Circular Statistics: An Analysis of Wind Direction Data

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Abstract: - An interest on renewable energy had encouraged some researchers to do research on wind energy. The focus of this paper was on the fitting of wind direction data which was one of the most important aspects in renewable energy. The main objective was to find the best distribution that can displayed the wind direction data. Three circular distributions were considered and they were Von Mises, Wrapped Cauchy and generalized Von Mises distributions. Data on the average daily wind direction were collected from Solar Energy Research Institute (SERI) in Universiti Kebangsaan Malaysia. It was fitted and compared to describe the best distribution. The suitability of the distribution was judged from the value of the mean circular error. Result showed that the Von Mises distribution fit the data very well compared to the generalized Von Mises and Wrapped Cauchy distribution. Thus, the Von Mises distribution can be used to model the wind direction in this particular station.

Key-Words: Wind direction, circular data, Von Mises distribution, generalized Von Mises distribution, wrapped Cauchy distribution

1 Introduction
Wind direction is the direction from which the wind is blowing. This direction is reported in cardinal directions or in azimuth degree. This type of data is categorized as circular data. The aim of this study is to find the circular distribution that best described wind direction of a particular station. For this purpose, the Von Mises, generalized Von Mises and Wrapped Cauchy distribution were applied and the best distribution that fit the data was judge through the goodness of fit test. The parameters for the distribution were estimated using the maximum likelihood estimation. Statistical softwares were used for this computing purpose.

2 Wind Direction Data
In this study, data of particular interest was obtained from a station in the Solar Energy Research Institute (SERI), Universiti Kebangsaan Malaysia (UKM). Data of daily average wind direction from the station was collected starting from 1st June 2007 to 14th October 2007. Electronic anemometer was used to get the reading of the observations. This
instrument contained electronics that can translate the wind directional data from the vane into the formats in which computers can interpret. Some of the missing values from the data was estimated from the monthly average of the wind direction.

3 The Probability Distributions

Three probability distributions were considered and they were described in the following subsections.

3.1 Von Mises distribution

The probability distribution function of the Von Mises distribution is given by

\[
f(\theta; \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\theta - \mu)}
\]

with \(0 \leq \theta < 2\pi\), where \(\kappa\) is the concentration parameter and \(\mu\) is the mean direction. The normalizing constant, \(I_0(\kappa)\) in the denominator is the modified Bessel function of the first kind and order zero. The Von Mises distribution is unimodal and symmetric at \(\theta = \mu\). The maximum likelihood estimation was used to estimate these parameters. By using statistical software, it was found that the estimate of the parameter \(\kappa\) is equal to 6.1715 while the estimation of \(\mu\) is 5.5741 radian.

3.2 Generalized Von Mises distribution

According to [3], the generalized Von Mises densities can be symmetric, asymmetric, unimodal or bimodal. In order to be more flexible in terms of asymmetry and bimodality, the generalized Von Mises distribution was also fitted. The form of generalized Von Mises distribution can be express as

\[
f(\theta; \rho, \mu, \kappa_1, \kappa_2) = \frac{1}{2\pi G_1(\delta, \kappa_1, \kappa_2)} e^{[\kappa_1 \cos(\theta - \mu_1) + \kappa_2 \cos(\theta - \mu_2)]}
\]

with \(0 \leq \theta < 2\pi\), where \(\rho\) is the scale parameter and \(\mu\) is the location parameter. It is unimodal and symmetric about the value \(\theta = \mu\). The maximum likelihood estimates of the parameters can be obtained by a recursive algorithm given by [7]. Using this algorithm, the estimates of \(\mu\) and \(\rho\) were found to be 5.6463 radians and 0.7974 respectively.

3.3 Wrapped Cauchy distribution

This distribution can be expressed as

\[
f(\theta; \rho, \mu) = \frac{1}{2\pi 1 + \rho^2 - 2\rho \cos(\theta - \mu)}
\]

with \(0 \leq \theta < 2\pi\), where \(\rho\) is the scale parameter and \(\mu\) is the location parameter. It is unimodal and symmetric about the value \(\theta = \mu\). The maximum likelihood estimates of the parameters can be obtained by a recursive algorithm given by [7]. Using this algorithm, the estimates of \(\mu\) and \(\rho\) were found to be 5.6463 radians and 0.7974 respectively.

4 Goodness of Fit Test

The assessment was accomplished by using the mean circular error (MCE) which is given by

\[
MCE = \frac{1}{n} \sum_{i=1}^{n} \left[1 - \cos(O_i - E_i)\right]^2
\]

For circular data, MCE is an analogy to the mean standard error, MSE. From (4), \(n\) is the number of observations, which are 136. \(O_i\) and \(E_i\) are observed and expected frequency respectively and these value can be obtained from the cumulative distribution function.

As the Von Mises and Wrapped Cauchy distribution include other functions like the Bessel function, it is hard to get the cumulative distribution function. Thus numerical integration was applied to get the value of \(O_i\) and \(E_i\). From the numerical integration, area under the curve of both distribution was divided into 7 subinterval. Each interval has same width and integration was done to each subinterval. So that the value of \(O_i\) and \(E_i\) for each subinterval can be obtained.
Hence the value of MCE for the Von Mises distribution is 1.0776, 2.1984 for the wrapped Cauchy distribution and 1.3781 for the generalized Von Mises distribution, respectively. Since the lowest value of MCE is 1.0776, so the best fitted distribution is the Von Mises.

5 Conclusion
Result from using the statistical test shows that the MCE for the Von Mises distribution is smaller than MCE for the Wrapped Cauchy and generalized Von Mises distributions. Thus it is clear that the Von Mises distribution provides a better fit to the data compared to two other probability distributions.

This research conclude that the wind direction data in this station can be modelled by using the Von Mises distribution with the parameter estimates \( \hat{\kappa} = 6.1751 \) and \( \hat{\mu} = 5.5741 \). The probability density function for this data is given by

\[
f(\theta, \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{6.1715\cos(\theta-5.5741)}
\]

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