Second Law Analysis of MHD Flow over Open Parallel Microchannels Embedded in a Micropatterned Surface

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Abstract: The purpose of this work is to investigate the entropy generation in a laminar, electrically conducting fluid flow past open parallel microchannels embedded in a horizontal stationary surface subject to a transverse magnetic field at prescribed surface temperature (PST). The entropy generation number is formulated by an integral of the local rate of entropy generation along the width of the surface based on an equal number of microchannels and no-slip gaps interspersed between those microchannels. The velocity, the temperature, the velocity gradient and the temperature gradient adjacent to the surface wall are substituted into this equation resulting from the momentum and energy equations obtained numerically by the Dormand-Prince pair and shooting method. The entropy generation number, as well as the Bejan number, are calculated, depicted graphically and discussed in detail. It is concluded that an increase in the width of the open microchannels tends to decrease entropy generation number along surface.

Key-Words: MHD, EBSM, hydrodynamic slip.

1 Introduction
Magnetic field applied by generating Lorenz force can control an electrically conducting fluid flow and heat transfer [1]. Moreover, the transporting of biological liquids via an applied magnetic field is a significant topic in the micro scale systems [2] where it is essential to consider the influence of the velocity slip at the boundaries, specially along hydrophobic microchannel walls. Consequently, many researchers have investigated the slip boundary layer problems along various surface configurations [3-8]. Most of the methods developed for transporting particles and cells are typically appropriate only for closed microchannels, with transportation in open microfluidic systems seldom being reported. Recently, Wu et al. [9] have evaluated a new method of transportation for particles, cells, and other microorganisms by rectified ac electro-osmotic flows in open microchannels. The authors propose using open microchannels instead of usual closed microchannels, since the former are open to the ambient air at the top, which can provide advantages, such as maintaining the physiological conditions for normal cell growth and introducing accurate amounts of chemical and biological materials [9]. The technique of EBSM (entropy based surface micro-profiling) has been developed for the first time by Naterer [10], who proposed embedded surface microchannels to reduce exergy losses in convection. This method includes local slip conditions within the open microchannels and thus tends to drag reduction and lower energy availability losses along the plate. However, in a subsequent work, Naterer et al. [11] have applied this method to special purpose of aircraft intake de-icing. Similarly, Yazdi et al. [12] have studied liquid fluid flow past embedded open parallel microchannels within the surface using EBSM. There have been many theoretical models developed particularly for second law analysis of MHD boundary layer flow [13-15]. However, to the best of our knowledge, no investigation has been made yet to evaluate the entropy generation of MHD flow.
over open microchannels embedded in a stationary surface at PST.

2 Problem Formulation

The 2-D, steady, laminar MHD flow past open parallel microchannels embedded within a horizontal surface at prescribed surface temperature (PST) is considered. The embedded surface microchannels are illustrated in Fig. 1. It is assumed that the width of the surface consists of a specific number of open microchannels and the base sections \( m' \), each of which has its own width. Furthermore, a no-slip boundary condition is applied between open microchannels, whereas a slip condition is applied to the open parallel microchannels.

![Fig.1. Schematic of embedded surface microchannels.](image)

The fluid is incompressible and Newtonian. The magnetic Reynolds number is assumed small and the induced magnetic field is neglected. We consider a transverse magnetic field with strength \( B(x) \) which is applied in the vertical direction, given by the special form:

\[
B(x) = B_0 x^{1/2}, B_0 \neq 0
\]

where \( x \) is the coordinate along the plate measured from the leading edge. The positive \( y \)-coordinate is measured normal to the \( x \)-coordinate in the outward direction towards the fluid. The corresponding velocity components in the \( x \) and \( y \) directions are \( u \) and \( v \), respectively. The surface is at prescribed surface temperature (PST), \( T_w \) given as [7]:

\[
y = 0, \quad T = T_w (= T_\infty + Ax^k)
\]

where \( A \) is a constant and \( k' \) is the surface temperature parameter at the prescribed surface temperature (PST) boundary condition. Within the framework of the above noted assumptions, the steady two-dimensional boundary layer equations are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} (u - u_\infty)
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\sigma}{\rho} \left( \frac{\partial u}{\partial y} \right)^2
\]

The associated boundary conditions are

\[
y = 0 \Rightarrow u = u_\infty, \quad v = 0, \quad T = T_w (= T_\infty + Ax^k)
\]

\[
y \to \infty \Rightarrow u = u_\infty, \quad T = T_\infty
\]

where \( \rho \) is the fluid density, \( \alpha \) is thermal diffusivity, \( \sigma \) is the electrical conductivity of the fluid and \( u_\infty \) is the velocity slip, assumed to be proportional to the local wall shear stress as follows:

\[
u_\infty = \left( \frac{\partial u}{\partial y} \right)_{w}
\]

where \( l \) is slip length, which is for Newtonian fluids usually expressed as a direct proportionality between the slip velocity and the shear rate at a wall [7]. To design a micropatterned surface in the presence of applied magnetic field, this slip boundary condition is considered inside the open microchannels [11]. Empirical evidence suggests that, for water flowing through a microchannel, the surface of which is coated with a 2.3 nm thick monolayer of hydrophobic octadecyltrichlorosilane, an apparent velocity slip is measured just above the solid surface. This velocity is approximately 10% of the free-stream velocity and yields a slip length of approximately 1 mm [16]. We introduce now the following non-dimensional similarity variables:

\[
f'(\eta) = \frac{u}{u_\infty}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = y \sqrt{\frac{\nu_\infty x}{\nu}}
\]

Here, the slip coefficient using similarity variables is given by [4]:

\[
f'(0) = K f''(0)
\]

where \( K \) is the slip coefficient defined by:

\[
K = \frac{1}{2} \sqrt{\frac{u_\infty}{x \nu_\infty}}
\]

The fundamental partial differential equations (4) and (5) are transformed to ordinary differential
equations by substituting dimensionless similarity variables (8) into equations (4) and (5) as follows:

\[ f'' + \frac{1}{2} ff' - M^2 f' + M^2 = 0 \]  
(11)

\[ \theta'' + \frac{\Pr}{2} \theta f' - Pr k f' \theta + Ec \Pr x^{k} (f^*)^2 = 0 \]  
(12)

The associated boundary conditions are:

\[
\begin{align*}
\eta &= 0 \Rightarrow f(0) = 0 \\
\theta(0) &= 1
\end{align*}
\]

\[
\eta \rightarrow \infty \Rightarrow \left\{ \begin{array}{l}
\theta'(\infty) = 1 \\
\theta(\infty) = 0
\end{array} \right.
\]

where \( \Pr, \Ec, \) and \( M \) show the Prandtl number, the Eckert number and the magnetic parameter respectively:

\[ M^2 = \frac{\sigma \beta_k^2}{\rho u_o^2}, \quad \Ec = \frac{u_o}{Ac_p}, \quad \Pr = \frac{\nu_o}{\alpha} \]  
(14)

The equations (11) and (12) are solved numerically by using the explicit Runge-Kutta (4, 5) formula, the Dormand-Prince pair and shooting method, subject to the boundary conditions (13). Finally, the skin friction coefficient and the local Nusselt number can be explained as follows:

\[ C_f = \frac{\tau_w}{\frac{1}{2} \rho u_o^2} = \frac{\mu}{\frac{1}{2} \rho u_o^2} = 2 \Re \frac{1}{2} f''(0), \]  
(15)

\[ Nu_x = \frac{\frac{x}{\tau_w}}{T_w - T_y} = |\theta'(0)| \left( \frac{u_o}{\nu_o} \right)^{1/2} \]  
(16)

The results of the numerical solutions to the problem are subsequently substituted into the entropy generation analysis.

3 Entropy Generation Analysis

Entropy generation analysis concerned with the MHD flow over open microchannels embedded within a horizontal surface at prescribed surface temperature (PST) is considered here. Heat transfer \( (S_T'^*) \), friction \( (S_F'^*) \), and magnetic irreversibilities \( (S_M'^*) \) are included within the local volumetric rate of entropy generation. The rate of entropy generation will be obtained based on the previous solutions of the boundary layer for fluid velocity and temperature. According to Yazdi et al.[15], the local volumetric rate of entropy generation in the presence of a magnetic field is given by:

\[ S^* = \frac{k_a}{T_w} \left[ \frac{\partial T}{\partial x} \right]^2 \left( \frac{\partial T}{\partial y} \right)^2 + \frac{\mu_a}{T_w} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2}{T_w} u^2 \]  
(17)

In order to for include the effect of the embedded open parallel microchannels within the surface, integration over the width of the surface is applied over the local rate of entropy generation adjacent to the wall. The cross-stream \( (z) \) dependence arises from interspersed no-slip (subscript \( ns \) ) and slip-flow (subscript \( s \) ) solutions of the boundary layer equations. Therefore, the integration over the width of the surface from \( 0 \leq z \leq W \) consists of \( m' \) separate integrations over each microchannel surface width, \( 0 \leq W_z \leq W \), as well as the remaining no-slip portion of the plate, which is interspersed between these microchannels and covers a range of \( 0 \leq W \sim W_m \) (see Fig.1). The previous correlations for the convection coefficient based on the velocity, temperature, velocity gradient and the temperature gradient adjacent to the wall are substituted into this equation. Thus, by performing the integrations, and assuming an equal number of microchannels and no-slip gaps interspersed between those microchannels, it can be shown that:

\[ S_{g}^* = S_{T}^* + S_{F}^* + S_{M}^* \]  
(18)

where

\[ S_{T}^* = \int_{0}^{m(W_z+2d)} S_{T,ns}^* dz + \int_{W_m}^{W-nmW_z} S_{T,slip}^* dz \]  
(19)

\[ S_{F}^* = \int_{0}^{m(W_z+2d)} S_{F,slip}^* dz + \int_{W_m}^{W-nmW_z} S_{F,ns}^* dz \]  
(20)

\[ S_{M}^* = \int_{0}^{m(W_z+2d)} S_{M,slip}^* dz + \int_{W_m}^{W-nmW_z} S_{M,ns}^* dz \]  
(21)

Finally, the dimensionless local entropy generation rate is defined as a ratio of the local entropy generation rate and a characteristic entropy generation rate. Here, the characteristic entropy generation rate is defined as:

\[ S_{go}^* = \frac{k_a}{L^2 T_w^2} \Delta T^2 W \]  
(22)

where \( L \) is characteristic length scale. In addition, the non-dimensional geometric parameters are defined as:

\[ \lambda = \frac{W_z + 2 d}{W}, \quad \zeta = \frac{d}{W} \]  
(23)
Consequently, the entropy generation number is expressed as:

\[ N_e = \frac{S_g}{S_{gs}} = \left\{ \begin{array}{l} \frac{k^2}{X^2} \theta_s^2(0)[m'\lambda] \\ + \frac{k^2}{X^2} \theta_{ns}^2(0)[1 + 2m'\zeta - m'\lambda] \\ + \frac{Re}{X} \theta_s^2(0)[m'\lambda] \\ + \frac{Re}{X} \theta_{ns}^2(0)[1 + 2m'\zeta - m'\lambda] \\ + \frac{Br \cdot Re}{\Omega} f_s^2(0)[m'\lambda] \\ + \frac{Br \cdot Re}{\Omega} f_{ns}^2(0)[1 + 2m'\zeta - m'\lambda] \\ + \frac{Br \cdot M^2 \cdot Re}{\Omega^2} f_s^2(0)[m'\lambda] \\ + \frac{Br \cdot M^2 \cdot Re}{\Omega^2} f_{ns}^2(0)[1 + 2m'\zeta - m'\lambda] \end{array} \right\} \] (24)

where \( X, Re, Br \) and \( \Omega \) are, respectively, the non-dimensional surface length, the Reynolds number, the Brinkman number and the dimensionless temperature difference. These parameters are given by the following relationships:

\[ Br = \frac{\mu_e(u_e)^2}{k_e \Delta T}, \quad Re = \frac{u_e L}{v_e}, \quad X = \frac{x}{L}, \quad \Omega = \frac{\Delta T}{T_e} \] (25)

As explained before, a laminar boundary layer flow is also considered in this research. It should also be noted that, although the entropy generation number is a non-dimensional parameter, the surface length should be selected in order to ensure that the Reynolds number remains below the point of transition to turbulence at \( Re_c = 5 \times 10^5 \), as in contrast to the external convective heat transfer problem, the critical Reynolds number within an open microchannel is 1800 [10]. This Reynolds number is based on the microchannel depth or hydraulic diameter. In this study, the Bejan number is defined as the ratio of heat transfer irreversibility to total irreversibility due to heat transfer, fluid friction and magnetic field for the laminar MHD boundary layer flow. Bejan number is given by [15]:

\[ Be = \frac{\text{Heat transfer irreversibility}}{\text{Entropy generation number}} \] (26)

4 Results and Discussion

Table 1 shows a comparison between the results of the present work and that of the previous works, obviously indicating a good agreement.

Table 1. Comparison of \( \theta'(0) \) between the present results and those obtained previously when \( M=0, Ec=0, K=0. \)

<table>
<thead>
<tr>
<th>Pr</th>
<th>( k' )</th>
<th>Ibrahim [17]</th>
<th>Present Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0</td>
<td>0.64591</td>
<td>0.64591</td>
</tr>
<tr>
<td></td>
<td>-0.4</td>
<td>0.23430</td>
<td>0.23430</td>
</tr>
<tr>
<td>1/3</td>
<td>0.23430</td>
<td>0.81911</td>
<td>0.81913</td>
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<td>1.04353</td>
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<tr>
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<td>0</td>
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<td>0.29266</td>
</tr>
<tr>
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<td>1</td>
<td>0.48032</td>
<td>0.48033</td>
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</table>

Fig.2 can be compared with previous work of Rahman [18] when \( M=0.5 \). Thus, in the presence of the slip, the magnetic parameter can increase fluid velocity both inside the boundary layer and adjacent to the wall effectively. This is the reason why the magnetic field has potential to control an electrically conducting fluid in micro scale systems.
Fig. 3 illustrates that an increase of the magnetic parameter $M$ tends to increase the wall shear stress, whereas the wall shear stress decreases in the presence of a high slip coefficient.

![Graph](image1)

(a)

Fig. 3 (a) Variation of $f'(0)$ and (b) $f''(0)$ as function of $K$ for various values of $M$

Fig. 4 illustrates the combined effect of the surface temperature parameter $k'$, slip coefficient $K$ and the magnetic parameter $M$ on the heat transfer rate $|\theta'(0)|$. The results demonstrate that the heat transfer rate is increased by an increase of all three parameters ($k'$, $K$ and $M$). It is interesting to note that the slip as a necessary boundary condition in the micro scale systems has capability of increasing the heat transfer rate.

![Graph](image2)

(b)

Fig. 4. Variation of $|\theta'(0)|$ as function of $K$ for various values of $M$ and $k'$ when $Ec=0.1$ and $Pr=5$.

The combined effect of the slip coefficient and $\lambda$ on the entropy generation number is illustrated in Fig.5 (a). It is observed that the entropy generation number changes the trend, after reaching the minimum corresponding to a specific slip coefficient. It is interesting to note that, at the points to the left of the minimum; the slope of the tangent is negative, indicating that an increase in the $K$ tends to decrease $N_s$ due to a significant reduction in the friction irreversibility. Similarly, at the points to the right, the slope is positive, i.e. higher $K$ values yield higher entropy generation number. However, although the slip coefficient reduces friction irreversibilities by decreasing the wall shear stress, it shows an opposite effect on heat transfer irreversibilities. Thus, the slip coefficient can decrease the total irreversibilities ($N_t$) where the values of the friction irreversibilities are much more significant compared to the heat transfer irreversibilities. Further, it suggests that an increase in the width of the open microchannels tends to enhance the slip effects along the width of the surface, causing the entropy generation number to decrease. It also indicates that extra effort and cost associated with micromachining the surface to achieve a desired embedded microchannel surface cannot be warranted when there is no significant slip inside the microchannels. This phenomenon is much more pronounced when no-slip boundary condition ($K=0$) is assumed inside the embedded open microchannels. The combined effect of $K$ and $\lambda$ on the Bejan number is shown in Fig. 5 (b). It indicates that an increase in the width of the microchannels increases the Bejan number. Further, it is noted that an increase in the slip coefficient accompanies a rise in the Bejan number.

![Graph](image3)

(a)
5 Conclusion

The entropy generation in an electrically conducting fluid flow past open parallel microchannels embedded within a horizontal surface subject to a transverse magnetic field at prescribed surface temperature (PST) is evaluated. The results demonstrate that the heat transfer rate is increased by an increase of surface temperature parameter \( k' \), slip coefficient \( K \) and magnetic parameter \( M \). The Bejan number, \( Be \), increases with the increase of \( K \) and \( \lambda \), while it decreases with the increase of \( M \). Moreover, there is a minimum value for \( N_s(K) \) that leads the slip coefficient to exhibit decreasing (or increasing) effect at different values. Consequently, an increase in the width of the open microchannels tends to enhance the slip effects along the width of the surface, causing the entropy generation number to decrease. The present study can be applied in variety of micro scale systems such as micromixing technology.

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