called scaling exponent) identifying and quantifying the strength of the inner correlations [2].

The estimation of the scaling exponent can be performed by means of several methods. But, recently, the detrended fluctuation analysis (DFA), developed by Peng et al. [3], has been extensively used for its low computational effort, effectiveness and independence of possible nonstationarities with unknown origin and cause [4]. Several applications in geophysics [5-9], environmental sciences [10], economics [11], biology and medicine [12, 13, 14] were performed using the DFA, which permits the identification of scaling behavior in monofractal series or to investigate the monofractality of a series. A single scaling exponent is enough to completely describe a monofractal process. Monofractals are homogeneous objects, in the sense that they have the same scaling properties, characterized by a single singularity exponent [15]. More than one scaling exponent can be necessary when the process presents some crossovers separating dynamical regimes with different scaling behaviors [16, 17]; or when different segments of the same series are characterized by different type of correlations indicating a time variation of the scaling behaviour [18]: in these cases, the object is still monofractal. But, in other cases, different scaling exponents can be revealed for many interwoven fractal subsets of the signal; in this case the process is not a monofractal but multifractal, which requires many indices to be fully characterized. Multifractals can be decomposed into many-possibly infinitely many-sub-sets characterized by different scaling exponents. Thus multifractals are intrinsically more complex and inhomogeneous than monofractals and characterize systems featured by a spiky dynamics, with sudden
plane waves of infinite horizontal extent, Eq. (1) can be used to obtain the apparent resistivity and phase values:

\[
\begin{bmatrix}
Z_{xx}(\omega) & Z_{xy}(\omega) \\
Z_{yx}(\omega) & Z_{yy}(\omega)
\end{bmatrix} = \frac{1}{\mu_0 \epsilon_0} \begin{bmatrix}
E_x(\omega) \\
E_y(\omega)
\end{bmatrix}
\]

(1)

With the usual MT assumption, the off-diagonal elements of \(Z\) are small compared to source length scales. Under such conditions, the off-diagonal elements of \(Z\) can be used to obtain the apparent resistivity and phase values:

\[
\rho_{ij} = \frac{1}{\mu_0 \epsilon_0} |Z_{ij}|^2, \quad \phi_{ij} = \tan^{-1}\left(\frac{\text{Im}(Z_{ij})}{\text{Re}(Z_{ij})}\right)
\]

(2)

Here, the apparent resistivity values were estimated using the Robust Transfer Function Estimation Program for data reduction described in [20].

The analysed data were recorded from the 9 of July 2007 to September 2011 at the permanent MT station installed in Tramutola site (LAT. 40.297 LONG. 15.805, 700 m a.l.s.) in south-eastern flank of the Agri Valley (Southern Italy). The valley is a NW-SE intermontane basin, affected by a coherent fault system N120 trending left-lateral strike-slip fault. The MT station is equipped with a receiver MT24LF (Magnetotelluric 24-bit A/D Low Frequency system) which records the magnetic fields by means two induction coils (EMI Inc., BF4), and the electric fields by means different electrical dipoles. The sampling frequency was set to 6.25 Hz. Subsets of 5.4 -105 values were used to calculate the MT estimates for T ranging from 0.75 s to 238 s. Fig. 1 shows, as an example, the resistivity time series in xy direction corresponding to T=0.75 s.

3 The multifractal method

The Multifractal Detrended Fluctuation Analysis (MF-DFA) represents the well-known tool for investigating multifractality in nonstationary signals [R1]. Assuming that \(x(k)\) is the time series and \(x_{ave}\) its mean, let’s calculate the “profile” by integrating the signal,

\[
y(i) = \sum_{k=1}^{i} [x(k) - x_{ave}] 
\]

(2)

The integration has the effect of reducing the level of measurement noise present in observational and finite records. The profile \(y(i)\) is divided into nonoverlapping \(N_S = \text{int}(N/s)\) segments of equal length \(s\) (the operator \(\text{int}(\ast)\) furnishes the inter part). Since the length \(N\) of the signal could not be an integer multiple of the time scale \(s\), a short part at the end of the profile \(y(i)\) may remain. Therefore, the same procedure is repeated starting from the opposite end of the series, producing \(2N_S\) segments. For each of the \(2N_S\) segments a linear least square fit is obtained. Then the variance is calculated by using

\[
F^2(s) = \sum_{i=1}^{s} \{y[i] - y_{ave}\}^2.
\]

(3)

for each segment \(v\), \(v=1,\ldots,N_S\) and

\[
F^2(s,v) = \frac{1}{s} \sum_{i=1}^{s} \{y[i] - y_{ave}\}^2.
\]

(4)

for \(v=N_S+1,\ldots,2N_S\). Here, \(y_{ave}\) represents the line fitting the profile in segment \(v\). Then, averaging over all the segments, we get the \(q\)-th order fluctuation function

\[
F_q(s) = \left[ \frac{1}{2N_S} \sum_{v=1}^{2N_S} F^2(s,v) \right]^{1/q}
\]

(5)

where, in general, the index variable \(q\) can take any real value except zero. For \(h(0)\) a logarithmic averaging procedure is used

\[
F_q(s) = \exp\left[ \frac{1}{2N_S} \sum_{v=1}^{2N_S} \ln F^2(s,v) \right] \approx s^{h(q)}
\]

(6)

If the series \(x_i\) is long-range power-law correlated, \(F_q(s)\) increases for large values of \(s\) as a power-law

\[
F_q(s) \propto s^{h(q)}.
\]

(7)
For monofractal signals \( h(q) \) is independent of \( q \). For multifractal signals, characterized by the different scaling of small and large fluctuations, \( h(q) \) will significantly depend on \( q \). For positive \( q \), the segments \( \nu \) with large variance (i.e. large deviation from the corresponding fit) will dominate the average \( F_q(s) \). Therefore, if \( q \) is positive, \( h(q) \) describes the scaling behaviour of the segments with large fluctuations; and generally, large fluctuations are characterized by a smaller scaling exponent \( h(q) \) for multifractal time series. For negative \( q \), the segments \( \nu \) with small variance will dominate the average \( F_q(s) \). Thus, for negative \( q \) values, the scaling exponent \( h(q) \) describes the scaling behaviour of segments with small fluctuations, usually characterized by a larger scaling exponents. The multifractal degree of the signal can be indicated by the range of the \( h_q \) exponents. The larger the \( h_q \)-range, the higher the multifractality.

4 Results

We analyzed the multifractal properties of the daily time series of apparent Earth’s resistivity and phase calculated from the electromagnetic field measured at Tramutola station, located in southern Italy, corresponding to 25 sounding periods (0.74473s, 0.93091s, 1.17029s, 1.46286s, 1.86182s, 2.40941s, 3.15077s, 4.0965s, 5.28516s, 6.82667s, 8.62316s, 10.92267s, 13.65333s, 16.384s, 21.14064s, 27.30667s, 34.49263s, 43.69067s, 54.61333s, 65.536s, 84.56258s, 113.97565s, 154.20235s, 163.84s, 238.31273s). Fig. 1 shows, the daily variation of the Earth’s apparent resistivity and phase in the directions \( xy \) and \( yx \). We applied the MF-DFA to each time series. Fig. 2 shows an example the fluctuation functions for two different values of \( q \) (-10, 10) of the apparent resistivity \( \rho_{xy} \) corresponding to the sounding period \( T=0.744736s \). The different slopes \( h_{10} \) and \( h_{10} \) of the two fluctuation curves suggest that the series is multifractal. Fig. 3 shows the relationship \( h_q \sim q \) of the resistivity \( \rho_{xy} \) corresponding to the period \( T=0.744736s \) for \( q \) ranging between -20 and 20. The multifractality of the series is evidenced by the nonlinearity of the \( h_q \sim q \) relationship, with a decreasing behavior of the exponent \( h_q \) with the increase of \( q \); a measure of the multifractal degree is given by the range of the \( h_q \) exponents, which is \(~0.98\). Figs. 4 and 5 shows the variation with the sounding period \( T \) of the \( h_q \)-range of the resistivities and phases in directions \( xy \) and \( yx \) respectively. The resistivities \( \rho_{xy} \) show an almost constant value of the multifractal degree up to a sounding period of about 65 s; while for higher sounding periods it decreases with the period. The phases \( \varphi_{yx} \) reveal a general decreasing trend with the sounding period \( T \). The phases \( \varphi_{yx} \) are characterized by a general decreasing behavior with the period quite similar to the behavior of phases in the \( xy \) direction. The resistivities \( \rho_{yx} \) are, instead, featured by a rather singular pattern, with a minimum at a sounding period around 5 s.

5 Conclusions

The daily time series of the phases and the apparent resistivities computed from the magnetotelluric field measured at station Tramutola in southern Italy show multifractal structure. The multifractal degree, quantified by the range of the \( h_q \) exponents (for \( q \) ranging between -20 and 20), gives information about the degree of homogeneity of the series. The higher the multifractal degree, the more heterogeneous the series. In our study, we found that the phases in both the directions shows almost similar behavior, which is generally decreasing with the increase of the sounding period. This pattern suggests that the time series corresponding to the higher sounding periods tend to be more homogeneous that those corresponding to the smaller sounding periods. Due to the relationship between the sounding period and the depth, this means that the phases estimated for the shallower layers are more heterogeneous that those estimated for deeper ones. More complex is the multifractal behavior shown by the resistivity, which evidence a more homogeneous structure around 10 s.

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References:


Fig. 1 Daily time variation of the resistivity $\rho_{xy}$ corresponding to the period T=0.74473s.
Fig. 2 Fluctuation functions for $q=-10$ and $q=10$ for the daily time series of the resistivity $\rho_{xy}$ corresponding to the period $T=0.74473s$.

Fig. 3 Relationship $h_q-q$ of the daily time series of the resistivity $\rho_{xy}$ corresponding to the period $T=0.74473s$.

Fig. 4 Variation with the sounding period $T$ of the range of the exponents $h_q$ for the resistivities $\rho_{xy}$ (a) and phases $\phi_{xy}$ (b).
Fig. 5 Variation with the sounding period $T$ of the range of the exponents $h_q$ for the resistivities $\rho_{yx}$ (a) and phases $\phi_{yx}$ (b).