On One Method of Accidental Leak Localization in the Branched Main Gas pipeline

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Abstract: In this paper a mathematical model (an algorithm) defining a placement of a section having gas accidental escape in complex main gas pipeline with several sections and branches is suggested. The algorithm does not required knowledge of corresponding initial hydraulic parameters at entrance and ending points of each sections of pipeline. The algorithm is based on mathematical model describing gas stationary movement in the simple gas pipeline and upon some results followed from that analytical solution and computing calculations.

Key-Words: - leak detection, branched gas pipeline, mathematical method

1 Introduction
At present pipelines represent the main useful means for liquid and gas substances transportation. Indeed oil, water and gas transportation by pipelines is the safest and cheapest method compared with other (rail, trailers and marine) transportation systems. That is why a large number of pipeline networks were constructed worldwide during the last 50 years for oil and especially natural gas transportation [25]. Nowadays pipelines become the main helpful and practical means for natural gas transportation worldwide [11, 16, 25], but it should be noted that at the same time the gas delivery infrastructure is rapidly aging [28]. The main fault of the outdated pipelines is leak and as a consequence deterioration of environment [6,24,25]. For instance methane emissions from leaking pipelines is a serious problem related to the environment as methane is one of the most principal greenhouse gases contributing to climate change [18,19]. In pipeline networks that transport gas or oil leaks may occur at any time and location [25], therefore, timely detection of leaks can stop or minimize contamination of environment and leak detection and location is important for the safe operation of pipelines [25]. Pipelines prognostic leak detection systems play a key role in minimization of the occurrence of leaks probability and their impacts on environment [16]. So elaboration the leak detection and location system in a large scale water, oil and gas pipeline system networks is an urgent and sensitive issue of nowadays [6,7,10,16]. That is why a great attention is paid to finding out the new methods and techniques for the leak fast detection and location in the pressurized pipeline and fortunately this process for the last three decades is in progress [1-33]. There are many different approaches and techniques for leak detection and location. They are mostly coming from the problem of oil and gas transportation by pipelines as the consequences from oil and gas leaks are the most severe (related not only to the environment pollution but also to the economical point of view) [25,28]. A spectrum of the methods for detection and location of the leaks in the natural gas pipelines is wide from using trained dogs to advanced satellite based hyper spectral imaging [9, 26]. But there are fundamental theoretical and technical methods for leak detection and location in natural gas pipelines and the various methods can be classified into non-optical and optical methods [4]. Optical monitoring methods (active and passive) have high special resolution, sensitivity and they accurate monitor as pipelines as well moving.
vehicles but at the same time it should be noted that they are expensive and characterized by false alarm signals [28]. The primary non-optical methods are classified as acoustic monitoring [6,17,20], gas sampling [27], soil monitoring [30], flow monitoring [5,12], and software based on dynamic modeling [4,15,16,21,22,24,26]. Each method has its advantages and disadvantages. Acoustic monitoring method is simple and it is able to accurately detect the waves generated from the gas escape crack but this method needs arrangement a large number of detectors along the pipeline. That is way for the long pipelines this method is too expensive [20,32], also this technology is useless for the tiny leaks [28]. In spite of these disadvantages for testing unsteady-state and integrity of the single pipe the method has been successfully used [7]. The sampling method uses gas sampling and the device carrying along a pipeline [28]. This method also can detect very small leaks but the response time of the detection system is usually from several hours to days and this method is also rather expensive [28]. Flow monitoring method monitors pressure change or mass flow and gives alarm when differences between measured and established rates are occurs [28]. Based on flow method some approaches and results are presented in [1,2,12]. For example a new one-dimensional flow analysis method for leak detection in pipelines is presented in [2]. The accuracy in calculations performed for detection leak position has shown physical realism of the mathematical model used in [2]. A flow model based on the Lyapunov stability criteria for evolving the criterion is presented in [1]. The magnetic flux leakage method for assessing the conditions of the pipeline uses a magnetizer-sensor device “pig” which is launched in pipeline [25,28]. Unfortunately this method is functioning well only for seamless pipelines, which is rarity in practice (transition pipelines) [22,28]. Dynamic method based on modeling of gas flow and parameters of flow within a pipeline. The method requires flow, pressure, temperature measurements at the inlet and outlet of a pipeline, and for some models the measurements at several points along the pipeline also are needed [15]. For leak detection differences between calculated and measured values is used [25,26]. There are some available assortments of modern technologies in the domain of leak detection [11,16]. To anticipate the leak magnitude the adaptive multi-model state estimation technique is based on a modified extended Kalman filter in conjunction with feed forward computations [14]. Results of numerical calculations showed that the developed state estimation scheme had effectively detected tiny leaks within short time duration [14]. In [17] leak detection and isolation techniques are based on non-linear mathematical model of the pipeline network, where data of the pressure measurements were obtained by sensitivity analysis [32]. There are some approaches of the leak detection and location based on the dynamic and the filter methods [8,22,33]. The pipeline models with a leak and using the strong tracking filters are presented in [33]. An adaptive particle filter method for fast and accurate leak detection is presented in [22]. This article provides the proper methodology for leak detection in branched main gas pipelines. The methodology does not required knowledge of corresponding initial hydraulic parameters at entrance and ending points of each sections of pipeline (receiving of this information is rather difficult without using telemetric informational system) [32]. The methodology is based on an algorithm describing gas stationary movement in the simple and branched gas pipelines and upon some results followed from the analytical discussions and computing calculations.

2 Problem Formulation

Analyses of a reliability of the main gas pipeline’s exploitation has shown high probability of the main gas pipeline’s some sections damage and gas leakage and as a result the gas pressure and expenditure alteration when non-stationary processes are in progress. After some time gas leakage (under some conditions), it is possible establishment a new stationary state of gas movement in the pipelines has stationary character. That is way it is necessary to study as a non-stationary stage as well the stationary stage of gas movement in the pipelines having gas escape in the some sections of the main gas pipeline. In this article we study only large-scale gas leakage problem from the main gas pipeline and we consider this question as a reverse task of hydraulic calculation problem. It is known analytical method of determination a large-scale gas escape location on the simple section of main gas pipeline [8,31], using data of the gas pressures and expenditure at the entrance and ending of the gas pipeline. But this method cannot be used for main gas pipelines with several sections and branches if previously would not be discovered the location of the section with gas escape. The method offered by us is devoid from this default. So the problem can be formulated as follows: In the complex main gas pipeline with several branches and sections first of all the placement of the section having accidental gas
escape is determined using minimal information (data of the gas pressures and expenditure at the main gas pipeline’s entrance and ending points before and after gas escape) and then defined location of the accidental gas escape in the determined section of main pipeline.

2.1 Mathematical statement of the problem

Thus suppose that there is a complex main gas pipeline having \( n-1 \) off-shots, with expenses \( q_k \) \((k = 1, n-1)\) and the pipeline is divided by off-shots on \( n \) simple sections with length \( L_k \) \((k = 1, n)\). If at the entrance of pipeline gas expanses in unit of time is \( M_0 \), then at the entrance of the per simple sections the gas expanses are calculated in the following way

\[
M_1 = M_0
\]

\[
M_k = M_{k-1} - q_{k-1}, \quad k = (2, n)
\]

where numbering is performed from the beginning of the pipeline to the ending.

As it is known in case of gas stationary movement in the horizontal gas pipeline exist the following equality [8,31]:

\[
P_1^2 - P_2^2 = \sum_{k=1}^{n} M_k^2 \beta_k L_k, \quad (1)
\]

where \( \beta_k = \frac{\lambda_k ZRT}{F_k^2 D_k} \), \( P_1 \) and \( P_2 \) are values of the pressures at the entrance and at the ending of the main gas pipe-line, respectively; \( M_k \)- are expenses of gas in the unit area of pipe-line for unit time in the branches; \( L_k \)- are lengths of simple section \( k \) of the main pipe-line; \( Z \) is a coefficient expressing deviation of natural gas from ideal gas; \( \lambda_k \) is a hydraulic resistance of a gas; \( T \) is an absolute temperature; \( R \) is a gas constant; \( D_k \) are diameters of pipelines; \( F_k \) are areas of branches profile sections. Suppose that at the entrance of the main gas pipeline in the unit of time through pipe passes \( M_0 \) mass of gas, and at the ending of pipeline instead of gas mass \( Mn \) expenditure of gas is \( Mn - Q \), which indicates that gas with mass \( Q \) is loosen, although the consumers (users) are getting the same mass of gas \( q_k \) \((k = 1, n-1)\) which is conditioned by gas distributive stations (service management). Let us suppose that gas leakage is placed on the section \( i \) and gas escape is located on the distance \( x \) \( (0 \leq x \leq L_i) \) from the entrance of the section \( i \). Also we suppose that accidental gas escape represents additional ramification of the main gas pipeline with expenditure \( Q \). It is evidence that expenditure of gas is remained the same in the ramifications located before the section \( i \) but after the section \( i \) instead of expenditure \( M_k \) it will be \( M_k - Q > 0 \) \((k = 1, n)\). In analogously of the right side of the equation (1) let us initiate the following functions \( f_i(x) \):

\[
f_1(x) = \sum_{k=1}^{n} [M_k - Q]^2 \beta_k L_k + Q[2M_1 - Q] \beta_1 x, \quad \text{when} \quad (0 < x \leq L_i)
\]

\[
f_i(x) = \sum_{k=1}^{n} M_k^2 \beta_k L_k + \sum_{k=1}^{n} [M_k - Q]^2 \beta_k L_k
\]

\[
\quad + Q[2M_i - Q] \beta_i x, \quad i \in (2,3,\cdots,n-1). \quad \text{when} \quad (0 < x \leq L_i);
\]

\[
f_n(x) = \sum_{k=1}^{n} M_k^2 \beta_k L_k + [M_n - Q]^2 \beta_n L_n
\]

\[
\quad + Q[2M_n - Q] \beta_n x, \quad \text{when} \quad (0 < x \leq L_n).
\]

Let us assume that after gas escape \( \overline{P}_1 \) and \( \overline{P}_2 \) are values of the gas pressures, at the entrance and ending of main pipeline, respectively (which are obtained by the measuring instruments).

Therefore, analogously of the equation (1) we have:

\[
\overline{P}_1^2 - \overline{P}_2^2 = f_i(x). \quad (2)
\]

So for detection of the section of accidental gas escape and the point of gas escape in this section we have the following mathematical model (algorithm): first of all it is required to search such kind value \( i_0 \) from the sequence \( i = \{1, 2, \cdots, n\} \) and then the value of the \( x \) from the interval \( [0, I_{i_0}] \) which will satisfy the equation (2).
3 Problem Solution
For convenience here and further we are defining some properties of the above mentioned function \( f_i(x) \):

1. Every function \( f_i(x) \) \((i = 1, n)\) represents linear increasing functions of \( x \) forasmuch as
\[
Q[2M_i - Q] \beta_i > 0, \quad (i = 1, n).
\]

2. It is easy to prove that for the function \( f_i(x) \) the following equalities are correctness:
\[
f_{i-1}(L_{i-1}) = f_i(0), \quad (i = 2, n)
\]

3.1 An algorithm defining a placement of the damage section
Now arrange (on the axis) the segments with length \( L_i, i = (1, n) \) step by step exactly in such a way that right tail-end point of the segment \( i-1 \) and left tail-end point of the segment \( i \) will be coincide with each other. Let us define function \( f_i(x) \) on the each segment \( i \) in such a way, that beginning of the calculation for the argument \( x \) will be the left point of the segment \( i \). In such a way arranged functions \( f_i(x) \) represent continuous, sectional increasing linear functions. From physical point of view above mentioned properties of the functions \( f_i(x) \) means that the more is distance of the location of the accidental gas escape from the begging of the main gas pipeline, the bigger difference between the values of pressures’ squares. Moreover this difference continuously defends on the distance in which the accidental gas escape is located from the begging point of the main gas pipeline. Using properties of the functions \( f_i(x) \) it is possible to construct algorithm which gives possibility to find such kind values of \( i_0 \) and \( x \) which will satisfy equation (2). For achievement of this aim first of all it is necessary to check up endings of branches. If for any value of \( i_0 \) from \( i \) the equality
\[
f_{i_0}(L_{i_0}) = \overline{P}_1^2 - \overline{P}_2^2
\]
is true, then gas accidental escape is located at the simple endings sections of the main pipeline. If the equality is not fulfilled for any value of \( i \), then extracting the values of \( i \) from the sequence \( i=1,2,..n \), it will be possible to find the least value of \( i_0 \) which will satisfy the following inequality
\[
f_{i_0}(L_{i_0}) > \overline{P}_1^2 - \overline{P}_2^2
\]

In that case gas accidental escape is located within the section \( i_0 \). If such kind inequality is not fulfilled for any values of \( i_0 \) from the sequence \( i=1,2,..n-1 \), then gas accidental escape is located on the last simple ending section numbered by \( n \). In that case the following inequality will be true
\[
f_{i_0}(L_{n}) \leq \overline{P}_1^2 - \overline{P}_2^2
\]
and \( i_0 = n \).

3.2 Definition of the placement of the gas escape in the damaged section
Afterwards it is emplaced the location (number of the section \( i_0 \)) of the gas accidental escape the appropriate distance \( x \) can be defined by the solution of the following equation
\[
f_{i_0}(x) = \overline{P}_1^2 - \overline{P}_2^2
\]
Namely if we have \( i_0 = 1 \) then
\[
x = \overline{P}_1^2 - \overline{P}_2^2 - \sum_{k=i-1}^{n} (M_k - Q)^2 \beta_k L_k
\]
/ \( Q(2M_i - Q) \beta_i \)

If fulfilled the following inequality
\[
2 \leq i_0 \leq n - 1
\] then
\[
x = \overline{P}_1^2 - \overline{P}_2^2 - \sum_{k=i}^{n-1} (M_k - Q)^2 \beta_k L_k - \sum_{k-i}^{n} (M_k - Q)^2 \beta_k L_k
\]
/ \( Q(2M_{i_0} - Q) \beta_{i_0} \)

And at last if \( i_0 = n \), then
\[
x = \overline{P}_1^2 - \overline{P}_2^2 - \sum_{k=1}^{n-1} M_k^2 \beta_k L_k - (M_n - Q)^2 \beta_k L_k
\]
/ \( Q(2M_n - Q) \beta_n \)
4 Conclusion
In this paper a new method (an algorithm) for defining a placement of a section and then defined location of the accidental gas escape in the determined section of main pipeline is suggested. The algorithm does not required knowledge of corresponding initial hydraulic parameters at entrance and ending points of each sections of pipeline (receiving of this information is rather difficult without using telemetric informational system). The methodology is based on an algorithm describing gas stationary movement in the simple and branched gas pipelines and upon some results followed from the analytical discussions and computing calculations. The structure of defining a placement of a section and a point in the section is simple to implement.

References:


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