Experimental Validation of Theoretical Model for Centric Dumped Collision between Two Balls

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Abstract: In everyday life one can meet collision phenomena characterized by sudden variation of kinematical and dynamical parameters in diverse aspects. An important feature of these consists in high values of impact forces occurring during the contact time. In order to estimate the values of these forces it is required to know the time variation of characteristic parameters. Recent researches present models, from simple to complex, of impact phenomenon. An important issue is experimental validation of theoretical models. The present work treats some qualitative aspects concerning experimental validation - using a shock sensor, of a centric dumped collision model for two balls.

Keywords: dumped collision, shock sensors, experimental tests

1 Introduction
The interaction between two bodies may happen by direct contact or via a field. In the case of direct mechanical contact, it can be obtained by approaching the bodies until they come into contact. In the instant the contact is made, regardless the value of relative velocity of theoretical contacting points, a sudden velocity variation occurs, having as direct consequence the appearance of important impact forces. The static contact problem, from elastostatics, mentions that the bodies come into contact with infinitely low velocity, [1]. The fundamental contact problems were solved by Hertz, [2], more than a century ago, but the impact phenomena studied as elastodynamics problems were later approached. Goldsmith, [3], gave one of the most important monographs treating the collision mechanics. The theoretical study and models of collision phenomena were substantially made undemanding for researchers due to increasing computers performances, but the experimental research remains prohibitive due to equipments. Another barrier between the theoretical and experimental work consists in the simplifying assumptions on which the theoretical models are founded. From the simplifying hypothesis frequently accepted in collision mechanics, two main can be mentioned: the smoothness of boundary surfaces of contacting bodies and the constant value of coefficient of restitution (COR).

2 Theoretical aspects
For the simplest collision case, namely two perfect elastic balls, the solution was provided by Timoshenko, [4], who gave the expressions for maximum approach between bodies, contact time and maximum collision force. For centric dumped collisions, following the paper of Hunt and Crossley, [5], Lankarani and Nikravesh, [6], model the centric dumped collision of two balls. The weak point of the model consists in different value for COR, providing a higher value for output COR, compared to the initial input. The model of oblique impact with dry friction is a more complicated case. Kane, [7], underlines that, using for this situation a kinematical COR according to Newton, defined as the ratio with changed sign between normal components of relative velocities of contacting points after and before collision, respectively, can lead to a paradoxical situation, when the final kinetics energy is greater that the initial one. With the aim of surmounting this not viable situation, Wang and Mason, [8], show the necessity of defining COR on dynamic basis, as Poisson proposed, as the ratio between the normal impacts from restitution and compression phases, respectively. Resuming to centric collisions, recently, Flores, [9], improves the equation from Lankarani, [6], and proposes a form describing the whole impact range, from elastic to plastic ones. This equation of Flores is:
\[ F = K\delta^a \left[ 1 + \frac{8(1-e)}{5e} \frac{\dot{\delta}}{\delta(\cdot)} \right] \]  

(1)

where \( F \) is the collision force, \( K \) a constant concerning the geometry and elastic properties of contacting regions, \( \delta \), the distance between the initial contacting points, \( \dot{\delta} \) the velocity between the initial contacting points, \( \delta(\cdot) \) the initial velocity between contacting points, and \( e \), the coefficient of restitution (COR). In a recent paper, a semi-analytical method for integrating the equation (1) was proposed, [10]. Figure 1 presents, [10], the collision force during the impact between a steel ball (with diameter \( d = 1.75\text{inch} \), and an immobile flat steel body for \( \delta(\cdot) = 1\text{ m/s} \), and \( e = 0.8 \).

![Fig.1. Collision force versus time](image1)

On this plot, there are emphasized the two phases of collision period: compression, (red line) and restitution (blue line). It is noticeable the asymmetry of the graph and the fact that the instant of maximum approach is delayed with respect to the moment of maximum collision force. Therefore, the presence of hysteresis loop is explained, in the diagram of force-deformation coordinates, Fig. 2. For comparison, on the same plot, the curve of perfectly elastic collision, \( e = 1 \), was represented, (black line). It is observed that the maximum force for dumped collision is smaller than the force for elastic case, because the initial kinetics energy is divided into two components and one is transformed into elastic potential energy and the second, is the lost energy as dumping work and as value, proportional to the enclosed area of hysteresis loop.

![Fig.2. Hysteresis loop for a dumped collision and perfect elastic collision line](image2)

3 Experimental validation of theoretical model

The authors proposed the experimental confirmation for solution of equation (1). In a first stage, a pendulum made from an inextensible wire 1, length \( \ell = 0.7\text{m} \), with a bearing ball 2, \( d = 1.75\text{inch} \) in diameter, was used as seen in Fig.3. An acceleration sensor 3 was used for collision force measurements, attached to the ball. The signal is transmitted from the sensor via a conductor wire, 4. During tests, it was observed that the conductor wire produced a spin motion of the ball and thus, a tangential impact occurred. The signal obtained on the oscilloscope is seen in Fig.4. The shape of the curve from Fig.4 is not similar to the theoretical one, and after a series of tests, this variant was abandoned. A new test rig option was designed in order to eliminate the spin motion. To this purpose, another mathematical pendulum consisting in a prismatic body with two identical bearing balls symmetrically attached was used. The swinging is obtained using four wires of equal length, this ensuring a circular translation of the pendulum, Fig.5.

![Fig.3. The first experimental validation device](image3)
4 Results and discussions

For relative large angular launching amplitudes of the pendulum, the signal at the oscilloscope, Fig.7, has the shape as the theoretical model from Fig.1, but only for the early instants of collision. The shape of the signal from Fig.7 presents a cusp and afterward it differs from theoretical model as it decreases in a wavy manner. The results presented in Fig.7 were obtained by collisions between the pendulums and hardened steel flat body and aluminum flat body, respectively. From Fig.7 it can be seen that the at angular point the signal has the same magnitude, showing that in that point the sensor reaches the protection limiting value. In this point the sensor is obliged to discharge and it does not process the input signal from the impact phenomenon.

The above remarks led to the conclusion that the launching amplitude must be significantly decreased. For low angular amplitudes, $\psi_{0} < 5^\circ$, the signal obtained is presented in Fig.8 and it can be observed the qualitative similarity of the shape with the curve from Fig.1. For a third set of experiments, the collision steel-rubber was considered. In this case, the values of impact forces are considerably smaller as seen on the experimental curve from Fig.9. Another aspect of this set of tests is the occurrence, on the last region of the impact curve, of a positive force...
proving the adhesion between rubber and the steel ball, [11]. Revisiting the curve from Fig. 8, one can observe the ratio between the duration of the increasing and decreasing periods of collision force, corresponding to a lower COR than the theoretical one from Fig.1.

Fig.8. Experimental signal for the steel-steel impact

As regards the quantitative concordance between the theoretical model and the experimental curves, supplementary tests are needed. To this purpose, a new shock sensor must be used having a much higher locking protection limit, (over 50,000g [12].) compared to the one used in the present paper. The lower blocking limit of the current sensor does not allow working with higher angular amplitudes required for a contact between real surfaces for which the internal envelopes coincide with theoretical smooth surfaces. The beyond consideration is proved by the impact images from Fig.10, where in Fig. 10.a is the contact mark from the steel flat block observed with optical microscope and in Fig. 10.b, the actual print obtained using the Nanofocus laser scanner, the latter showing that the contact actually takes place between the tips of the asperities of the contacting surfaces.

Fig.9. Experimental signal for the steel-rubber impact

Fig.10. Contact prints from microscope (a), and Nanofocus scanner (b)

A second difficulty to overcome concerns the value to adopt for COR, \( e \). In a previous work, [13], it was shown that the coefficient of restitution varies significantly with initial impact velocity value.

5 Conclusions
The paper presents the qualitative experimental validation of theoretical impact model for dumped collision between two balls, namely, the corroboration of differential equation integration solution with experimental results. In a first attempt, a spherical pendulum having attached a shock sensor is used but the results were altered by an unwanted spin motion of the ball. A second device, based on a circular translation motion and with an acceleration sensor attached, was used. A good agreement between the theoretical model and the experimental signals was obtained for lower angular amplitudes. The low value of the locking limit of the sensor didn’t allow for quantitative correlation of results. A shock sensor with high locking limit is required
for quantitative comparison, as well as using colliding bodies with super-finished surfaces, since the surfaces must be considered smooth and ensuring constant values for the coefficient of restitution.

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